Notre Dame Journal of Formal Logic
Volume VII, Number 3, July 1966

SOME THINGS DO NOT EXIST

R. ROUTLEY

The main objects of this paper are to suggest a definition of 'exists', to propose solutions to difficulties raised within restricted predicate logic with identity by failures of existential presuppositions of purportedly referring expressions such as individual constants and definite descriptions, to develop within a semantical system **R***, with the syntax of a restricted applied predicate calculus, the logic of 'exists', and to unify within =**R***, i.e. **R*** with identity, several hitherto distinct logical theories, to construct theories of definite descriptions, and to criticize certain widely accepted criteria for the ontological commitment of a theory. The logical developments in this paper are limited almost entirely to those that can be carried out in a first-order predicate logic with identity but without modal or intensional functors.

On the meaning of the predicate 'exists'. 'Exists' is gramatically a predicate, and the predicate seems to demarcate a property which Russell has, Socrates had, and Pegasus lacks. If, at a given time or atemporally, a domain D' of items, represented by names or referring expressions referring or purportedly referring to these items, is selected, then the property of existence, like other properties, can be represented over D' by a subdomain of D', by the class of its instances. For example given the domain [Churchill, Russell, the present king of France, Pegasus] 'exists' is represented by the subdomain [Churchill, Russell]. 'Exists', like any other property-word, has various designation-domains, the main special feature of which is that whereas the actual designations or extensions of other predicates, like '(is) red' or 'walks', are proper subclasses of the class G of all actual (or existent) items the extension of 'exists' coincides with G. The sense of 'exists' can also be explained [see below] in ways resembling explanations of the sense or meaning of other property-demarcating predicates, though admittedly the explanation is more like that for predicates also cast under suspicion, such as 'is true' and 'is good', than that for paradigmatic property-demarcating predicates such as predicates which demarcate simple descriptive properties. But, without drastic legislation on the meaning of 'property', these differences would at

Received November 30, 1964

most show that 'exists' does not mark out a property of a certain sort, e.g. that it marks out an ontological, not a descriptive property. There is, in short, because of sense and designation features of 'exists', a prima facie case for claiming that 'exists' demarcates a property. Moreover, widely deployed arguments designed to show that 'exists' does not demarcate a property fail, I contend, to establish their conclusion. Although moves useful in disabling these arguments appear in the sequel, the thesis that existence is a property is not actively defended[1]; and for much of what follows it is enough that 'exists' is a predicate which functions in some ways like a property-demarcating predicate.

The full sense of (or dictionary data on) the predicate 'exists' is given by giving

(i) its grammatical-range: namely all (singular) grammatical substantival expressions. That is, '*a* exists' is well-formed or grammatical if '*a*' is a singular substantival expression.

(ii) its significance-range. 'Exists' is exceptional in that its significance-*range coincides, or almost coincides, with its grammatical-range.* Whether it exactly coincides turns on the controversial issue as to whether sentences like 'most tame tigers exist' are significant or not. It seems, however, despite celebrated arguments[1] to the contrary, that English sentences like 'some tigers exist', 'Churchill exists', 'This exists', 'Red exists ' are certainly significant.

(iii) its concentrated sense, under which conditions for its correct applications (among its significant uses) are specified. Although these correctness conditions do not in the case of 'exists' quite coincide with specific criteria for assessing content values such as truth and falsity of "*a* exists", they are nonetheless closely bound up with truth and falsity conditions. Now criteria used for assessing the truth or falsity of "*a* exists" notoriously vary. Most frequently used criteria are those linked with spatio-temporal or temporal locatability or observability, but that these are far from exhausting employed criteria such examples as "Santa Claus exists", "Red exists," "Natural numbers exist," "Ideas exist," "Electrons exist," "Battles exist," "Tame tigers exist," "God exists" make clear. A definition of the concentrated sense of 'exists' which is neutral as between various rival criteria must allow for two degrees of freedom; for criteria for assessing truth or falsity of "a exists" depend both on 'a' and on the context in which the sentence appears. There is a further minor complication: once temporal variations are admitted 'exists' is no longer unambiguous. On the other hand when temporal variables are bound as in *'exists at some time' or explicitly introduced as in 'exists at t', 'exists'* does not seem to have several senses. (Whether it does have several senses or not rests in the end of criteria for sameness of sense agreed upon). Then 'exists', or 'exists at *t*', appears to be a one-sense multi-criterion expression, i.e. an expression with one sense but with several

1. Some standard arguments are criticized by M. Kiteley 'Is Existence a Predicate?' *Mind* LXXIII (1964), 364-373.

different criteria, or, more narrowly, truth-conditions, linked under the one sense. My suggestion is that criteria (and test-procedures) are combined under a single sense by a designation requirement as follows: 'u exists (at t)' can be expanded or defined '$\mathsf{qu}(u)$[2] is a referring or specifying expression which has, in the context, a suitable designation (at t)' - where the designation is suitable in usual non fictional contexts in the case of a medium sized material object, e.g. my house, Churchill, if it is (at t) spatio-temporally locatable and observable, in the case of an observable physical property if it has observable instances, in the case of a theoretical item if it is referred to in a true scientific theory and is tied theoretically, e.g. by correspondence rules, with several observable phenomena, and so on through quite a long and open-ended list of cases; and where the designation is suitable is a special context like a fictional one if the fictional or legendary item is mentioned as existing in fiction or in legendary stories, and in a special context like an intuitionistic mathematical one if the designed item is effectively constructible proofwise or is constructed in some typical instances. Which case is at hand is determined both by the sentence in which 'exists' occurs and by its context of appearance. Whether special contexts occur where ordinary criteria are supplaned by other criteria as sketched above is debatable. It seems that although special contexts do occur the relevant sentences can always be satisfactorily paraphrased by sentences set not in the special contexts but in usual contexts. For many sorts of designating expressions what, if anything, counts as a suitable designation even in usual contexts is also debatable, and the above examples are not intended as more than illustrative of various criteria. In providing an account of the sense of 'exists (at t)' these issues need not be resolved any more than it need be settled in explaining the sense of 'good' what is good. Someone who simply asserts that u exists does not say what he counts as a suitable designation, what criterion he is using, but he gives it to be understood or gives the impression that he could if challenged. Compare with 'exists' and 'existent' on these matters 'good' and 'true'. Some features of the definition are worth emphasizing: first that use/mention difficulties are avoided through use of a quotation-function '**qu**'; second that the definition is not implicitly circular since 'has' need not carry existential import (the definitions can be expanded using the 'Σ' introduced below); third that 'suitable designation' can, like 'sufficiently many good-making characteristics' in a definition of 'good', be given independent elaboration; and fourth that an intermediate course between one-sense one-criterion and several sense accounts of 'exists' is adopted. The definition also goes some distance towards meeting Leibnitz's requirement that the existent is what is possible and something more since an expression can only have a suitable designation if it has a possible designation.

2. On quotation function '**qu**' see L. Goddard and R. Routley 'Use, Mention and Quotation' *Australasian J. of Philosophy* (May 1966). The function '**qu**' is so defined that whatever the expression value of the argument the function value is the quotation-expression of that value.

On the elementary logic of 'exists'. With sketchy preliminaries over a start can be made a logical development. A restricted predicate logic **R*** is obtained from the usual restricted predicate logic by adding individual constants and a predicate constant 'E', read 'exist(s)', and by changing the interpretation of quantifiers and of free and bound variables. For completeness the primitive frame of **R*** is sketched. The primitive symbols of **R*** are:

(improper symbols)	$\supset$	$\sim$	π	(	)	'
(variables)	x	y	z	f	g	h
(constants)	a	b	c			
	f_0	g_0	h_0	E		

From recursion rules like: If u is a (predicate or individual) variable (constant) then u' is a variable (constant), further variables and constants can be generated. The formation rule for 'E' runs: If u is an individual variable or constant then $E(u)$ is well-formed. The postulate set of **R*** is as follows:

R0: *If A is truth-functionally valid, then A.*

R1: $(\pi x)(A \supset B) \supset . A \supset (\pi x)B$, *provided individual variable x does not occur free in A.*

R2: $(\pi x)A \supset \check{S}^{x}_{y} A|$, *where y is an individual variable or a consistent individual constant*[3].

RR1: $A, A \supset B \rightarrow B$ (modus ponens)

RR2: $A \rightarrow (\pi x)A$ (generalization)

Quantifier 'π' is read 'for all' or 'for all possible'. A given constant a is *consistent* if it is possible that 'a' has a referent. The restriction to consistent individual constants can be eliminated and replaced by another qualification once identity is introduced. Individual constant expressions go proxy for any individual (referring) expressions, e.g. 'Churchill', 'Pegasus', 'the least rapidly converging sequence', 'the round square cupola on St. Paul's'; but constants can only be used for instantiation in **R2** if they refer to consistently describable or possible individuals.

Further quantifiers can be introduced by definitions

$$
\begin{aligned}
(\Sigma x)A &\equiv_{Df} \sim(\pi x)\sim A \\
(\exists x)A(x) &\equiv_{Df} (\Sigma x)(A(x) \,\&\, E(x)) \\
(\forall x)A(x) &\equiv_{Df} \sim(\exists x)\sim A(x)
\end{aligned}
$$

3. The substitution notation, the extrasystematic notation, and some terminology and abbreviations are adapted from A. Church *Introduction to Mathematical Logic:* Vol. I, Princeton (1956). The explanation of '$\check{S}^{x}_{y} A|$' parallels Church's explanation of $\check{S}^{P}_{B} A|$, p. 192. To avoid confusion Church's symbols 't' ('true') and 'f' ('false') are replaced, respectively, by '$\not t$' and '$\not f$'.

