Notre Dame Journal of Formal Logic
Volume VII, Number 1, January 1966

PROPOSITIONAL CONNECTIVES, SUPPOSITION, AND CONSEQUENCE IN PAUL OF PERGOLA

IVAN BOH

Studying the rules of those consequentiae the validity of which depends on the internal structure of propositions, one quite often comes across Paul's remarks on various suppositions of terms; an understanding of his supposition theory is therefore essential for understanding such rules. However, reading of the tract on supposition itself presupposes a grasp* (a) of propositional connectives 'or', 'and', and 'if'; (b) of modal determinations; and (c) of the validity-conditions of consequence in general; and (d) of some specific rules of consequence.

The propositional connectives mentioned under (a) have been discussed by Paul in his *Summulae*,[1] a tract which treats of the topics of the *logica vetus* and the *logica nova;* and the modal functors were at least enumerated in the same tract.[2] His discussion of consequence and of the rules of consequences, however, follow the tract on supposition and should not have been used at this stage. But Paul can be excused from the charge of assuming a knowledge of what he will discuss later on the ground that even the *logica nova* should give the reader an idea of 'logically following from' and of the validity conditions for an inference.

I. Propositional Connectives. Having defined conditional proposition as an expression in which several propositions are conjoined by the conditional sign[3] (*Cpq*, where the functor '*C*' is to be understood not truth-functionally but ambiguously, just as the ordinary 'if') and having made

---

*Paul of Pergola was born in the end of the 14th or the beginning of the 15th century, probably in Italy, and died in 1451. He studied under Paul of Venice (d. 1429) who lectured at the Universities of Padua and Siena. Paul of Pergola was a successful teacher of philosophy; although a member of the order of Augustinian Hermites, he was a publicly paid lecturer in Venice. His numerous works include *Logica,* found in at least 10 mss. and 8 printed editions; *De sensu composito et diviso; Dubia super consequentiis Strodi*; and his commentaries on the insolubles and the sophismata of Heytesbury. Cf. the introduction to his *Logica* by the editor Sister Mary Anthony Brown, ofm, pp. v-xiii, published by The Franciscan Institute, St. Bonaventure, N.Y., 1961.

Received March 23, 1964

clear what he means by negative conditional (*NCpq*, and not for example *CNpq*), Paul points out the necessary and the sufficient conditions for truth and falsity of such compounds. 'A conditional', he says 'is true when the contradictory of the consequent is repugnant to the antecedent'[4] and 'For truth of a conditional it is required that the opposite of the consequent does not stand with the antecedent'.[5] He does not explicate the notion of 'being repugnant to', but it is certain that it does not mean merely 'is not true together with', because he concludes his remarks on conditionals with the statement that any true conditional is necessary and any false conditional impossible and none which would be contingent.[6] A partial symbolic representation of the metatheorem in question might then be stated with the help of the modal functor:

*1.01 T'*Cpq*' if and only if *NMKpNq*

The corresponding metatheorem for falsity is

*1.02 F '*Cpq*' if and only if *MKpNq*

Conjunction he defines as an expression in which several propositions are conjoined by the sign of conjunction 'and' (*Kpq*).[7] Again he distinguishes between affirmative (*Kpq*) and negative conjunctions (*NKpq* and not, e.g. *KNpNq*). The truth and falsity conditions for conjunction can be stated as follows:

*1.03 T '*Kpq*' iff T '*p*' and T '*q*'
*1.04 F '*Kpq*' iff F '*p*' or F '*q*'

Paul considers 'true' and 'false' as the fifth and the sixth modes which proposition may have. He is also concerned with the other four modal determinations of conjunction. 'For possibility of conjunction it is required that each part be possible and that it not be repugnant to another part or be incompossible with it'.[8] Similarly for impossibility, necessity, and contingency:

*1.05 If *MKpq*, then *Mp* and *Mq* and *NCpNq*
*1.06 If *NMp* or *NMq*, then *NMKpq*
*1.07 If *NMNKpq*, then *NMNp* and *NMNq*
*1.08 If *MNp* or *MNq*, then *MNKpq*

Unlike for conditionals, Paul has stated here merely the necessary conditions for possibility and necessity and the sufficient conditions for impossibility and contingency.

