Introduction to the Special Issue on Singular Cardinals Combinatorics

Matthew Foreman

During May 1–6, 2004 a workshop on Singular Cardinals Combinatorics sponsored by the Pacific Institute of Mathematical Sciences (PIMS) was held at the Banff International Research Station (BIRS). The workshop was organized by Claude Laflamme, Stevo Todorčević, and the author. The topic was an outgrowth of major recent advances in forcing and combinatorics and the discovery of a new "natural structure" in ZFC.

Tools that were originally developed to study the Singular Cardinals Hypothesis, such as Forcing and PCF theory, have revealed extensive, surprising new relationships. Explorations of "lower bounds" via inner model theory have suggested new squarelike principles that have, in turn, fed back into topics related to PCF theory and forcing. Perhaps most strikingly, new objects have been discovered whose existence depends on the Axiom of Choice but are nonetheless definable. These include examples of "scales", canonical stationary sets and ideals, and superatomic Boolean Algebras. These discoveries have led to a host of new problems, related to but distinct from the classical questions surrounding the PCF theory.

The conference gathered experts in the various subdisciplines to discuss the interconnecting results with the goal of organizing the area and setting programs for continued research. The participants at the workshop were Uri Abraham, David Asperó, Joan Bagaria, James Cummings, Mirna Džamonja, Todd Eisworth, Matthew Foreman, Moti Gitik, Andreas Hajnal, Tetsuya Ishiu, Menachem Kojman, Claude Laflamme, Jean Larson, Andreas Liu, Menachem Magidor, Juan Carlos Martínez, William Mitchell, Ernest Schimmerling, Masahiro Shioya, John Steel, Katherine Thompson, Stevo Todorčević, Philip Welch, and Hugh Woodin.

At the conference, it was felt that collecting papers summarizing the results of the talks and collections of questions would further the goals of the conference. This volume is the result. The participants would particularly like to thank BIRS and PIMS for sponsoring the conference and the *Notre Dame Journal of Formal Logic* for publishing the proceedings.