$[(\forall x) A(x) \equiv (\pi x)(E(x) \supset A(x))]$ follows. The symbol 'Σ' is read 'for some' or 'for at least one', '$\exists$' is read 'there exist(s)' or 'for some existing', and '$\forall$' 'for all existing' or 'for all actual'.

With this little apparatus several sentences usually judged to lie beyond the scope of the formalism of predicate calculus can be symbolised; e.g. 'Churchill exists' can be written '$E(c)$' and 'something exists' '$(\Sigma x)E(x)$'. Substitution in theorem $[f(y) \supset (\Sigma x) f(x)]$ gives $[E(c) \supset (\Sigma x)E(x)]$, i.e. if Churchill exists then something exists. All the usual predicate inferences can be specialized in this way for the predicate 'E'; e.g. from $[(\pi x) f_0(x) \supset h_0(x))]$, (say, all unicorns are one-horned) and $[(\Sigma x)(f_0(x) \;\&\; E(x))]$ (some unicorns exists) follows $[(\Sigma x)(h_0(x) \;\&\; E(x))]$, i.e. $[(\exists x)h_0(x)]$ (there exist one-horned things). A generalization of 'Round squares do not exist' can be symbolized '$(\pi x)(f(x) \;\&\; \sim f(x) \supset . \sim E(x))$', and in view of the equivalence: $(\pi x)(f(x) \;\&\; \sim f(x) \supset . \sim E(x)) \equiv \sim (\exists x)(f(x) \;\&\; \sim f(x))$, can alternatively be written in the regular way as '$\sim (\exists x)(f(x) \;\&\; \sim f(x))$'. 'Some things do not exist' is symbolized '$(\Sigma x) \sim E(x)$'; its equivalent 'not every item exists' by '$\sim (\pi x)E(x)$'. These sentences do not yield contradictions; a point about which there need be no difficulty so long as it is remembered that '*a* does not exist' can be explicated by ' '*a*' is a referring expression without a suitable referent'. Thus $[(\pi x)E(x)]$ is not universally true[4], unless the class of domains with respect to which interpretations are allowed is severely curtailed, and is *not* a theorem, as can be demonstrated using a decision procedure for monadic predicate calculus under which 'E' is treated as an ordinary predicate. But "There are things that don't exist," i.e. $[(\exists x) \sim E(x)]$ is impossible since it is equivalent to $[(\Sigma x)(E(x) \;\&\; \sim E(x))]$. Thus $[(\forall x)E(x)]$ is a theorem. "Some things exist", i.e. $[(\Sigma x)E(x)]$, which is equivalent to $[(\exists x)E(x)]$, does *not*, however, follow from $[(\forall x)E(x)]$. $[(\Sigma x)E(x)]$, like $[(\Sigma x) \sim E(x)]$, is not a theorem of **R***. Whether these statements are universally true depends both on the width of the domain of individuals and on the criteria for existence admitted. If properties such as non-existence, for example, are admitted as individuals then it is demonstrable in unrestricted predicate logic that something does not exist. On the other hand it seems, under the criteria for existence I favour, that $[(\Sigma x)E(x)]$ is not analytic, even though the statement, through occurrences of its representative sentence, is contextually self-supporting. The strengthened system **R$_1$*** is obtained from **R*** by adding the axioms

R3: $\sim (\pi x)E(x)$

R4: $\sim (\pi x) \sim E(x)$

---

4. On the strong case for rejecting "Everything exists" see N. Rescher 'The Logic of Existence' *Philosophical Review* LXVII (1959) 160-2. Note that Rescher's two-sorted logic (p. 174-6) can be readily set up in **R***.

Among the theorems of **R*** are Leonard's principle[5] **L6**, i.e. $[f(y) \,\&\, E(y) \supset . \,(\exists x)\, f(x)]$, but not $[f(y) \supset (\exists x)\, f(x)]$; $[(\forall x)\, f(x) \,\&\, E(y) \supset f(y)]$, but not $[(\forall x)\, f(x) \supset f(y)]$. Consider now that subsystem **FR*** of **R***, the wff of which consists of all wff (or definitional contractions of wff) which contain no quantifiers other than '$\forall$' or '$\exists$' and no constants. **FR*** is a free logic[6], i.e. logic free of existential presuppositions: it can be axiomatized using a postulate set consisting of **R0, R1'**, **R2'**, **RR1**, and **RR2'**, where **R1'** and **RR2** are obtained from **R1** and **RR2** by replacing occurrences of 'π' by '$\forall$', and **R2'** is: **(R2')**: $(\forall x) A \,\&\, E(y) \supset \check{\mathsf{S}}^{x}_{y}\, A|$

Theorem: All theorems of **FR*** *are theorems of* **R***.

Proof: It suffices to show that the quantificational rules and axioms of **FR*** are derivable given **R***.

(i) **RR2'** (generalization). Since $[A(x) \supset . \,E(x) \supset A(x)]$, $[(\pi x)A(x) \supset . \,(\pi x)E(x) \supset A(x)]$.

Thus from the rule: $A(x) \rightarrow (\pi x)\, A(x)$
the rule: $A(x) \rightarrow (\forall x)\, A(x)$, is derivable.

(ii) **R1'**. $[(E(x) \supset . A \supset B] \supset . A \supset . E(x) \supset B]$. Thus $[(\pi x)(E(x) \supset . A \supset B) \supset . A \supset (\pi x)(E(x) \supset B)]$, provided x is not free in A.

Therefore, under the same proviso, $[(\forall x)(A \supset B) \supset . A \supset (\forall x)\, B]$

(iii) **R2'**.
$$\begin{aligned}(\forall x) A \,\&\, E(y) &\supset . \,(\pi x)(E(x) \supset A) \,\&\, E(y)\\ &\supset . \,(E(y) \supset \check{\mathsf{S}}^{x}_{y}\, A|\,) \,\&\, E(y)\\ &\supset . \,\check{\mathsf{S}}^{x}_{y}\, A|\end{aligned}$$

Theorem: Every theorem of **R*** *which is (or the definitional abbreviation of which is) a wff of* **FR*** *is a theorem of* **FR***.

Proof: Let B be a theorem of **R*** which is a wff of **FR***. Since B is a theorem of **R*** there is a sequence of wff of **R***

$$B_1, B_2, \ldots B_n, \text{ such that } B_n = B,$$

which represents a proof of B. It needs to be shown that given this sequence a new sequence can be constructed which constitutes a proof of B in **FR***. A proof of this result (which can alternatively be stated: For every π-Σ quantifier free theorem of **R*** there is a proof which is π-Σ quantifier-free, where a wff is π-Σ quantifier-free if every occurrence of

5. Leonard's replacement for PM ***10.24**. See H. S. Leonard 'The Logic of Existence' *Phil. Studies*, VII, 4 (June 1956) 49-64. An equivalent replacement was advocated much earlier by G. E. Moore 'Facts and Propositions' *Arist. Soc. Supp.* Vol. VII (1927), 204.

6. It coincides with Lambert's free logic: See T. C. Lambert 'Notes on E! III; A Theory of Descriptions' *Phil. Studies* XIII 4 (June 1962), 51-59. Though free of existential presuppositions **FR*** is not free of possibility presuppositions.

quantifiers 'π' or 'Σ' can be definitionally replaced by an occurrence of '$\forall$' or '$\exists$' respectively) is sketched. Replace both **R*** and **FR*** by equivalent Gentzen sequents systems. To be explicit **R*** is replaced by a system **G1R*** obtained from Kleene's system **G1**[7] by suppressing all individual constants. by adding predicate constant 'E', and by replacing '$\forall$' and '$\exists$' respectively by 'π' and 'Σ'. **FR*** is replaced by a system **G1FR*** obtained from **G1** by suppressing all individual constants, by adding predicate constant 'E' and by replacing rules $\forall \rightarrow$ and $\rightarrow\exists$, respectively, by:

$$\frac{\rho \rightarrow E(y),\ H\ /\ A(y),\ \rho \rightarrow\ H;}{(\forall x)A(x),\ \rho \rightarrow\ H} \qquad \frac{\rho \rightarrow E(y),\ H\ /\ \rho \rightarrow H,\ A(y)}{\rho \rightarrow H,\ (\exists x)A(x)\ .}$$

Now consider the proof of B i.e. the derivation of $\rightarrow B$, in **G1R***. It needs to be shown that there is a derivation of $\rightarrow B$ in **G1FR***. A proof can be obtained by induction over the number of occurrences of quantifiers $\forall$ & $\exists$ in B when B is written in π-Σ quantifier free form. If A contains no occurrences of $\forall$ or $\exists$ then the same proof suffices as a derivation of $\rightarrow A$ in **G1FR***. If B contains $k+1$ occurrences of $\forall$ and $\exists$ consider the last introduction of a quantifier in the proof of B. Either $\forall$ or $\exists$ is introduced and since the cases are similar it suffices to consider introduction of $\forall$. By hypothesis of induction the proof to that stage can be transformed into a proof in **G1FR***. If at the last introduction $\forall$ is introduced in the succedent the same step will suffice in **G1FR***, and if in fact π was introduced in the succedent in **G1R*** $\forall$ could equally well have been introduced. If $\forall$ is introduced into the antecedent in the last introduction, e.g., by $\pi \rightarrow$ and definitional abbreviation, then a further premiss,

$$\rho,\ \Lambda \rightarrow \Delta,\ H,\ E(y)$$

where y is the variable free in $\pi \rightarrow$, i.e. in:

$$\frac{A(y),\ \rho \rightarrow H,}{(\pi x)A(x),\ \rho \rightarrow H}$$

must be supplied in the proof otherwise the proof will not be a proof of the required $\rightarrow B$, for it will not be possible to ensure that B is π-Σ free. But if the additional premiss is available it can be used in rule $\forall \rightarrow$ of **G1FR***. This completes the sketch of the proof.