Disjunction is defined as 'an expression in which several propositions are conjoined by the sign of disjunction'.[9] From Paul's specification of truth and falsity conditions it is obvious that an inclusive disjunction is meant. 'For truth of a disjunction it suffices that one part be true . . . . , and for falsity it is required that each part be false';[10]

*1.09 If T'*p*' or T'*q*', then T '*Apq*'
*1.10 If F'*Apq*' then F'*p*' and F'*q*'

In addition, we have a characterization of modal determinations of disjunctions:

*1.11 If *Mp* or *Mq*, then *MApq*
*1.12 If *NMApq*, then *NMp* and *NMq*
*1.13 If *NMNp* or *NMNq* or *EpNq*, then *NMNApq*
*1.14 If *MNApq*, then *MNp* and *MNq* and *NNMKpq* and *NNMKpNq*

The last of these,[11] if it has correctly depicted Paul's intention, entails that if we admit that a disjunctive proposition is contingent, we must also admit the existence of two propositions which are both consistent and independent. The "existence postulate" is here at least foreshadowed, if not stated as such. Paul adds three statements concerning the relationship between disjunctions and conjunctions and between the various modals. The first expresses a form of duality-principle: 'Conjunction and disjunction with contradictory components contradict each other'.[12] (Other forms of DeMorgan's principles are expressed by *1.04 and *1.10) In the object language:

*1.15 *EKpqNANpNq*
*1.16 *EApqNKNpNq*

The relations among modals are conceived as follows:

*1.17 *CEpNqENMNpNMq*
*1.18 *CEpNqEMNpMNNq*

These laws reflect the commonly accepted view of the medievals that if a proposition is of natural matter[13], its denial will be in the remote matter; and if a proposition is of contingent matter (where a predicate can be predicated of the subject either affirmatively or negatively) its denial is also not self-contradictory and is of the same matter.

Paul has thus given the reader a sufficient eclucidation of the propositional connectives to understand the examples of various types of descent in the supposition theory. As for modal determinations, he leaves the reader very much to his intuitions and to his grasp of the semantic relations of terms. A statement is necessary not because the universal quantifier precedes it, but because it predicates an essential character or a necessary property of the subject. 'Man is mortal' is just as universally true and necessary as 'Every man is mortal'. As a matter of fact, the latter, if intended to be historically limited to the existing men[14] will cease to be necessary, since the existence of individual man is merely contingent.

The minimum knowledge of *consequentia* which Paul presupposes at this state is (a) that it is an inference (*illatio*) which is invalidated as soon as the antecedent is true and the consequent false, or the antecedent necessary and the consequent contingent or the antecedent possible and the consequent impossible; and (b) that it may be valid on material grounds, i.e. because the antecedent is impossible or the consequent necessary.

II. *Supposition Theory.* Like many of his predecessors, Paul too explicated the various suppositions which terms may have in propositions[15] by specifying the various types of reduction to singulars. It is not relevant to the purposes of this paper to go into detail; what is needed is an elucidation of the notion of the three basic types of descent and of the manner in which some common syncategorematics exhibit their force.

The three types of descent to singulars admitted by Paul are the following: the conjunctive, the disjunctive (*disiunctivus*), and the disjunct (*disiunctus*) descent. One may make a conjunctive descent from a term if that term has distributive supposition. Take for example the subject term of universal affirmative propositions, or the subject or predicate terms of universal negative propositions; they are so used as to refer to each individual which they have been instituted to signify. If we should limit the universe to two individuals, the proposition 'Every man is running', for instance, would refer to both of them, so that both 'this is running' and 'that is running' would be true.

