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LEŚNIEWSKI'S ANALYSIS OF WHITEHEAD'S THEORY OF EVENTS

VITO F. SINISI

Logic prescribes the shapes of metaphysical thought.[1]
(Whitehead)

Stanisław Leśniewski (1886-1939) was a leading member of the famous Warsaw school of logicians which flourished between the two Wars. The works of Lejewski and Sobociński have made many readers of this journal familiar with Leśniewski's three systems of logic: *protothetic*, *ontology*, and *mereology*. What does not seem to be generally known is that in the course of setting forth *mereology* [1]: Leśniewski proved that A. N. Whitehead's axiomatic basis for the concept of event is an inadequate foundation for Whitehead's theory of events.

The purpose of this note is to recapitulate Leśniewski's analysis (available only in Polish) of Whitehead's theory of events. Perhaps a knowledge of this analysis will be of value not only to those interested in Leśniewski's work but to that growing number of philosophers concerned with Whitehead's metaphysics and philosophy of science.

In *An Enquiry Concerning the Principles of Natural Knowledge* (Cambridge, 1919) Whitehead set forth a theory of events. According to Whitehead:

> Every element of space or of time (as conceived in science) is an abstract entity formed out of this relation of extension (in association at certain stages with the relation of cogredience) by means of a determinate logical procedure (the method of extensive abstraction). The importance of this procedure depends on certain properties of extension which are laws of nature depending on empirical verification. There is, so far as I know, no reason why they should be so, except that they are. These laws will be stated in the succeeding parts so far as is necessary to exemplify the definitions which are there given and to show that these definitions really indicate the familiar spatial and temporal entities which are utilized by science in precise and determinate ways. Many of the laws can be logically proved when the rest are assumed. But the proofs will not be given here,

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as our aim is to investigate the structure of the ideas which we apply to nature and the fundamental laws of nature which determine their importance, and not to investigate the deductive science which issues from them. (*Op. cit.*, pp. 76-77)

Later in the *Enquiry* Whitehead sets down the fundamental properties of the relation of extension, and defines "intersection," "separation," and "dissection."

27. *The Relation of Extension, Fundamental Properties.*

27.1 The fact that event a extends over event b will be expressed by the abbreviation aKb. Thus 'K' is to be read 'extends over' and is the symbol for the fundamental relation of extension.

27.2 Some properties of K essential for the method of extensive abstraction are,

(i) aKb implies that a is distinct from b, namely, 'part' here means 'proper part':

(ii) Every event extends over other events and is itself part of other events: the set of events which an event e extends over is called the set of parts of e:

(iii) If the parts of b are also parts of a and a and b are distinct, then aKb:

(iv) The relation K is transitive, i.e. if aKb and bKc, then aKc:

(v) If aKc, there are events such as b where aKb and bKc:

(vi) If a and b are any two events, there are events such as e where eKa and eKb.

It follows from (i) and (iv) that aKb and bKa are inconsistent. Properties (ii) and (v) and (vi) together postulate something like the existence of an ether; but it is not necessary here to pursue the analogy.

28. *Intersection, Separation and Dissection.*. 28.1 Two events 'intersect' when they have parts in common. Intersection, as thus defined, includes the case when one event extends over the other, since K is transitive. If every intersector of b also intersects a, then either aKb or a and b are identical.

Events which do not intersect are said to be 'separated.' A 'separated set' of events is a set of events of which any two are separated from each other.

28.2 A 'dissection' of an event is a separated set such that the set of intersectors of its members is identical with the set of intersectors of the event. Thus a dissection is a non-overlapping exhaustive analysis of an event into a set of parts, and conversely the dissected event is the one and only event of which that set is a dissection. There will always be an indefinite number of dissections of any given event.

If aKb, there are dissections of a of which b is a member. It follows that if b is part of a, there are always events separated from b which are also parts of a. (*Op. cit.*, pp. 101-102)

In 1926 Tarski called Leśniewski's attention to this theory of events and to its relation with Leśniewski's general theory of sets which was first published in 1916.[2] Tarski expressed the belief that neither "If every intersector of b also intersects a, then either aKb or a and b are identical" nor "If b is part of a, there are always events separated from b which are also parts of a" follows from Whitehead's definitions and statements (i) - (vi), and that consequently (i) - (vi) do not provide an axiomatic basis for Whitehead's theory of events.