On the interpretation of logic **R***. The intended extensional interpretation of **R*** is, formally at least, straightforward once a non-null discourse-universe or individual-expression domain D is selected. D is a class of names or referring expressions each of which refers or putatively refers to an individual item. If D is non-null some among these expressions, e.g. 'Churchill', 'Pegasus' either have or possibly have referents; such expressions are called *possibly referring*. But D may also include expressions, e.g. 'Primecharlie', the name of the first even prime greater than two, which do not even possibly have referents. The items referred to by

7. Set out in S. C. Kleene *Introduction to Metamathematics* Amsterdam (1952) 442-3.

expressions which have or could have referents are called *possible items* (with respect to D). These items are, if you like, the items which would be actually referred to by the expressions concerned if possibilities were actualities. Expressions of D which could not possibly have a referent are sometimes said, purely for convenience, to refer to *impossible items*. Any discourse-universe D will have an associate *individual item domain* D' of possible items with respect to D. The intended extensional interpretation of **R*** with respect to non-null domain D' associated with D is then exactly the same as intended interpretations of restricted predicate calculus with respect to some non-empty individual domain[8]: in particular: -

(i) Individual variables are variables having elements of D', or possible items, as their (designation) range.
(ii) For a given set of range-values of the free variables of $(\pi x)A$, the true-value of $(\pi x)A$ is $\nleftarrow$ if the truth-value of A is $\nleftarrow$ for every range-value of x, and if $\not f$ otherwise. Thus $[(\pi x)\, f(x)]$ is true if every element of D' has $\not f$, e.g. belongs to the class assigned to f. Validity and satisfiability can, therefore, be defined in pretty much the usual way. Though associated item domains may be empty they must not be null, where emptiness and nullity are distinguished as follows:

Item domain w is *empty* $\equiv_{Df}$ $\sim(\exists x)(x \varepsilon w)$
Item domain w is *null* $\equiv_{Df}$ $\sim(\Sigma x)(x \varepsilon w)$

D is non-null if its associated domain D' is non-null.

The interpretation may be presented more satisfactorily if, in place of designation-ranges of variables, substitution-ranges of variables, i.e. the classes of expressions with which variables can be replaced, are taken. Then an interpretation of **R*** will read as follows:

(i) Individual variables are variables having possibly-referring expressions of D as their substitution-range.
(ii) Monadic predicate variables are variables having as their substitution-range singulary predicates whose field is D and which are represented by subclasses of D. [Or: Singularly functional variables are variables having as their substitution-range singularly (sentential) functions from possibly-referring expressions to truth-values]. And so on. The semantical rules for the predicate constant '*E*' are as follows:

(Ei) E is a monadic predicate constant which is represented by the class E° of expressions of D which have (suitable) existent referents, i.e. which actually have referents. E° is a subclass of the class of possibly-referring expressions.
(Eii) If x has substitution-range value a then $E(x)$ has truth-value $\nleftarrow$ if a belongs to E°, and has truth-value $\not f$ otherwise.

Under the interpretation sketched, there are no longer obstacles to

8. For instance, that given by Church, *op. cit.*, 175.

including in domains of individual items ideal or abstract items such as bodies not acted on by external forces, point masses, and Pegasus, or to taking as discourse-universes purely abstract or fictional universes. If such domains are selected certain standard difficulties about universal generalizations whose antecedent referring expressions have no actual referents can be evaded. For Newton's first law of motion, 'All bodies not acted on by external forces continue in their state of rest or uniform rectilinear motion', can now be symbolized by '$(\pi x)(f(x) \supset h(x))$'. That there are, in fact, no such bodies does not matter; for although such paradoxes of implication as $[\sim(\Sigma x) f(x) \supset (\pi x)(f(x) \supset h(x))]$ are still theorems, $[\sim(\exists x) f(x) \supset (\pi x)(f(x) \supset h(x))]$, *i.e.* $[\sim(\Sigma x)(f(x) \;\&\; E(x)) \supset (\pi x)(f(x) \supset h(x))]$, is *not* a theorem, as can be shown by using again a decision procedure. In other words so long as we are prepared to assert that $[(\Sigma x) f(x)]$ is true; i.e. that some (ideal) items are bodies not acted by external forces, predicate calculi as here reinterpreted can be used for the formulation of all scientific laws. $[(\pi x)(f(x) \supset h(x))]$ is not automatically true. Not that I want to pretend that this situation is entirely satisfactory: for, first, suitably large domains have to be selected and the premisses of the law sentences qualified; and second, with a little ingenuity conditionals at least as satisfactory as formal implications can be defined.

On domains which include impossible items. The domains selected may also include impossible items, which can be represented by constants '*a*', '*b*', '*c*' etc; but quantifiers take no account of them and free variables of **R*** may not be replaced by them, i.e. expressions without possible referents do not lie within the substitution-range of free variables or the class of range-values of quantified expressions. The restriction to possibly-referring expressions of range-values of bound variables, or, put differently, the restriction to possible items on the interpetation of quantifiers 'π' and 'Σ' cannot be lifted, unless quantification theory is radically amended, in such a way that contradictions do not spread. Extension of range-values of quantifiers to all possibly-referring expressions is the *maximum admissable extension* within the framework of standard quantification logic. For an extension to all possibly-referring expressions as substitution-range-values can be made consistently; for **R*** is consistent and its interpretation is consistent since it can be mapped into the system. But a further extension cannot be consistently made. If 'Primecharlie' (Primecharlie is the first even prime greater than two; Joesquare is the round square at t_1 at p_1) were within the substitution-range then for some f, f(Primecharlie) and $\sim f$(Primecharlie). For, unless predicate and sentence negation are distinguished, either "Primecharlie is not prime" and "Primecharlie is prime" are both true or they are both false. If they are both true 'is prime' provides a suitable predicate; if they are both false 'is not prime' is suitable. Since then for some f [f(Primecharlie) & $\sim f$ (Primecharlie) would be true, neither $[\sim(f(x) \;\&\; \sim f(x))]$ nor $[(\pi x)\sim(f(x) \;\&\; \sim f(x))]$ would be universally valid: the laws of non-contradiction and excluded middle would fail. The system would be inconsistent under interpretation.

In contrast with '$\forall$' and '$\exists$', 'Σ' and 'π' are not replaceable by or definable in terms of near regular quantifiers with more extensive ranges. For quantifiers '$\forall$' and 'E' the equivalences $[(\forall x)(E(x) \supset A(x)) \equiv (\forall x)A(x)]$ and $[(\exists x)(E(x) \,\&\, A(x)) \equiv (\exists x)A(x)]$ are provable. In contrast for quantifiers 'π' and 'Σ' the relations $[(\Sigma x)(\Diamond(x) \supset A(x)) \equiv (\pi x)A(x)]$ and $[(\Sigma x)(\Diamond(x) \,\&\, A(x)) \equiv (\Sigma x)A(x)]$, where '$\Diamond$' is a predicate constant read 'is possible' or 'is a possible item', though they hold under the intended interpretation, are not derivable from relations connecting them with more extensive unrestricted quantifiers of a consistent standard system. The further relations specified do, however, underlie the intended interpretation of 'π' and 'Σ'. More extensive quantifiers 'A' and 'S' can be introduced in non-standard systems which restrict the application of the classical laws of non-contradiction and exclude middle. For example, this may be achieved by distinguishing sentence and predicate negations, and qualifying the classical laws for predicate negation. Important among such systems are *Meinongian* systems, that is systems for which $\sim(f(x) \,\&\, \bar{f}(x))$, where '-' represents predicate negation, holds only for possible x. Not only are unlimited quantifiers available in such systems; also the problem of the null domain can be easily resolved.

Although the value-range for bound variables, or quantifiers, cannot be further extended consistently and within the framework of a logic approximating to standard quantification logic, the substitution-range of *free* variables can be further extended by slightly modifying quantification logic. There are various alternative ways in which this extension can be carried through. In each case extension of ranges of free variables has only a limited effect on the class of theorems; but free variable formulations of usual logic laws are no longer unconditionally asserted as theorems. There are in particular two roughly equivalent ways in which the extension can be made:

(a) By adding a new predicate constant '$\Diamond$' to the primitive symbols of **R***, and by modifying the postulate set of **R***. The resulting system **UR*** has the following postulate set:

UR0: *If A is truth-functionally valid, then αA.*
UR1: $\alpha((\pi x)(A \supset B) \supset . A \supset (\pi x) B)$, *provided x is not free in A.*
UR2: $\alpha(\Diamond(y) \supset . (\pi x) A \supset \check{\mathsf{S}}^{x}_{y} A|)$ *where y is an individual variable or an constant.*
UR3: $(\pi x) \Diamond (x)$

URR1 and **URR2** are the same as **RR1** and **RR2**.

If $x_1, x_2, \ldots x_n$ are all the distinct free variables in A, which are not $\Diamond$-qualified, then αA is $(\Diamond(x_1) \supset . \Diamond(x_2) \supset . \ldots \supset . \Diamond(x_n) \supset A)$. ($x_1, \ldots x_n$ are here $\Diamond$-qualified in αA). From **UR0** it follows that quantifier-free laws containing free variables only hold on condition that substitution-ranges of these variables are restricted to possibly referring expressions; and from **UR2** that instantiations are permitted only for variables with substitution-ranges restricted to possibly referring expressions or for constants with possibly referents, i.e. for consistent constants. In fact no theorems of **UR*** contain variables which are neither bound nor unqualified by '$\Diamond$'. It is, however, possible to lift some of the qualifications on **UR1** and **UR2.**

(b) By replacing theorems of **R*** by their closures whenever theorems would otherwise fail because of the extension in the substitution-range of free variables. One system which results by this strategy is **UR$_1$***, a system which differs from **UR*** in that **UR3** is dropped and 'α' is replaced throughout the postulate sentences by '$\vdash$', where $\vdash A$ is the universal (π-) closure of A.