Let the form of any *de inesse* proposition be represented by the schema '*Xab*' and the four standard types of the square by '*Aab*', '*Eab*', '*Iab*', and '*Oab*'. Let the subscripts indicate the ostensive use of terms to which they are attached, e.g. 'Xa_1b' for '*Hoc a est b*'. The descent from the subject of an *A* proposition can then be exhibited by the schema[16]:

*2.01 $Aab \therefore Xa_1b \ \& \ Xa_2b$

The descent demanded is conjunctive and '*a*' is said to have confused distributive supposition. Generally, the term 'ϕ' has distributed supposition if all the values of 'ϕx' which make 'ϕx' true are referred to by the statement in which it occurs. In an *E* proposition both '*a*' and '*b*' have such supposition and the following schemata of descent are valid[17]:

*2.02 $Eab \therefore NXa_1b \ \& \ NXa_2b$
*2.03 $Eab \therefore NXa_1b_1 \ \& \ NXa_1b_2 \ \& \ Na_2b_1 \ \& \ Na_2b_2$

Similarly we can exhibit the patterns for exclusive propositions in which the predicate term has confused distributive supposition:

*2.04 Only $Xab \therefore Xab_1 \ \& \ Xab_2$

which is equivalent to[18]

*2.05 $Aba \therefore Xb_1a \ \& \ Xb_2a$

On the other hand, the descent which is permitted under the subject of an *I* proposition is disjunctive rather than conjunctive. It is said to have determinate supposition. Generally, a term 'ϕ' is said to have determinate supposition if at least one of the values of 'ϕx' which make 'ϕx' true is referred to by the statement in which it occurs. The following schema of descent is valid:

*2.06 $Iab \therefore Xa_1b$ *or* Xa_2b

The predicate term of an *I* proposition also has determinate supposition:

*2.07 *Iab* .·. Xab_1 *or* Xab_2
*2.08 *Iab* .·. Xa_1b_1 *or* Xa_1b_2 *or* Xa_2b_1 *or* Xa_2b_2

Since the medievals distinguished indefinite propositions from those which are quantified, the question arises as to what kind of supposition *its* terms have. Two cases come to mind, the examples of which might be: 'Man is running' and 'Man is an animal'. The latter is often given as an example of scientific law[19] and could therefore not be counted as an indefinite proposition; the former would, even if it should happen to be the case that every man is running. It is the semantically undeterminate propositions (propositions of contingent matter) that we must have in mind when we state the rule of supposition of terms of indefinite propositions. This rule is identical with that for *I* propositions (The case is parallel for *O* and the indefinite negative proposition).

Observing **2.01* and **2.06* makes it obvious why the inference '*Aab* .·. *Iab*' would be admitted as valid in Paul's system. But while the question of the relationship between universal and particular propositions is thus settled, the proper analysis of universal propositions is not.

There is a third kind of descent, called the disjunct descent,[20] which is reducible, however, to the determinate supposition and is required only in the first stage of making a descent under the predicate term of an *A* proposition. Take 'Every man is an animal'; it does not follow from it: 'Every man is this animal or every man is that animal'. What follows, rather, is this: 'Every man is either-this-or-that animal', since this disjunct predicate is true of each individual man. We can exhibit such a descent by the following schemata:

*2.09 *Aab* .·. $X(a_1\&a_2)$ (b_1 *or* b_2)
2.10 *Aab* .·. Xa_1(b_1 *or* b_2) & Xa_2(b_1 *or* b_2) (by the principle of distribution)
2.11 *Aab* .·. (Xa_1b_1 *or* Xa_1b_2) & (Xa_2b_1 *or* Xa_2b_2) (by the principle of distribution)

Sometimes a term has confused distributive supposition but no descent is allowed. This happens in three cases[21], all of them very relevant to understanding of certain consequential rules:

(a) with the subject of an exceptive proposition, eg. 'Every *man* except Socrates is running';
(b) with the subject of a modal proposition in the composite sense, e.g. 'Necessarily every *man* is an animal';
(c) with the subjects of both the antecedent and the consequent of a conditional, e.g. 'If every *man* is running, every *man* is running'.

