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ON THE DEFINITION OF MEREOLOGICAL CLASS

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Consider mereology axiomatized as in [1]*. Sobociński has posed the question, "If the usual definition of class, DMI, is replaced by

$[Aa] \therefore A \varepsilon \mathsf{Kl}(a) . \equiv : A \varepsilon A : [B] : a \subset \mathsf{el}(B) . \equiv . A \varepsilon \mathsf{el}(B)$,

is the resulting system equivalent to the original?". This note gives a negative answer. Theses *A12* and *A13*, together with the two trivial models which follow them, show where the resulting system is weaker than mereology.

Consider the axiom system A consisting of *A1-A6*; *DA1*.

A1 $[A] : A \varepsilon A . \supset . A \varepsilon \mathsf{el}(A)$

A2 $[AB] : A \varepsilon \mathsf{el}(B) . B \varepsilon \mathsf{el}(A) . \supset . A = B$

A3 $[ABC] : A \varepsilon \mathsf{el}(B) . B \varepsilon \mathsf{el}(C) . \supset . A \varepsilon \mathsf{el}(C)$

A4 $[AB] : A \varepsilon \mathsf{el}(B) . \supset . B \varepsilon B$

DA1 $[Aa] \therefore A \varepsilon \mathsf{Cl}(a) . \equiv : A \varepsilon A : [B] : a \subset \mathsf{el}(B) . \equiv . A \varepsilon \mathsf{el}(B)$

A5 $[Aa] : A \varepsilon a . \supset . [\exists B] . B \varepsilon \mathsf{Cl}(a)$

A6 $[ABa] : A \varepsilon \mathsf{Cl}(a) . B \varepsilon \mathsf{Cl}(a) . \supset . A = B$**

DA2 $[Aa] \therefore A \varepsilon \mathsf{Kl}(a) . \equiv : A \varepsilon A : [D] : D \varepsilon a . \supset . D \varepsilon \mathsf{el}(A) : [D] : D \varepsilon \mathsf{el}(A) . \supset . [\exists EF] .$
$E \varepsilon a . F \varepsilon \mathsf{el}(D) . F \varepsilon \mathsf{el}(E)$

A7 $[Aa] : A \varepsilon \mathsf{Kl}(a) . \supset . [\exists B] . B \varepsilon a$ [*DA2, A1*]

A8 $[Aa] : A \varepsilon \mathsf{Kl}(a) . \supset . [\exists B] . B \varepsilon \mathsf{Cl}(a)$ [*A7, A5*]

A9 $[ABa] : B \varepsilon \mathsf{Kl}(a) . A \varepsilon \mathsf{Cl}(a) . \supset . A \varepsilon \mathsf{el}(B)$

PF $[ABa] : \mathrm{Hp}(2) . \supset .$

3) $a \subset \mathsf{el}(B) .$ [*DA2, 1*]

$A \varepsilon \mathsf{el}(B) .$ [*DA1, 2, 3*]

A10 $[ABDa] : A \varepsilon \mathsf{Cl}(a) . B \varepsilon \mathsf{Kl}(a) . D \varepsilon \mathsf{el}(A) . \supset . [\exists EF] . E \varepsilon a .$
$F \varepsilon \mathsf{el}(D) . F \varepsilon \mathsf{el}(E)$ [*A9, DA2*]

A11 $[Aa] : A \varepsilon \mathsf{Cl}(a) . \supset . a \subset \mathsf{el}(A)$ [*DA1*]

A12 $[Aa] : A \varepsilon \mathsf{Cl}(a) . ! \{\mathsf{Kl}(a)\} . \supset . A \varepsilon \mathsf{Kl}(a)$ [*DA2, A11, A10*]

*Refer to [1] for the definitions of terms used in this note.

**This system is not independent.

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To show that $!\{\mathsf{Kl}(a)\}$ may fail, consider the model for A consisting of four names A, B, C, D with the relations, $A \neq B$, $B \neq C$, $A \neq C$, $\mathsf{dscr}\{A \cup B \cup C\}$, $D \varepsilon \mathsf{Cl}(A \cup B \cup C)$. Then $\mathsf{Kl}(A \cup B)\, o \wedge$

$A13$ $[Aa]: A \varepsilon \mathsf{Kl}(a) . \rightarrow \{\mathsf{Kl}(a)\} . \supset A \varepsilon \mathsf{Cl}(a)$

PF $[Aa]: \text{Hp}(2) . \supset .$

$[\exists B] .$

3) $B \varepsilon \mathsf{Cl}(a)$. [$A8$, 1]

4) $B \varepsilon \mathsf{Kl}(a)$. [$A12$, 3]

5) $A = B$. [2, 1, 4]

$A \varepsilon \mathsf{Cl}(a)$ [3, 5]

To show that $\rightarrow\{\mathsf{Kl}(a)\}$ may fail, consider the model for A consisting of the two names A, B with the relations, $A \neq B$, $A \varepsilon \mathsf{el}(B)$. Then $A \varepsilon \mathsf{Kl}(A)$ and $B \varepsilon \mathsf{Kl}(A)$.

REFERENCE

[1] R. E. Clay: The relation of weakly discrete to set and equinumerosity in mereology, *Notre Dame Journal of Formal Logic*, Vol. VI, 1965, pp. 325-340.

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