Theorem: All theorems of **UR$_1$*** *are theorems of* **UR*.**

Proof: It suffices to derive **UR$_1$0**, **UR$_1$1** and **UR$_1$2**; and derivation of **UR$_1$0** may serve as typical. Suppose, $x_1, x_2 \ldots x_n$ are all the distinct free variables in A. Then, if A is truth functionally valid $\Diamond(x_1) \supset . \Diamond(x_2) \supset \ldots \supset . \Diamond(x_n) \supset A$ is a theorem of **UR***. By repeated application of **URR2** and **UR1** it follows:

$$(\pi x_1) \Diamond(x_1) \supset . (\pi x_2) \Diamond(x_2) \supset . \ldots \supset . (\pi x_n) \Diamond(x_n) \supset . (\pi x_1) \ldots (\pi x_n) A$$

By **UR3** and **URR1** $(\pi x_1) \ldots (\pi x_n) A$, results as required. Note that:

$$\alpha (\pi x)(\Diamond(x) \supset A(x)) \equiv \alpha(\pi x) A(x)$$

is a theorem of **UR***.

Theorem: If the symbol '$\Diamond$' is so introduced into **UR$_1$*** *that the axiom schemes*

UR$_1$3: $(\pi x)\Diamond(x)$
UR$_1$4: $(\pi x) A(x) \supset . \Diamond(y) \supset A(y)$, *where x is the only free variable in $A(x)$,*

are satisfied then all theorems of **UR*** *are theorems of* **UR$_1$***

Theorem: The closures of all theorems of **R*** *are theorems of* **UR*** *and* **UR$_1$***

For **UR$_1$*** differs from **R*** only in range of free variables. It is almost immediate that the postulate set of **UR$_1$*** is derivable from that of **R***.

A system embracing **R*** is obtained from **UR*** by replacing **UR3** by

UR4: $\Diamond(x)$

and replacing the hypothesis $\Diamond(y)$ where y is a constant, in **UR2** by the proviso: provided y is a consistent constant.

Since an interesting axiom set for '$\Diamond$' does not seem to be attainable without the introduction of a higher order calculus or a calculus of individuals, it is worth investigating replacements for '$\Diamond$'. Once identity is introduced a simple resolution can be made: for

$$\Diamond(x) \equiv (x{=}x \;\&\; \Diamond(x)) \equiv (\Sigma y)(y{=}x \;\&\; \Diamond(y)) \equiv (\Sigma y)(y{=}x).$$

In **=R*** the condition $(\Sigma y)(y{=}x)$ will be used in place of $\Diamond(x)$. In a modalized logic the equivalence $[\Diamond(x) \equiv \Diamond E(x)]$ may be used. In higher predicate logic In higher predicate logic it is tempting to define:

$$\Diamond(x) \equiv \sim(\Sigma f)(f(y) \;\&\; \sim f(y)).$$

From such equivalences $[(\pi y)\Diamond(y)]$, i.e., everything is possible, or, more exactly, every possible item is possible, follows. The false statement "Absolutely everything is possible" could not be adequately represented in **R*** even if the symbol '$\Diamond$' were added to the primitive symbols, but it can be represented in **UR*** using free variables thus: $\Diamond(x)$. An advantage of Meinongian systems is that they permit the formulation of such sentences: 'Absolutely everything is possible' is symbolized '$(Ax)\Diamond(x)$'

On null domains. Although **UR*** seems reasonable as a venture towards a system which allows for domains which include impossible items (or for substitution-ranges which include expressions like 'Primecharlie' which lack even possible referents) the system fails for null domains. Strictly two cases should be disentangled:
(i) a domain is selected but contains only impossible items such as Primecharlie
(ii) no domain is selected.
UR* fails in case (i) because $[(\Sigma x)\Diamond(x)]$ and, therefore $[\sim(\pi x)(f(x) \,\&\sim f(x))]$ (compare $[\sim(\pi x)\not f]$) are theorems of **UR*** though they are not theorems if a domain of type (i) is selected. More comprehensive systems **VR*** and **VR$_1$*** which include the null domain as in (i) can be reached from **UR*** and **UR$_1$***. **VR*** differs from **UR*** only in that **VR2** which replaces **UR2** has the added proviso: provided x is free in A. **VR3** can be admitted since $[(\pi x)\Diamond(x)]$ is true if no item of the domain is possible. **VR$_1^*$** differs from **UR$_1^*$** only in that **VR$_1$2** carries a similar proviso to **VR2**. Theorems analogous to those relating **UR*** and **UR$_1$*** relate **VR*** and **VR$_1$***.

Theorem: System **UR$_1$*** *results from* **VR$_1$*** *by addition of* $\sim(\pi x)\sim(p\supset p)$ *(or:* $\sim(\pi x)\not f$*)*.

Proof: It has to be shown that the qualification on **VR$_1$2** can be lifted given the additional axiom.
Now if x is not free in A,

$$\begin{aligned}
&\sim A \supset . A \supset \sim(p\supset p)\\
&\qquad \supset . (\pi x)(A \supset\sim(p\supset p)); &&\text{by } \mathbf{VRR_12} \text{ and } \mathbf{VR_11}\\
&\qquad \supset . (\pi x)\, A \supset\sim(\pi x)\sim(p\supset p); &&\text{from } \mathbf{VR_11} \text{ and } \mathbf{VR_10}.\\
&\sim(\pi x)\sim(p\supset p)\supset(\pi x)A\supset \check{S}^x_y\, A\Big|\,; &&\text{by } \mathbf{VR_10} \text{ \& since } \check{S}^x_y\, A\Big| \text{ is } A.\\
&\sim(\pi x)\sim(p\supset p)\supset . (\pi y_1)(\pi y_2)\ldots(\pi y_n)(\pi x)A\supset \check{S}^x_y\, A\Big|)
\end{aligned}$$

where $y_1, y_2 \ldots y_n$ are all the free variables in A; by **VR$_1$1**

By **VRR$_1$1**

$$(\pi x)A\supset \check{S}^x_y\, A\Big|, \quad x \text{ not free in } A.$$

Note that a move similar to that preceding the last application of **VRR$_1$1**, i.e. modus ponens, should strictly precede each of the other applications of modus ponens.

Even though a system containing (interpretation-) unrestricted free variables may not hold for interpretations over null domains, a system which holds over null domains will contain unrestricted free variables if it contains free variables at all. Hence the relations between **U** and **V** systems.

Theorem: System **UR*** *results from* **VR*** *by addition of* $[(\Sigma x)\Diamond(x)]$

Proof: $(\pi x)(\Diamond(x) \supset (p \supset p))$ by **VR0**
$(\pi x)(\sim(p \supset p) \supset \sim \Diamond(x))$ by **VR0, VR1, VRR1**
$\sim (\pi x) \sim \Diamond(x) \supset \sim (\pi x) \sim (p \supset p)$ by **VR0, VR1, VRR1**
$\sim (\pi x) \sim (p \supset p)$

The theorem then follows as in the preceding theorem.

Theorem: A system including **R*** *results from* **VR*** *by addition of* $\Diamond(x)$;

specifically the system obtained from **R*** *by adding the primitive symbol '$\Diamond$' and the above axiom.*

For: $\Diamond(x) \supset \sim (\pi x) \sim \Diamond(x)$
By the above theorem **UR*** results, and as before **R*** + $\Diamond(x)$ results.

Theorem: **R*** *results from* $\mathbf{VR_1}$* *by addition of:*

$$(\pi x)A \supset \overset{\vee}{\mathsf{S}}{}^{x}_{y}\, A \Big| , \text{ where } y \text{ is a variable or consistent context}$$

For: $(\pi x) \sim (p \supset p) \supset \sim (p \supset p)$
$p \supset p \supset \sim (\pi x) \sim (p \supset p)$
$\sim (\pi x) \sim (p \supset p)$

Therefore $\mathbf{UR_1}$* results from $\mathbf{VR_1}$*. Then **R*** results from $\mathbf{UR_1}$* by application of the given schema.

If the symbol '*E*' is omitted from these systems, so that systems **R*-E**, $\mathbf{VR_1}$***-E** etc., result, then various completeness theorems can be proved. In particular: **R*-E** is complete, by a syntactical variant of Gödel's completeness theorem, $\mathbf{UR_1}$***-E** is complete in the sense that every closed wff of the system valid in a non-null domain is a theorem of the system; and $\mathbf{VR_1}$***-E** is complete, i.e. every closed wff of the system valid in any domain, null or not, is a theorem of the system[9]. Also appropriate completeness results can be established for **R*+R3+R4** and the **U** and **V** systems obtained from this system.

If, however, case (ii) obtains, if a domain of individual items is not selected, or a context not indicated, it is indeterminate without further stipulation or interpretation what statements, if any the closed symbolic sentences of systems **R***, **UR*** or **UR*** yield, since it is not clear what referents any referring expressions have or could have or could not have. There are various ways of slicing through this difficulty or of introducing an interpretation; for example:

9. A proof of this result can be adapted from the proof of theorem 2 in T. Hailperin 'Quantification Theory and Empty Individual-Domains' *J.S.L.* 18 (1953) 197-200.

(a) An interpretation which treats the case as similar to that of a domain empty of existing items or, better, null of possible items. Case (ii) is assimilated under case (i).
(b) A move according to which closed sentences are not assigned an interpretation at least in case (ii) (in case (i) and other cases also for sentences of **R***, it may be argued) because they are not significant.
(c) An interpretation under which all non-compound quantified wff are false and the inference:

$$\sim (\Sigma x)\, A \supset \sim (\pi x)\, A$$

is retained. Under this interpretation though all non-compound quantified wff not all wff are false, e.g. truth-functional compounds such as:

$$(\pi x)A \equiv (\Sigma x)A; \quad (\pi x)(A \supset B) \supset .\ (\pi x)A \supset (\pi x)B$$

are true
(d) A ploy according to which the closed sentences are statement-incapable at least in case (ii), and perhaps also for a large class of selected domains or non-usual contexts. How this move can be implemented should emerge from my suggestion below for completing the theory of descriptions proposed. Other interpretations remain, e.g. analogues of those made in different connections by Mostowski and Strawson.