Also, a term may have merely confused supposition, yet no descent is allowed.[22] This, too, happens in three cases:

(d) with both the undistributed subject and predicate of a modal

proposition in the composite sense, e.g. 'Necessarily *man* is an *animal*'.
(e) with the subject and predicate terms of antecedent and consequent of a conditional, e.g. 'If a *man* is not, *man* is not'.
(f) with the predicate of a reduplicative proposition, e.g. 'Man as man is an animal'.

Paul calls these suppositions immobile. His discussion of them shows that the supposition theory had a function to deal with the problems of proper inferences involving analyzed propositions, in addition to other possible functions such as that of clarifying the reference of general terms in various types of propositions, or that of describing the necessary conditions for the truth of predicative propositions.[23] Take (a) '$(x)\,[(Hx \cdot x \neq s) \supset Rx]$'; if a descent is attempted, the result will be an improper exceptive proposition, e.g. 'Plato except Socrates is running'. Take (b): '*LAab*'; no specification by subscripts is allowed because the implicans would be true and the implicate 'LAa_1b' false. The example of (c) might have seemed more puzzling to Paul's students. As it stands, it is a logically true sentence, being a substitution instance on *Cpp*, and if someone should try to make a descent under both the antecedent and the consequent in such a way as to preserve the identify of the individual, e.g. 'CXa_1bXa_1b', the inference would be from tautology to tautology. Paul's point, however, is that there is no reason why this identity should be preserved; as far as the application of the *de omni* rule goes, one could just as well infer 'CXa_1bXa_2b', and this is a false conditional for someone who, like Paul, held every conditional to be necessary (cf. **1.01* above)

Similar points of quantificational logic are supposed to be brought out by the remaining three rules of immobilization: (d) is clear: (e), however, does not make sense to someone who considers sentences of the form '$Ex(x)$' equivalent to '$(\exists y)(y=y)$' and consequently tautologous ('*Ex*' for the denotative term existence), and their denials self-contradictory. But let the reader judge Paul's illustration:

> 'If man is not, then man is not'; the term 'man' has merely confused immobile supposition, because it does not follow: 'If man is not, man is not, and these are all the men there are, therefore if this man is not, man is not', because the antecedent is possible and the consequent impossible - being a conditional whose implicans is possible and the implicate impossible.[24]

One could argue for a different appraisal of this inference. Since 'This man is not' - the implicans of the consequent - is self-contradictory, and since from the impossible anything follows (materially for Paul),[25] the consequent is always true regardless of the implicate's truth-value or modal status. Now since the antecedent ('If man is not, man is not, and these are all the men there are') is proclaimed to be possible and since the consequent is always true, the exclusion of such inferences as the example in question does not seem warranted. In order to substantiate his claim that the undistributed general terms in conditionals have merely confused *immobile* supposition and that no descent is allowed Paul should have used as illustrations ordinary predicative (*de inesse*) propositions, and not propositions *de secundo adiacente.*

Example in (*f*) is adequate. According to Paul's pattern,[26] 'Man as man is an animal' is explicated by the following three propositions: (i) 'Man is a man', (ii) 'Man is an animal', (iii) 'If something is a man, that same thing is an animal'. If we take the conjunction of the three as the antecedent and attempt to make a descent to 'Man as man is this-or-that animal' (as we wound under the predicate of an *A* proposition), we get a false consequent which could not possibly follow from true premisses.

III. *Rules of Consequences, Part A.* The numerous consequential rules listed by Paul can be divided into two sub-classes: (a) those that govern relations among unanalyzed propositions, and (b) those that govern relations among analyzed propositions. This paper is concerned more with the latter than with the former; but because of the close dependence of term-logic on propositional logic and because of the inherent historical interest, a collection of rules of propositional logic is given in this section. Since Paul considers *consequentia* to be an illation or an inference rather than a conditional proposition, the horizontal arrow '→' will have the same logical force as Paul's signs *'ergo'* and *'igitur'*.

