IN MEMORIAM SUMNER B. MYERS
1910-1955

Colleagues, students and friends have joined to dedicate this volume of the Michigan Mathematical Journal to the memory of Sumner B. Myers.

Sumner B. Myers was born to Dr. Solomon and Mrs. Nettie Myers on February 19, 1910. He graduated from Harvard University, *summa cum laude*, in 1929, and wrote his dissertation (1932) at Harvard, under the direction of Marston Morse. After a year in Europe, on a Harvard Traveling Fellowship, a year as Instructor at Harvard, and two years at the Institute for Advanced Study, on a National Research Council Fellowship, he came to the University of Michigan in 1936. He married Alison Tennant in 1942, and thereafter the Myers home played an important role in the life of the Mathematics Department. Young faculty members and students found there wise counsel in mathematical and personal problems; Department members hard pressed by illness or by the housing shortage after the war found shelter and help.

Sumner Myers took a deep interest in problems of the Department (to which he devoted a large portion of his time and energy in various offices, including that of Acting Chairman), the University and the world-at-large. He had strong principles, and whenever he saw injustice done, his sense of moral responsibility forced him to speak out and to take whatever action was possible. At the same time he was full of warmth and humor, and he was a master at finding the right word to relieve a tense situation.

But his main concern was mathematics, both research and teaching. A brief survey of his investigations will be given below. As a teacher, he was magnificent. In his lectures, which were very well planned, in spite of their informal appearance, he knew how to stimulate the students, undergraduates as well as graduates. Beyond the classroom, he was generous with his time and with his fund of mathematical knowledge; the many theses written under his direction attest to his ability in leading students to active research.

During a period of great fruitfulness, he died of a heart attack on October 8, 1955. His loss is deeply felt by his family and closer friends, by the other members of his university, and by all mathematicians who had known him.

In his first papers, particularly in his thesis, Sumner Myers dealt with the calculus of variations in its classical form. His early mastery of the calculus of variations, and especially of the second variation, served him well in the subject which attracted his attention next: differential geometry in the large. In [6] and [7], he studied the minimal locus of a point on a two-manifold, and he derived topological conclusions from its structure. This notion goes back to Poincaré, and was independently and simultaneously developed by J. H. C. Whitehead and Myers in 1935. It is defined as follows: Let A be a point on the complete Riemannian manifold M_n. A point P on a geodesic ray g issuing from A is called a minimum point with respect to A, provided P is the last point on g such that AP furnishes an absolute minimum to the arc length of curves joining A to M. The set of all such points in M_n is precisely the minimal locus of A (we shall denote it by m_A). Myers discusses the analytic, two-dimensional case completely: the set m_A is always a graph, and its complement a two-cell; the homology of M_2 is determined by m_A; the order of a vertex P of m_A is precisely the number of geodesics joining A to P in

M_2; the end points of m_A are conjugate points of A, and they are cusps (turned towards A) of the locus of first conjugate points to A. There are corresponding relations, not quite as satisfactory, in the nonanalytic case. Some of these theorems have been obtained by J. H. C. Whitehead for n dimensions; but their complete analogues have not yet been proved, and they seem very difficult.

As another example of Myers' work in this direction, we cite the following theorem which he published in 1941: A complete Riemannian manifold, of dimension n and with positive mean curvature at least e^2, is compact, and its diameter does not exceed $\pi\sqrt{n-1}/e$. In his surprisingly simple proof, he again makes expert use of the second variation.

We can not discuss all the other contributions of Myers to the field of Riemannian geometry, but we could hardly fail to mention the joint paper of N. Steenrod and Myers [10], which contains one of the basic theorems on the subject. This theorem states that the group of isometries of a compact Riemannian manifold is a Lie group. In contrast to the classical theorems about germs of groups acting isometrically on germs of Riemannian manifolds, this theorem is truly a proposition of geometry in the large.

Myers' later work dealt with topological questions in function spaces. His first paper in this direction [13] was probably motivated by his interest in existence theorems for geodesics. In it he investigated the relation between various compactness and continuity properties of a set of mappings of a topological space into a metric space.

After this paper, Myers' attention turned to the relation of spaces of continuous functions to the spaces on which the functions are defined. The type of problem he treated in a number of papers may be described as follows: Let X be a topological space, and B(X) the Banach space of bounded continuous functions b on X with norm $||b|| = \sup_{x\in X} |b(x)|$. It had been known (Banach, Stone, Eilenberg) that if X is compact, the topology of X is determined by B(X). Myers showed that already certain closed linear subspaces of B(X), which he called completely regular, suffice to determine the topology of X.

Alaoglu had already answered in the affirmative the "inverse question:" If B is a Banach space, does there exist a compact space X such that B is equivalent to a closed linear subspace G of B(X)? Myers treated the question whether there exist such subspaces G of certain simple kinds; for example, he gave necessary and sufficient conditions for the existence of a compact space X such that G is "completely regular over X."

Particularly important, of course, is the following question: when is G identical with B(X)? This is the question of the characterization of those Banach spaces which are equivalent to a space of continuous functions on a compact space. Such characterizations had been given earlier in terms of ring theory (Gelfand) and of linear lattice theory (Kakutani and Krein); but the characterization given by Myers is totally in terms of Banach-space notions. (As to the relation of Myers' characterization to those of R. Arens and J. L. Kelley and of J. A. Clarkson, see [17, p. 237].)

While it is impossible to give here in detail Myers' answers to the questions raised above, it may be worthwhile to mention two concepts introduced by him which proved particularly useful in this kind of investigation. These are the concept of a T-set in a Banach space B and a functional $F_T(b)$ defined for all b in B. The formal definitions are as follows: a T-set is a point set of B which is maximal with respect to the following property: for any finite set $\{b_1, \cdots, b_n\}$ of points of T, the equality

$$\sum_{1}^{n} ||b_i|| = ||\sum_{1}^{n} b_i||$$

holds. (For example, if X is compact, then the set of all functions in B(X) taking their maximum at a fixed point x_0 of X is a T-set.) $F_T(b)$ is defined as

$$\inf_{t \in T} [||b + t|| - ||t||].$$

Nearly all of Myers' answers to the problems above are in terms of these two notions. As an example, we mention the following result: A Banach space B is equivalent to a B(X) with compact X if and only if the following three conditions are satisfied:

(i) all functionals $F_T(b)$ are linear;

(ii) there exists an element e in B such that, for every $b \in B$, $||b|| + 1 = ||b + e||$ or $||b|| + 1 = ||b - e||$;

(iii) for each b, there exists a b' such that $F_T(b') = |F_T(b)|$ for all F_T for which $F_T(e) = 1$.

This survey, by no means complete, of Myers' work shows a diversity of interest. Yet, except possibly in his earliest work on the calculus of variations, his primary concern was always the relationship between geometric structure and the topology of the underlying space. In this sense, he was truly representative of the spirit of modern geometry.

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