On empty domains. Quantifiers '$\wedge$' and '$\vee$' of **R*** are next defined:

$$(\wedge x)A \equiv_{Df} (\Sigma y)E(y) \supset (\pi x)A$$
$$(\vee x)A \equiv_{Df} \sim (\wedge x) \sim A$$

Therefore $[(\vee x)A \equiv .\ (\Sigma y)E(y) \,\&\, (\Sigma x)A]$ and $[(\exists x)A(x) \supset (\vee x)A(x)]$ '$\wedge$' & '$\vee$' are frequently employed quantifiers. The added condition simply requires that the domain or universe has an existent member, i.e. that the domain of individuals is non-empty. So the condition makes explicit a presupposition made in usual interpretations of restricted predicate logic, viz. $[(\Sigma y)E(y)]$. Consider that subsystem $\mathbf{IR}_1$* of **R*** the wff of which consists of the wff of **R*** which contain no quantifiers other than '$\wedge$' and '$\vee$'. Now what happens when the domain of individuals of $\mathbf{IR}_1$* is empty. Since $[(\Sigma y)E(y)]$ is false when no individuals of the indicated domain exist, it follows that all non-vacuously $\wedge$-quantified wff are true and non-vacuously $\vee$-quantified wff false; and in particular that $[(\wedge x) \not f]$ is false. This suggests that $\mathbf{IR}_1$* is syntactically isomorphic with inclusive quantification theory, i.e. with quantification logic when empty as well as non-empty domains are admitted.

Theorem: $\mathbf{IR}_1$* *can be completely axiomatized using the* $\wedge$*-closures of*

IR1: $(\wedge x)(A \supset B) \supset (\wedge x)A \supset (\wedge x)B$
IR2: $A \supset (\wedge x)A$, *provided x is not free in A*
IR3: $(\wedge x)A \supset \check{S}^{x}_{y}\, A \big|$, *provided x is free in A,*

together with **RR1** & **IR0**: *If A is truth-functionally valid and A' is the* $\wedge$*-closure of A, then A'* (*'A' & 'B' represent wff of* $\mathbf{IR}_1$*).

Proof: It suffices to show that $\mathbf{IR}_1$-$\mathbf{IR}_3$ are derivable given $\mathbf{R}^*$, and that the axiomatization is complete. First **IR1-IR3** will be derived. When $[(\Sigma y)E(y)]$ is true $[(\wedge x)A(x) \equiv (\pi x)A(x)]$. Since **IR1-IR3** hold when '$\wedge$' is supplanted by by 'π', **IR1-IR3** hold when $[(\Sigma y)E(y)]$ is true. So suppose $[(\Sigma y)E(y)]$ is false. **IR1**, expanded by definition, is of the form: $(\not{f} \supset p) \supset ((\not{f} \supset q) \supset (\not{f} \supset r))$; **IR2** is of the form: $A \supset . \not{f} \supset (\pi x)A$, i.e. $\not{f} \supset . A \supset (\pi x)A$. Thus **IR1** and **IR2** hold. **IR3** expands to the closure of: $(\not{f} \supset (\pi x)A) \supset \check{S}^x_y A|$, i.e. $\not{t} \supset \check{S}^x_y A|$, i.e. $\check{S}^x_y A|$. Since $\check{S}^x_y A|$ contains y free its closure is the closure of $(\wedge y) \check{S}^x_y A|$ i.e. of $(\Sigma x)E(x) \supset (\pi y) \check{S}^x_y A|$. Since this is of the form: $(\not{f} \supset p)$, **IR3** holds. The point of this shuffle with **IR3** is clarified if **IR3** is replaced by closures of

IR4: $(\wedge x)A \supset (\wedge y) \check{S}^x_y A|$

IR5: $A \supset (\wedge x)B \supset . (\wedge x)(A \supset B)$, provided x is not free in A.

IR* can be alternatively and equivalently axiomatized using **IR4** and **IR5** in place of **IR3**, so an alternative is to show that **IR4** and **IR5** are derivable. Since they both hold when 'π' supplants '$\wedge$', consider only the case where $[(\Sigma x)E(y)]$ is false. Since **IR4** is then of the form: $(\not{f} \supset p) \supset (\not{f} \supset q)$, it holds. Also

$$\mathbf{IR5} \equiv (A \supset (\not{f} \supset (\pi x)B)) \supset (\not{f} \supset (\pi x)(A \supset B))$$

$$\equiv (\not{f} \supset (A \supset (\pi x)B)) \supset (\not{f} \supset (A \supset (\pi x)B); \quad \text{by Comm. and since } x \text{ is not free in } A.$$

$$\equiv \not{t}$$

The completeness of $\mathbf{IR}_1$* as an axiomatization of $\wedge$-$\vee$ quantificational wff valid in both empty and non-empty domains can be shown by adapting a completeness proof used by Hailperin[10]. The main change, apart from the systematic introduction of '$\wedge$' throughout the proof, is the replacement of '$\sim (\wedge x)\not{f}$' by '$(\vee y)(p \supset p)$'. Since

$$\sim (\wedge x)\not{f} \equiv \sim ((\Sigma y)E(y) \supset (\pi x)\not{f})$$

$$\equiv (\Sigma y)E(y) \quad \text{(upon choice of a non-null domain for } \mathbf{R}^*\text{).}$$

addition of '$(\wedge x)\not{f}$' is tantamount to addition of '$(\Sigma y)E(y)$'.

The system $\mathbf{IR}_2$* obtained from $\mathbf{IR}_1$* by adding **R4** as a further condition on the interpretation (within **R***) is therefore, like **R***, a syntactical variant

---

10. T. Hailperin *op. cit.* Any dispute over the truth-values of vacuous quantifications would, in the above setting, be automatically resolved in Hailperin's way: for if the domain is empty

$$(\wedge x)\not{f} \equiv - (\Sigma y)E(y) \equiv \not{t}$$

of ordinary exclusive quantification theory. Further $\mathbf{IR}_1{}^*$ is formally equivalent to a syntactical variant of $\mathbf{VR}_1{}^*$. While $\mathbf{IR}_1{}^*$ is a syntactical variant of Quine's inclusive quantification theory[11], it differs from that system in interpretation. A system approximating well to Quine's in interpretation can be obtained from $\mathbf{IR}_1{}^*$ by replacing '$\wedge$' throughout by '$\forall$'. This system can be restored to exclusive quantification theory by addition of $[(\pi y)E(y)]$. From $[(\pi y)E(y)]$ using $[(\pi x)(E(x) \supset . (E(x) \supset A) \equiv A)]$ it follows:

$$(\forall x)A \equiv (\pi x)A \equiv (\wedge x)A.$$

On interpretations of predicate calculus. It is hardly surprising, in view of the above conditional equivalences, that various interpretation-schemes for quantifiers have not often been disentangled. But for philosophical applications of the calculus it is most important to distinguish interpretation-schemes. In particular three standard interpretation-schemes can be distinguished - where standard interpretations are opposed, for example, to geometrical and arithmetical interpretations -

(1) A weak interpretation like that of quantifiers 'π' and 'Σ' of $\mathbf{R}^*$, where quantifiers do not carry existential assumptions. Here the domain of individuals need not be assumed to be non-empty but only to be non-null in order to retain syntactic variants of all the usual theorems.
(2) A medium interpretation, like that of quantifiers '$\wedge$' and '$\vee$' of $\mathbf{IR}^2{}^*$. In this case it is assumed (for all viable interpretations or applications of the calculus) that the domain of individuals is non-empty, but *not* that every member of the domain exists. If the non-emptiness requirement is abandoned $\mathbf{IR}_2{}^*$ gives way to $\mathbf{IR}_1{}^*$, a logic in which several of the usual relationships fail.
(3) A strong interpretation like that of quantifiers '$\forall$' and '$\exists$'. Under interpretations like Quine's it is presupposed that *all* the individuals of the non-empty domains selected exist: in effect the axiom $[(\pi x)E(x)]$ is built into the interpretation. The weak interpretation (1) is more satisfactory and more comprehensive than interpretations (2) and (3); and adoption of (1) permits an untrammelled treatment of many problems relating to existence.[12]

Thus issues often raised as to the logical truth or analyticity of statements $[(Ex)(f(x) \vee \sim f(x))]$ and $[(x)\, f(x) \supset (Ex)\, f(x)]$ can be illuminated by

11. W. V. Quine, 'Quantification and the empty domain' *J.S.L.* 19, (1954) 177-179.

12. Both these points are argued, skillfully by C. Lejewski in a paper, 'Logic and Existence' *Brit. J. Phil. Science*, V (1954-5), 104-119, which came to my attention after this paper had been written. Lejewski is not very explicit either about his quantificational system, which seems to be $\mathbf{R}_1{}^*$, or about the qualifications which have to be imposed upon interpretations of his unrestricted quantifiers if formal quantification theory is to be preserved. To count expressions which necessarily lack a reference as not meaningful, as Lejewski seems obliged to, seems to me disastrous. Nor am I at all happy about either Lejewski's or Leśniewski's analysis of 'exists'.

considering their analogues in **R***. Although $[(\Sigma x)(f(x) \vee \sim f(x))]$ is a theorem, neither $[(\exists x)(f(x) \vee \sim f(x))]$ nor $[(\vee x)(f(x) \vee \sim f(x))]$ are theorems. For

$$\begin{aligned}(\exists x)(f(x) \vee \sim f(x)) \equiv .\ &(\Sigma x)((f(x) \vee \sim f(x)) \;\&\; E(x)) \\ \supset .\ &(\Sigma x)(f(x) \vee \sim f(x)) \;\&\; (\Sigma x)E(x) \\ \textit{i.e.}\ &(\vee x)(f(x) \vee \sim f(x))\end{aligned}$$

$[(\vee x)(f(x) \vee \sim f(x))]$ is only true if $[(\Sigma x)E(x)]$ is true. Again, though $[(\pi x)\, f(x) \supset (\Sigma x)\, f(x)]$ is a theorem, neither $[(\forall x)\, f(x) \supset (\exists x)\, f(x)]$ nor $[(\wedge x)\, f(x) \supset (\vee x)\, f(x)]$ are theorems; for the first implies the second and the second is true only if $[(\Sigma x)E(x)]$ is true. But if $[(\Sigma x)E(x)]$ is rejected as not logically true so, by a rule of rejection, are those statements which imply it.