(To make sure that the method of presenting the rules of consequences listed below will not be misunderstood, a brief explanation is given. 'If', '&', and 'or' belong to the metalanguage; they are not to be understood as truth-functional connectives on the same level as *N, K, A,* etc. Letters *p*, *q, r* are schematic letters for *quoted* expressions. *3.01 - *3.17 are to be thus understood as compact statements of rules and not as logically true conditionals. If thellatter were desired, one could render, e.g. *3.01 as *CCNqNpCpq*. The unstarred rules are explicitly rejected.)[27]

*3.01 If $Nq \to Np$, then $p \to q$
*3.011 If $Nq \to Np$ is not valid, then $p \to q$ is not valid.
*3.02 If p is true and $p \to q$ is valid, q is true
*3.021 If q is false and $p \to q$ is valid, p is false
*3.022 If p is true and q false, then $p \to q$ is not valid.
*3.03 If Lp and $p \to q$ is valid, then Lq
*3.031 If NLq and $p \to q$ is valid, then NLp
*3.032 If Lp and NLq, then $p \to q$ is invalid
*3.04 If Mp and $p \to q$ is valid, then Mq
*3.041 If NMq and $p \to q$ is valid, then NMp
*3.042 If Mp and NMq, then $p \to q$ is invalid
*3.05 If $p \to q$ is valid and $q \to r$, then $p \to r$
*3.051 If $q \to r$ and $p \to q$, then $p \to r$
*3.052 If $p \to r$ and $q \to r$ and $r \to s$, then $p \to s$
*3.06 If $p \to q$ is valid and Kpr is the case, then Kqr is also the case
*3.061 If $p \to q$ is valid and $NKqr$, then $NKpr$

It may be that *'stat cum'* and *'repugnant'* should be understood as modal notions, the former as *MKpq* and the latter as *NMKpq* and that the rules

**3.06* and **3.061* should be revised accordingly. In this case we would have their analogues in Lewis & Langford theses 17.3: $p ⥽ q \cdot por : ⥽ \cdot qor$ and 17.31: $p ⥽ q \cdot \sim (por) : ⥽ \cdot \sim (por)$, the first of which is weaker than $p ⥽ q \cdot pr \cdot ⥽ \cdot qr$ and the second of which is stronger than $p ⥽ q \cdot \sim (qr) \cdot ⥽ \cdot \sim (pr)$.

The next two rules and their derivatives employ the notion of epistemic modality, 'known (to be true or valid)' and of the quasi-deontic modalities 'to be conceded', 'to be negated'. This shows that Paul conceived his general rules of consequences not only as prior to the rules for analyzed propositions but also as regulative of the obligation-procedures, a subject which he treats in the tract immediately following the tract on consequences. He observes, for example, that even the rustics concede 'The moon is eclipsed', but do not concede 'The earth is interposed between the sun and the moon', because the inferential connection from the first to the second is not *known by them* to hold.

*3.07 If $p \rightarrow q$ is valid and $p \rightarrow q$ is known to be valid and p is to be conceded, q is to be conceded

*3.071 If q is to be negated and $p \rightarrow q$ is known to be valid, p is to be negated

*3.072 If p is to be conceded, but q is to be negated, then either $p \rightarrow q$ is not valid or now known to be valid

*3.08 If $p \rightarrow q$ is known to be valid, and the p is known, q is known

*3.081 If q is not known and $p \rightarrow q$ is known to be valid, p is not known

*3.082 If p is known and q not known, then either $p \rightarrow q$ is not valid or not known to be valid

Although Paul continues at this point with the enumeration of rules of term-logic and only in the last section of the tract on consequences discusses rules for consequences which employ hypothetical (conjunctive, disjunctive, conditional) propositions, these latter obviously go together with the rules of unanalyzed propositions. It is very likely that the order of discussion has been imposed on Paul by the tradition.