Leśniewski followed up Tarski's conjecture, and despite certain difficulties due to Whitehead's use of colloquial language he managed to express

Whitehead's statements (i) - (vi) as well as the last two quoted statements above more perspicuously as follows:[3]

(1) if aKb, then a is different from b [$aKb \supset a \neq b$],
(2) if a is an event, then ((for some b — (b is an event and aKb)) and for some b — (b is an event and bKa))
[$Ea \supset ((\exists b)(Eb \cdot aKb) \cdot (\exists b)(Eb \cdot bKa))$],
(3) if a is an event, b is an event, (for all c—if c is an event and bKc, then aKc) and a is different from b, then aKb
[$(Ea \cdot Eb \cdot (c)((Ec \cdot bKc) \supset aKc) \cdot a \neq b) \supset aKb$],
(4) if aKb and bKc, then aKc [$(aKb \cdot bKc) \supset aKc$],
(5) if aKc, then for some b—(b is an event, aKb and bKc)
[$aKc \supset (\exists b)(Eb \cdot aKb \cdot bKc)$],
(6) if a is an event and b is an event, then for some e—(e is an event, eKa and eKb)
[$(Ea \cdot Eb) \supset (\exists e)(Ee \cdot eKa \cdot eKb)$],
(7) if a is an event, b is an event, and for all c and d—, if c is an event, d d is an event, cKd and bKd, then for some e—(e is an event, cKe and aKe), then (aKb or a is identical with b)
[$(Ea \cdot Eb \cdot (c)(d)((Ec \cdot Ed \cdot cKd \cdot bKd) \supset (\exists e)(Ee \cdot cKe \cdot aKe))) \supset aKb \vee a = b)$],
(8) if a is an event, b is an event and aKb, then for some c—(c is an event, not (for some e—(e is an event, cKe and bKe)) and aKc)
[$(Ea \cdot Eb \cdot aKb) \supset (\exists c)(Ec \cdot \sim (\exists e)(Ee \cdot cKe \cdot bKe) \cdot aKc)$].

Using the "method of interpretation" Leśniewski was able to show (contrary to Whitehead's claim) that neither (7) nor (8) follows from (1) - (6) provided that a theory T (whose axioms are given below) is a non-contradictory theory. (Expressions of the type "$a > b$" are to be read as "a is a rational number, b is a rational number and a is greater than b," while expressions of the type "Ra" are to be read as "a is a rational number"). The axioms of T are:

(α) $(\exists a)Ra$.
(β) $Ra \supset (\exists b)(Rb \cdot a > b)$,
(γ) $Ra \supset (\exists b)(Rb \cdot b > a)$,
(δ) $a > c \supset (\exists b)(Rb \cdot a > b \cdot b > c)$,
(ϵ) $a > b \supset a \neq b$,
(ζ) $(a > b \cdot b > c) \supset a > c$,
(η) $(Ra \cdot Rb \cdot a \neq b) \supset (a > b \vee b > a)$.

Replacing in (1) - (6) expressions of the type "Ea" by expressions of the type "Ra" and expressions of the type "aKb" by expressions of the type "$a > b$", we obtain the following six statements:

(A) $a > b \supset a \neq b$,
(B) $Ra \supset ((\exists b)(Rb \cdot a > b) \cdot (\exists b)(Rb \cdot b > a))$,
(C) $(Ra \cdot Rb \cdot (c)((Rc \cdot b > c) \supset a > c) \cdot a \neq b) \supset a > b$,
(D) $(a > b \cdot b > c) \supset a > c$,
(E) $a > c \supset (\exists b)(Rb \cdot a > b \cdot b > c)$,
(F) $(Ra \cdot Rb) \supset (\exists e)(Re \cdot e > a \cdot e > b)$.