On simplifications using identity. A restricted predicate logic with identity, =**R***, is got from **R*** by adding the binary constant '=' and the postulates:

R5: $x = x$
R6: $x = y \supset . A \supset B$, *where x and y are individual variables or constants, and B is obtained from A by replacing one particular occurrence of X in A by y, this particular occurrence of x being neither within the scope of quantifier* (πx) *nor of* (πy) [*nor within the scope of a non-extensional operator*].

Call the proviso on **R6** *proviso* (*a*). Some advantages of **UR*** are available in =**R*** without corresponding drawbackds. To explain the point some preliminary results are needed.

Theorem: (i) $(\pi x)A \;\&\; (\Sigma y)(y{=}a) \supset .\ \check{\mathsf{S}}^{x}_{a}\, A\,|$

(ii) $\check{\mathsf{S}}^{x}_{a}\, A\,| \;\&\; (\Sigma y)(y{=}a) \supset .\ (\Sigma x)A.$

Proof: Choose y so that y does not occur in A. Then

1. $y = a \supset .\ \check{\mathsf{S}}^{x}_{y}\, A\,| \supset \check{\mathsf{S}}^{x}_{a}\, A\,|$; by iteration of **R6**.
2. $(\pi x)A \supset \check{\mathsf{S}}^{x}_{y}\, A\,|$; **R2**
3. $(y{=}a) \supset .\ (\pi x)A \supset \check{\mathsf{S}}^{x}_{a}\, A\,|$; from 1 & 2 using Comm
4. $(\Sigma y)(y{=}a) \supset .\ (\pi x)A \supset \check{\mathsf{S}}^{x}_{a}\, A\,|$; since y is not free in A
5. $(\pi x)A \;\&\; (\Sigma y)(y{=}a) \supset \check{\mathsf{S}}^{x}_{a}\, A\,|$; changing bound variables if necessary

(ii) $\check{\mathsf{S}}^{x}_{a}\, A\,| \supset \sim((\pi x)\sim A \;\&\; (\Sigma y)(y{=}a))$; from (i)

$\supset .\ (\Sigma x)A \vee \sim(\Sigma y)(y{=}a)$ result by importation.

Consequently the requirement in **R2** that a be a consistent constant can be can be replaced by the requirement on a that $(\Sigma y)(y=a)$, and the symbol '$\Diamond$' of **UR*** can be defined for **=R*** thus:

$$\Diamond(x) \equiv_{Df} (\Sigma y)(y=x).$$

The postulates of **UR*** and of **UR$_1$*** are then immediately derivable. But though constants can be handled in **=R*** quite satisfactorily the ranges of free variables of **=R*** are still confined to consistent values. For instance, [Primecharlie = Primecharlie] does not follow from **R5**. These restrictions could be lifted, much as in **UR***, by adding explicitly a condition $(\Sigma y)(y=x)$ on each distinct variable x occurring free in all axioms of **=R*** except instances of **R5** and **R6**. Instead a system **=VR***, which allows for null domains, will be erected. Its primitive symbols are those of **=R***. '$\Diamond$' is defined as above. The postulates of **=VR*** are those of **VR*** together with **VR6**, which is the same as **R6** (except in allowing x and y to be descriptions) and

VR5: $x = x$, *where x and y are individual variables, individual constants (or definite descriptions).*

Various modifications of usual restricted predicate logic with identity designed to eliminate existential presuppositions of the usual logic can be developed in **=R***. For if restricted variables $w, w', \ldots$ such that $E(w), E(w') \ldots$ hold, are introduced the following relations result.

Theorem: (i) $E(x) \equiv (\exists y)(y=x)$

(ii) $(\forall x)A \;\&\; (\exists z)(z=y) \supset \check{S}^{x}_{y}\, A|$, $\quad$ y an individual variable or constant

(iii) $\check{S}^{x}_{y}\, A| \;\&\; (\exists z)(z=y) \supset (\exists x)A$, $\quad$ y an individual variable or constant

(iv) $(\forall x)A \supset \check{S}^{x}_{w}\, A|$

(v) $\check{S}^{x}_{w}\, A| \supset (\exists x)A$

(vi) $(\forall y)((\forall x)A \supset \check{S}^{x}_{y}\, A|)$

Proof: (i) $E(x) \supset .\ E(x) \;\&\; (x=x)$

$\supset .\ (\Sigma y)((y=x) \;\&\; E(y))$, i.e. $(\exists y)(y=x)$

$y = x \;\&\; E(y) \supset E(x)$; $\quad$ by **R6**

$(\Sigma y)(y=x \;\&\; E(y)) \supset E(x)$ $\quad$ by **R1**, **RR2**, since y not free in $E(x)$

(ii) & (iii) from

$(\forall x)A \;\&\; E(y) \supset \check{S}^{x}_{y}\, A|$

$\check{S}^{x}_{y}\, A| \;\&\; E(y) \supset (\exists x)A$ $\quad$ by (i)

(iv) & (v) from (ii) and (iii), using $E(w)$ and (i).

(vi) from $E(y) \supset .\ (\forall x)\, A \supset \check{S}^{x}_{y}\, A|$ $\quad$ by **RR1**

Theorem: The system H_1 *obtained from* $=\mathsf{R}^*$ *by replacing '*π*' throughout by '*$\forall$*' and taking the* $\forall$*-closure of every axiom is a subsystem of* $=\mathsf{R}^*$*, i.e. the notation of the resulting system can be defined using the notation of* $=\mathsf{R}^*$ *and every theorem of the resulting system is a theorem of* $=\mathsf{R}^*$*. The system* H_1 *is effectively the system obtained from Church's system* F^{1P} *by taking the* $\forall$*-closure of every axiom, and adding the symbol 'E' and individual constants.*

Theorem: The system H_2*, obtained from* H_1 *by deleting the primitive symbol 'E' and by deleting the clause 'or y is a consistent individual constant' from the second axiom scheme is a subsystem of* $=\mathsf{R}^*$*.*

The second axiom-scheme of H_2 is the $\forall$-closure of:

(H): $(\forall x)A \supset \check{S}^x_y A\Big|$, where y is an individual variable.

'$\forall\!\!\!/ A$' symbolizes the $\forall$-closure of A.

Theorem: A is a theorem of H_2 *if and only if* $(A \vee \sim A) \longleftrightarrow A$ *is a theorem of a notational variant of Hintikka's quantification theory without existential presuppositions*[13] *in which Hintikka's "free variables" are taken as individual constants.*

In both H_2 and Hintikka's system

$$\forall\!\!\!/ \quad \check{S}^x_a A\Big| \ \& \ (\exists x)(x=a) \supset (\exists x)A$$

is a theorem but

$$\forall\!\!\!/ \quad \check{S}^x_y A\Big| \supset (\exists x)A$$

is not a theorem scheme.

Theorem: The system H_3 *obtained from* $=\mathsf{R}^*$ *by replacing '*π*' throughout by '*$\forall$*', by replacing* $\mathsf{R2}$ *by* (H) *by deleting 'E' but restricting in interpretation free variables* x, y, z *. . . so that* $E(x)$*,* $E(y)$*,* $E(z)$ *. . . respectively hold, is a subsystem of* $=\mathsf{R}^*$*.*

H_3 differs from Leblanc's and Hailperin's system for singular inference[14] in only one major respect, namely that $[x=x]$ does not hold unconditionally, i.e. for any constant whether consistent or not. A system H_4 which is deductively equivalent to the Hailperin Leblanc system can be obtained by replacing **R5** of H_3 by **VR5**, only this move leads to complications in the

---

13. J. Hintikka 'Existential presuppositions and existential commitments' *J. of Philosophy* 56 (1959), 125-137. A symbol with the systematic rules of '$\longleftrightarrow$' can be defined for H_2. If $(f \vee \sim f)$ is added to Hintikka's system as an axiom, the logics are deductively equivalent (though there are notational differences). Both systems require non-null domains for interpretations.
14. H. Leblanc and T. Hailperin 'Non-designating singular terms' *Phil. Review* 68 (1959), 239-243.

theory of descriptions. Since $[a=a]$ holds unconditionally in **H**$_4$, **H**$_4$ is stronger than **H**$_3$ or **H**$_2$.

To axiomatize **H**$_3$ and **H**$_4$ satisfactorily a system allowing for restricted variables such as '$x/E(x)$', i.e. 'x such that x exists' is wanted. Systems containing restricted variables are most valuable here for two reasons: - First when it comes to developing subsystems like **FR*** of **R*** and **=VR***; for instance, '$(\forall x)B(x)$' could then be replaced by '$(\pi x/E(x))\ B(x/E(x))$'. Secondly, for the development of more comprehensive systems which permit, unlike the makeshift systems **UR**$_1$ and **VR**$_1$, an unfettered treatment of impossible items; more accurately of systems under the interpretations of which may be taken substitution-ranges for individual variables which include or even consist entirely of inconsistent referring expressions. It is possible, for instance, to construct a Meinongian system in which **=R*** can be embedded as that subsystem which holds for all consistent items. The presentation of such a system lies beyond the scope of the present venture.