The following propositional rules are stated:[28]

*3.09 If *Kpq*, then p

*3.091 If *Kpq*, then q.

3.092 If q, then *Kpq*, and

3.093 If q, then *Kpq*

are rejected, unless one of the components implies the other, as for example, 'You are running, therefore you are running and moving'. He also rejects

3.094 If *NKpq*, then p, and

3.095 If *NKpq*, then q,

but again admits inferences from *Apq* to q if it is the case that *Cpq;* e.g. 'You are running or you are moving, therefore you are moving'. Implicit in the assertion of the validity of this rule is the apprehension that *ApNp* is a logical truth and that whichever alternative is the case, q will follow either by *modus ponens* (*3.14) or by disjunctive syllogism (*3.11). Paul states next the rules of consequences which employ weak disjunctions:[29]

*3.10 If *p*, then *Apq*
*3.101 If *q*, then *Apq*
3.102 If *Apq* then *p* is explicitly rejected.
*3.11 If *Apq* and *Np*, then *q*
*3.12 If *NKpq*, then *ANpNq*
*3.121 If *ANpNq*, then *NKpq*
*3.13 If *NApq*, then *KNpNq*
*3.131 If *KNpNq*, then *NApq*

The rules of consequences which employ conditionals are given last:

*3.14 If *Cpq* and *p*, then *q*
*3.15 If *Cpq* and *Nq*, then *Np*

Note that if the last two rules were formulated in the object language as theses (*CKCpqpq* and *CKCpqNqNp*), they would be indistinguishable, apart from the commuted conjunctive antecedents, from **3.02* and **3.021.* Professor Otto Bird once suggested to me that it would be preferable, if the object language is to be used for representing the rules, to use a different symbol for the functor of an indeterminate implicative relation, e.g. *I* of Lewis. In this case, the theses corresponding to **3.02* and **3.14* would be *IKpIpqq* and *IKCpqpq* respectively, showing that the principle of conditionalization has been applied twice in the former and only once in the latter case, namely in the positions of the functor *I*. However, since the asserted occurrence of *C* in a logically true theses is always understood to be a result of at least implicit conditionalization, the presence of *I* in the places of conditionalization in subordinate places would suffice to distinguish **3.02* from **3.14;* for we would then have *CKpIpqq* and *CKCpqpq* respectively.

Paul also admits the inference from a conditional to a disjunction[30]

*3.16 If *Cpq*, then *ANpq*,

but he says that such an inference is valid only materially. What he means when he says this may be seen from the following considerations. On the assumption that *Cpq* is a true statement *KpNq* is false, being its contradictory. Now since by **1.17,* if one of a pair of contradictories is impossible, the other is necessary, and since *KpNq* is impossible (being the negation of a valid conditional which for Paul is always necessary), *ANpq* is necessary. But a necessary proposition is implied by any other according to the second rule of material consequence (*"Necessarium sequitur ad quodlibet"*[31]) and as it is also implied by *Cpq*. The final rule of this tract is:

*3.17 If *NCpq*, then *AKpAqMqKMpAqMq*

There is a striking similarity between this rule and **1.02.* The example of such a consequence given by Paul is: 'It is not the case that if you are a man you are awake, therefore although you be (*quamvis tu sis*) a man it is or could be the case that you are not awake'.[32] One might be tempted to represent the consequent by *KpANqMNq;* but this would be a mistake, since then one could infer from the denial of a necessary conditional its antecedent in a non-modal form. What ought to be possible to deduce from NCpq is this:

(a)	*MKpNq*	(by **1.02*)
(b)	*KKMpNqNCpNNq*	(from (a) by **1.05*)
(c)	*Mp*	
(d)	*MNq*	(from (b) by **3.09* and **3.091*)
(e)	*NCpNNq*	

(*c*) and (*d*) are in fact the consequents of the theses *19.76* and *19.77* respectively of Lewis S_2; (*e*) is equivalent to the antecedent itself, if the implicit rule of double negation can be assumed in Paul's system.

