ON THE DIRICHLET PROBLEM

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The purpose of this note is to present a very short proof of the following result due to W. Kaplan ([1], Theorem 7).

THEOREM. *Suppose that* $f(\theta)$ *is real, measurable, almost everywhere finite, and that* $f(\theta)$ *has period* 2π. *There exists a function* $u(z)$, *harmonic in* $|z| < 1$, *such that for almost all* θ

$$u(z) \to f(\theta) \tag{1}$$

as $z \to e^{i\theta}$ *along any nontangential path.*

This theorem is actually an extension of Kaplan's result. Kaplan concluded only that (1) holds when z approaches $e^{i\theta}$ along a radius; here we obtain (1) for all nontangential paths.

Proof. By a theorem of Lusin ([2], page 217) we can find a continuous function $F(\theta)$ such that

$$F'(\theta) = f(\theta) \tag{2}$$

for almost all θ. Let

$$U(re^{i\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(t)\frac{1 - r^2}{1 - 2r\cos(\theta - t) + r^2}\,dt$$

for $r < 1$. Next, by a well known result due to Fatou ([3], page 53),

$$\frac{\partial}{\partial\theta}U(z) \to F'(\theta) \tag{3}$$

as $z \to e^{i\theta}$ along any nontangential path, whenever $F'(\theta)$ exists. Now let

$$u(z) = \frac{\partial}{\partial\theta}U(z),$$

and our theorem follows from (2) and (3).

REFERENCES

1. W. Kaplan, *Approximation by entire functions*, Michigan Math. J. 3 (1955-56), 43-54.
2. S. Saks, *Theory of the integral*, Warsaw, 1937.
3. A. Zygmund, *Trigonometrical series*, Wilno, 1935.

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