On definite descriptions. Russell's analysis of definite descriptions (PM, ***14.01**) can be replaced (omitting scope indicators) by:

$$A((\iota x)B(x)) \equiv_{Df} (\Sigma y)((\pi x)B(x) \equiv x{=}y)\ \&\ A(y)).$$

Substituting 'E' gives:

$$E((\iota x)B(x)) \equiv_{Df} (\exists y)((\pi x)B(x) \equiv .\, x{=}y) \equiv (\exists !x)B(x),$$

a good analogue of ***14.11**, though reached without the need for the separate definition ***14.02**. To eliminate difficulties of scope binary quantifiers which satisfy equivalences:

$$(\pi x)(B(x),\ A(x)) \equiv (\pi x)(B(x) \supset A(x))$$
$$(\Sigma x)(B(x), A(x)) \equiv (\Sigma x)(B(x)\ \&\ A(x))$$

may be introduced. Then 'ι' can be defined as a binary quantifier thus:

$(D1)$: $(\iota x)(B(x),\ A(x)) \equiv_{Df} (\Sigma x)((\pi y)(B(y) \equiv .\ y{=}x),\ A(x)).$

When $A(x)$ is the smallest sentence context in which individual or descriptive expression 'x' may grammatically occur $(\iota x)(B(x),A(x))$ may be replaced by $A((\iota x)B(x)$. Not only can the theory of descriptions be simplified when constructed on $(D1)$; more important we are no longer forced into the embarrassing position of having to say that such statements as "Ponce de Leon sought the fountain of youth", "The King of France is the king of France" are false or else not immediately treatable under the theory. In fact, it follows $[(f((\iota x)\ f(x)) \equiv (\Sigma !x)\ f(x)]$ and as a special case $[E((\iota x)E(x)) \equiv (\exists !x)E(x)]$. Using $[E(y)\ \&\ (\iota x)(f(x),\ x{=}y) \supset .\ f(y)]$ it follows: $[(\iota x)(f(x), E(x)) \supset (\iota x)(f(x), f(x))]$ but the converse $[(\iota x)(f(x),f(x)) \supset (\iota x)(f(x), E(x))]$, i.e. (written in the usual way) $[f((\iota x)\ f(x)) \supset E((\iota x)\ f(x))]$ is not a theorem. Thus Lambert's requirements on descriptions[15] are satisfied,

15. T. C. Lambert *op. cit.* Some such requirements must be met if paradox is to be avoided.

and those versions of the Ontological argument which use the last statement as a premiss are undermined.[16] An additional gain is that arguments like:

> *Scott is the author of Waverley*
> *∴ Scott exists if and only if the author of Waverley exists,*

can be directly symbolized and their validity checked. Consider, e.g., the following formalization of a vogue solution of the Barber paradox. Let '**h**' symbolize the monadic predicate 'lives in Alcoa, is male, and is a shaver' and '**sh**' the binary predicate 'shaves'. Then given the definition of the barber, **b**, as:

1. $\mathsf{b} =_{df} (\iota x)(\mathsf{h}(x) \mathbin{\&} (\pi x)(\mathsf{h}(y) \supset .\ \mathsf{sh}(x,y) = {\sim}\mathsf{sh}(y,y)))$

it follows, using natural deduction:

$E(b)$
$E((\iota x)(\mathsf{h}(x) \mathbin{\&} (\pi y)(\mathsf{h}(y) \supset .\ \mathsf{sh}(s,y) \equiv {\sim}\mathsf{sh}(y,y))))$
$(\Sigma x)(\pi x)(\mathsf{h}(z) \mathbin{\&} (\pi y)(\mathsf{h}(y) \supset .\ \mathsf{sh}(z,y) \equiv {\sim}\mathsf{sh}(y,y)) \equiv (z=x)) \mathbin{\&} E(x))$
$\mathsf{h}(w)$ (instantiating twice and using simplication)
$\mathsf{h}(w)\ .\ \mathsf{sh}(w,w) \equiv {\sim}\mathsf{sh}(w,w)$ (instantiating and using $[w=w]$)
$\mathsf{sh}(w,w) \equiv {\sim}\mathsf{sh}(w,w)$
$\not f$

2. $E(b) \supset \not f$

3. ${\sim}E(b)$

This solution may be extended to apply to other paradoxes: set theoretical paradoxes can be eliminated by replacing the abstraction axiom by:

$(\Sigma w)(\pi x)(x \varepsilon w \equiv .\ A(x) \mathbin{\&} E(x))$, where A does not contain w.

For the full development of set theory from this main axiom further conditions specifying when items exist have to be added. It then emerges that a more attractive axiom, which does *not* imply the existence of any sets, is:

$(\Sigma w)(\pi x)(x \varepsilon w) \equiv .\ A(x) \mathbin{\&} \Diamond E(x))$; where A does not contain w.

Conditions for possible existence (or for existence) which suffice for most of set theory can be obtained using one of the following equivalences:

(i) $\Diamond E(x) \equiv x \varepsilon \mathrm{V}$
(ii) $\Diamond E(x) \equiv M(x)$

where, however, in the implicit definitions of 'V' (given by Quine[17]) and of 'M' (given by Gödel[17]) 'π' and 'Σ' replace usual

16. On such versions - historically the most important versions - see J. Berg 'An examination of the ontological proof' *Theoria* XXVII (1961) 99. Since the threat of the Ontological argument has been one of the main motives for excluding existence as a property, it is of no little importance that premisses of these versions fail. If Meinong's law $[(\pi x) \Diamond {\sim}E(x)]$ is correct all versions of the Ontological argument which appeal to logical necessity must fail.

17. W. V. Quine, *Mathematical Logic* Revised edn., Cambridge Mass. (1951); K. Godel, *The Consistency of the Continuum Hypothesis* Revised edn. Princeton (1951). I do not think the solutions of set-theoretical paradoxes suggested in the text are the best solutions.

quantifiers. Two important consequences ensue. First, it follows that the Russell set and related sets cannot possibly exist. Definitional versions of set-theoretical (and other) paradoxes may then be avoided by accepting Leśniewski's requirements on ontological definitions, where, however, his '$x \varepsilon x$' addition to definiens is replaced by an '$E(x)$' or, if the second abstraction axiom is preferred, an '$\Diamond E(x)$' addition to definiens. Secondly, if the second axiom scheme is used along with **=R*** most of set theory and classical mathematics can be developed without any commitment to the existence of classes or mathematical items such as numbers.

In both **=R*** and **=VR*** definite descriptions can be admitted together with and on a par with individual constants. Just as $[a=a]$ is not derivable in **=R*** unless a is a consistent constant, so $[(\iota x)\, f(x) = (\iota x)\, f(x)]$ is not derivable unless $(\iota x)\, f(x)$ is a consistent description: for (by definition)

$$\Diamond((\iota x)\, f(x)) \equiv (\Sigma y)(y = (\iota x)\, f(x)) \equiv (\Sigma ! x)\, f(x),$$

and

$$(\Sigma ! x)\, f(x) \quad \equiv .\ (\iota x)\, f\,(x) = (\iota x)\, f(x)$$

Because of these consequences a theory of descriptions based on *(D1)* cannot be employed in **=VR***. For $[\sim\Diamond(\Sigma x)(f(x) \,\&\, {\sim} f(x))]$, but it follows from **VR5** that $[(\iota x)(f(x) \,\&\, {\sim} f(x)) = (\iota x)\ (f(x) \,\&\, {\sim} f(x))]$ holds. A different, still not completely satisfactory theory of definite descriptions, can be constructed for **=VR*** as follows: - 'ι' is introduced as a primitive symbol: $(\iota x)A$ is wff if A is wf and contains x free. x is bound in $(\iota x)A$. From **VR5** it follows at once:

$(\iota x)A = (\iota x)A$; and from **VR6**:

$a = (\iota x)A \supset .\ B(a) \equiv B((\iota x)A)$

Thus an axiom scheme indicating at least when $a = (\iota x)A$ is wanted. The following scheme recommends itself:

VR7: $\alpha((\pi y)(y = (\iota x)A(x) \equiv .\ A(y) \,\&\, (\pi x)(A(x) \supset x=y)))$[18]

From **VR7** it follows using **VR2**:

$$\Diamond(y) \supset .\ y = (\iota x)\, f(x) \equiv .\ f(y) \,\&\, (\pi x)(f(x) \supset x=y)$$

Since $(\pi y)\Diamond(y)$ there follow:

$$(\Sigma y)(y = (\iota x)\, f(x)) \equiv (\Sigma y)(f(y) \,\&\, (\pi x)(f(x) \supset .\ x=y))$$
$$\Diamond((\iota x)\, f(x)) \equiv (\Sigma ! y)(f(y))$$
$$E((\iota x)\, f(x)) \equiv (\exists ! y)\, f(y)$$
$$\Diamond((\iota x)\, f(x)) \supset f((\iota x)\, f(x))$$
$$\Diamond(y) \supset .\ y = (\iota x)(x=y)$$
$$(\pi y)(y = (\iota x)(x=y)).$$

From **VR6** and **VR7** it follows, choosing y not free in $A(x)$ and $B(x)$:

18. Compare T. C. Lambert *op. cit.*

$$\alpha((\pi y)(y = (\iota x)A(x) \supset . B(y) \equiv B((\iota x)A(x)))$$
$$\alpha((\pi y)((B(\iota x)A(x)) \,\&\, y = (\iota x)A(x) \supset . B(y) \,\&\, A(y) \,\&\, (\pi x)(A(x) \supset . x{=}y)))$$
$$\alpha(B(\iota x)A(x) \supset . (\Sigma y)(y = (\iota x)A(x)) \supset (\Sigma y)(B(y) \,\&\, A(y) \,\&\, (\pi x)(A(x) \supset . x{=}y)))$$
$$\alpha(\Diamond((\iota x)A(x)) \supset . B((\iota x)A(x)) \supset (\Sigma y)((\pi x)(A(x) \equiv . x{=}y) \,\&\, B(y)))$$
$$\alpha((\pi y)(B(y) \,\&\, A(y) \,\&\, (\pi x)(A(x) \supset . (x{=}y) \supset . B((\iota x)A(x)))$$
$$\alpha((\Sigma y)((\pi x)(A(x) \equiv . y{=}x) \,\&\, B(y)) \supset . B((\iota x)A(x))$$

Thus the main equivalences of the theory of descriptions in =**R*** holds in =**VR*** under the condition $\Diamond(\iota x)A(x))$, provided the free variables are $\Diamond$-qualified. In particular, then:

$$(\exists ! x)A(x) \equiv E((\iota x)A(x)$$
$$(\exists ! x)E(x) \equiv E((\iota x)E(x))$$
$$\Diamond((\iota x)E(x)) \supset (\exists ! x)E(x)$$ [16]

The theory of descriptions constructed for =**VR*** works tolerably well for all consistent descriptions. It does not work so pleasingly when applied to inconsistent descriptions. This is the fault not so much of the theory of descriptions as of the quantifier-free logic of =**VR*** which, because it admits predicate negation only as sentence negation, excludes such results as:

$$f((\iota x)(f(x) \,\&\, \bar{f}(x))) \,\&\, \bar{f}((\iota x)(f(x) \,\&\, \bar{f}(x))).$$

The law of contradiction $[\sim(f(x) \,\&\, \bar{f}(x))]$ fails for inconsistent constants and descriptions.

So far Russell's case for introducing descriptions through contextual definitions has not been disputed. But Russell's case has been undermined by more recent work on descriptions. The Russellian approach depends on a sharp distinction between logically proper names and descriptions. Since definite descriptions have been admitted in the interpretation as terms on a par with both logical and non-logical proper names, Russell's distinction has been virtually abandoned. Descriptions are introduced by definitions chiefly for reasons of economy. It would accord equally well with the approach adopted here to introduce descriptions into **R*** by treating 'ι' along with '=' as a primitive and adding further postulates.

Improvements can be grafted onto the theory of descriptions outlined for =**R***. For the theory presented so far is incomplete in an important respect, namely with regard to existential import of definite descriptions. The following points have to be taken account of. Definite descriptions which do not carry existential import are often used; e.g. descriptions occurring in nonextensional sentence contexts, descriptions coupled with ontological predicates such as 'exists' and 'is impossible' and descriptions employed in fictional contexts. But very often when people make statements using 'the . . . ', 'some . . . ' and 'all . . . ' they presuppose or imply, even though they generally do not assert, that the items they refer to exist. These points cannot be adequately represented in classical logic. In classical logic furthermore there is a marked but scarcely vindicated differentiation between the existential import of 'the . . ' and 'some . . ' and that of 'all . . '. Notice, however, that it is in the general consistent with '$(\Sigma x)\, f(x)$', '$(\pi x)\, f(x)$' and '$(\iota x)\, f(x)$' to add on either '$\& E(x)$' or '$\& \sim E(x)$' though not both. Therefore the way for a more satisfactory formal treatment of existential import is clear: the way is open to adjoint further conditions which relate (the making of) 'all-' 'some-' and 'the-' statements to existence statements. Two ways of endeavouring to represent better the regular uses of definite and indefinite referring expressions are investigated.

The first way starts with the introduction of the symbol '$\lrcorner$' read 'yields the statement' or 'is used to make the statement [ ]'[19]. Two valued logic is restricted in its application to expressions filling statement brackets. The following condition is imposed:

$$[(\Sigma r)\,.\,\mathbf{qu}(p) \lrcorner [r]] \rightarrow [\mathbf{K}p(q)],$$

where, in the relevant cases, qu(q) is an existence sentence, i.e. one in which 'E' or '$\exists$' occurs essentially, and '$\mathbf{K}p$' is a function from sentences which depends on $\mathbf{qu}(p)$. An important case under the condition is

$$(D2)\colon\ [\mathbf{qu}((\iota x)(f(x), g(x)) \lrcorner [(\iota x)\, f(x), g(x))]] \rightarrow [\mathbf{K}g(\exists x)\, f(x)].$$

The theory of descriptions of the first way is given by ($D1$) and ($D2$) together with a more detailed specification of '$\mathbf{K}g$'. If qu(g) is a non-compound extensional predicate $\mathbf{K}g((\exists x)\, f(x)) = (\exists x)\, f(x)$. Thus if $[(\exists x)\, f(x)]$ is false (in the relevant context) $\mathbf{qu}((\iota x)(f(x), g(x)))$ does not yield the relevant statement. If $\mathbf{qu}(g)$, for a given value of the argument, is a propositional attitude or belief-weighted predicate like 'sought by a', 'believed by a' or 'thought by a' or an ontological predicate then $\mathbf{K}g((\exists x)\, f(x)) = \not{t}$ - though for some propositional attitude predicates a case can be made out for either $\mathbf{K}g((\exists x)\, f(x) = Ba((\exists x)\, f(x))$ or $\mathbf{K}g((\exists x)\, f(x)) = \sim Ba(\sim(\exists x)\, f(x))$. And so on. By introducing the function '$\mathbf{K}$' some deficiencies of recent theories about descriptions are removed. The current account approximates to Frege's natural theory and to Strawson's earlier theory. Other difficulties such as those presented by lying can be evaded by appropriately modifying the usual definition of *lying*. The symbol '$\rightarrowtail$' read 'use of . . . referentially implies' or 'presupposes', which separates off one of the uses of 'imply,' can be defined:

$$\mathbf{qu}(p) \rightarrowtail [\mathbf{K}p(q)] \equiv_{Df} [(\Sigma r)\,.\,\mathbf{qu}(p) \lrcorner [r]] \rightarrow [\mathbf{K}p(q)]$$

e.g. use of 'The king of France likes his beer cold' referentially implies that there is a king of France iff that the sentence yields a statement strictly implies that there exists a king of France. To elaborate these points the underlying three-valued logic, with third value: statement-incapable, and containing statement-brackets, '$\mathbf{qu}$' and '$\lrcorner$' would have to be presented.

The second way explicitly adds existential clauses to descriptions in

19. On the quotation-function 'qu' and statement-brackets see L. Goddard and R. Routley *op. cit.*

cases where import is indicated by the context of occurrence of the description. This existential import is indicated in many usual contexts. The more familiar description operator 'ι^E', where the superscript 'E' displays the existential loading, can then be defined:

$(D3)$: $(\iota^E X)A(X) \equiv_{Df} (\iota X)(A(X) \;\&\; E(X))$.

The description theory of the second way is given by $(D1)$ and $(D3)$ together with a more detailed specification of the contexts in which 'ι' and 'ι^E' respectively appear. This specification will parallel the specification of '$\mathsf{K}g$'.

On ontological commitment. If systems like some of those introduced are viable Quine's related criteria for ontological commitment of a theory[20]— (i) to be is to be the value of a bound variable, and (ii) entities of a given sort are assumed by a theory if and only if some of them must be counted among the values of the variables in order that the statements affirmed in the theory be true—are defective without repairs. For if a is a value of a variable bound by a Σ-quantifier or even of a variable bound by a $\forall$-quantifier it does not, in general, follow that a exists. Such criteria are only correct under the special condition that relevant domains or values of variables are non-empty, i.e. are so restricted as to include existent items; but in such an event the criteria are quite circular. In correcting (i) it may be replaced by (iii), to exist is to be the value of a variable bound by '$\exists$'. That (iii) is correct, if not very valuable in testing a theory for ontological commitment, can be shown by exemplifying it in (iv), a exists if and only if a is the value of a variable bound by '$\exists$'. For (iv) amounts to the theorem: $E(a) \equiv (\exists x)(x = a)$. Quine's earlier thesis to the effect that the only way a theorist commits himself ontologically is by use of existential generalization also directly reflects a theorem, viz. $[E(x) \supset . f(x) \supset (\exists x) f(x)]$, i.e. existential generalization holds only if the item generalized upon exists.

Other criteria are assimilated with (iii) under (i); in particular (v), to be possible (in one sense) is to be the value of a variable bound by 'Σ' (under the intended interpretation of 'Σ'). Criterion (ii) is also unspecific as to exactly what is assumed about the items postulated. For instance, in order that the statement $[(\Sigma x){\sim}E(x)]$ of R_1* be true it is not assumed that there exist entities which satisfy the statements, i.e. which don't *exist*, but only that some (possible) items do not exist or that not all possible items actually exist.

Finally there is no need for a theorist using extended predicate logics or set theories with quantifiers 'π' and 'Σ' to commit himself to the existence of any properties, classes or numbers. Such logics may be ontologically neutral - at least as far as what exists goes. Since the

20. See W. Quine *Word and Object* Cambridge Mass. (1960), 242-3; *Point of View* Revised edn. Cambridge Mass. (1961), 5-14, 103-107, 130-131; 'On Universals' *J.S.L.* 12 (1947), 74.

expression 'ontological commitment' is far from unambiguous, it should be emphasized that one who speaks of possible items does not thereby necessarily commit himself to various beings or entities. Not all possible items exist. One can convincingly argue that possible non-existent items are necessarily not entities and that they lack being.

The University of New England
Armidale, New South Wales, Australia