Similar substitutions into (7) and (8) yield, respectively,

(G*) $(Ra \cdot Rb \cdot (c)(d)((Rc \cdot Rd \cdot c > d \cdot b > d) \supset (\exists e)(Re \cdot c > e \cdot a > e))) \supset (a > b \vee a = b)$,

and

(H*) $(Ra \cdot Rb \cdot a > b) \supset (\exists c)(Rc \cdot \sim(\exists e)(Re \cdot c > e \cdot b > e) \cdot a > c)$.

As can be readily seen, (A), (D) and (E) are valid on the basis of T, being, respectively, axioms ϵ, ζ, and δ of T. The following proofs demonstrate that (B), (C) and (F) are also consequences of T.

(B) is a consequence of axioms β and γ.

The proof of (C):

a. $\sim(a > a)$ from axiom ϵ.
b. $(Ra \cdot (c)((Rc \cdot b > c) \supset a > c)) \supset \sim(b > a)$ from a.
c. $(Ra \cdot Rb \cdot (c)((Rc \cdot b > c) \supset a > c) \cdot a \neq b) \supset (Ra \cdot Rb \cdot a \neq b \cdot \sim(b > a))$ from b.

(C) is a consequence of c and axiom η.

The proof of (F):

d. $(Ra \cdot Rb \cdot \sim(a \neq b)) \supset (\exists e)(Re \cdot e > a \cdot e > b)$ from axiom γ.
e. $(Ra \cdot a > b) \supset ((\exists e)(Re \cdot e > a) \cdot a > b)$ from axiom γ.
f. $(Ra \cdot a > b) \supset (\exists e)(Re \cdot e > a \cdot e > b)$ from e and axiom ζ.

(F) follows from axiom η, d, and f.

The crucial part of Leśniewski's analysis now follows; he shows that the negations of (G*) and (H*) are consequences of T.

From axiom β is obtained

g. $(Ra \cdot Rc \cdot \sim(a \neq c)) \supset (\exists e)(Re \cdot c > e) \cdot a > e)$,

as well as

h. $(Ra \cdot c > a) \supset ((\exists e)(Re \cdot a > e) \cdot c > a)$,

and from h and axiom ζ it follows that

i. $(Ra \cdot c > a) \supset (\exists e)(Re \cdot c > e \cdot a > e)$.

From axiom η, g, and i is obtained

j. $(Ra \cdot Rc) \supset (\exists e)(Re \cdot c > e \cdot a > e)$,

and from axiom α and axiom γ it follows that

k. $(\exists a)(\exists b)(Ra \cdot Rb \cdot b > a)$.

From j and k, axiom ζ, a, and axiom ϵ follows

(G) $(\exists a)(\exists b)(Ra \cdot Rb \cdot (c)(d)((Rc \cdot Rd \cdot c > d \cdot b > d) \supset (\exists e)(Re \cdot c > e \cdot a > e)) \cdot \sim(a > b) \cdot \sim(a = d))$,

and from j and k is obtained

(H) $(\exists a)(\exists b)(Ra \cdot Rb \cdot a > b \cdot (c)((Rc \cdot a > c) \supset (\exists e)(Re \cdot c > e \cdot b > e)))$.

(G) and (G*) are mutually contradictory as are (H) and (H*). Since (G) and (H) are consequences of theory T, it follows that if T is a non-contradictory theory, neither (G*) nor (H*) is a consequence of T. Thus, using the method of interpretation applied in proofs of independence, it may be concluded that neither (7) nor (8) is (contrary to Whitehead's claim) a consequence of the statements (1) - (6), i.e., (i) - (vi) do not provide an adequate axiomatic foundation for Whitehead's theory of events.

NOTES

1. In the Forward to [3].
2. See [2].
3. Leśniewski did not completely symbolize his discussion but used a mixture of Polish and symbols, as in the Polish of (1) - (8) below. To facilitate reading I have followed each of (1) - (8) with its expression in Peano-Russell notation, and I shall follow this practice henceforth; I shall use logical notation in place of Leśniewski's Polish *cum* symbols.

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University of Wisconsin—Milwaukee
Milwaukee, Wisconsin
and
University of Iowa
Iowa City, Iowa