To deduce *KpANqMNq* from *NCpq* is in fact not warranted by Paul's phrasing of the rule: he uses the present subjunctive 'although you be' and not 'although you are'. The adversative proposition which we are allowed to infer must be a conjunction of *Mp* or, even better, of *ApMp* with *ANqMNq*. The assertoric parts of the disjunctions would account for the effect that the adversative functor 'although' has on the inferred proposition. Whereas **1.02* allows an inference to the weakest proposition which constitutes the necessary condition for the falsity of a conditional, **3.16* makes explicit that the denial of *Cpq* may be made when one of the several conditions is satisfied. For *KApMpANqMNq* can be distributed into *AA(KpNq)(KpMNq)A(KMpNq)(KMpNq)*.

IV. *Rules of Consequences, Part B.* Keeping in mind the above views on supposition of terms and on general consequential relations among propositions we can better understand Paul's phrasing of some special rules belonging to quantification theory (extended so as to include in some cases modal notions). Consider the following:

> There is a valid inference from an inferior to its superior affirmatively and without distribution sign, provided no confounding sign is impeding it. E.g. 'A man is running, therefore an animal is running'.[33]

Neither 'man' nor its logical superior 'animal' is preceded by the sign of universality; the two propositions constituting the inference are affirmative; and no confounding sign such as those mentioned in part II (e.g. 'necessarily') is present in either the premiss or the conclusion. The fact that 'animal' is a logical superior of 'man' will, of course, have to be stated explicitly. Symbolic rendition might then be given as

*4.01 If *Aab*, then if *Iac*, then *Ibc*

Apart from the question of the analysis of an *A* proposition, the rule has obviously an analogue in the lower functional calculus, namely $CC\Pi xC\phi x\psi xC\Sigma xK\phi x\theta x\Sigma xK\psi x\theta x$, which is a logically true expression. Although the fact that the latter is logically true is certifiable deductively and without any direct appeal to the supposition of terms, the distribution sign, the absence of confounding signs, etc., it must not be forgotten that even the elementary quantification theory had to settle these questions, in another and more compact way, by laying down the appropriate formation and transformation rules, and by making the appropriate restrictions on these rules. The notions of free and bound occurrences, however, are needed only because moderns work within the framework of an ideal and possibly uninterpreted "language," and not because of any special need of the natural language. The fact that medievals did not employ these notions in their quantificational logic is thus, of itself, not to be considered as indicative of any defect in their analysis.

Pergola's system also includes rules which *reject* various types of inference. One such rules is: "The inference from an inferior to its superior distributively is invalid; e.g. 'Every man is running, therefore every animal is running' ".[34]

4.02 If *Aab*, then if *Aac* then *Abc*

is thus rejected.

A third rule permits an inference from an inferior to its superior, provided the terms have merely confused *mobile* supposition; but if the terms are immobilized, the inference is not permitted.[35]

*4.03 If *Abc*, then if *Aab*, then *Aac*
4.04 If *Aab*, then if $C(x\epsilon a)(x\epsilon c)$, then $C(x\epsilon b)(x\epsilon c)$

The last inference pattern is invalid because the terms *a* and *c* and *b* and *c* in the two conditionals have, as was observed in part II, immobilized supposition. Paul could also have said that such inference is invalid because it assumes that whatever is implied by the antecedent is implied by the consequent, but he does not have the latter rejection rule recorded among his principal propositional rules; he does have, however, the corresponding valid rules **3.05* and **3.051* which could be invoked to validate the correctness of **4.03* without any reference to the supposition of terms. But such validation of **4.03* and the rejection of *4.04* would involve an implicit recognition of the transitivity (and possibly of asymmetry) of the *A* functor.

A rule may also be rejected for confusing *de dicto* and *de re* modalities (or for confusing the composite and the divisive sense of propositions).[36] Thus the inference from 'Necessarily man is an animal' and 'Socrates is a man' to 'Necessarily Socrates is an animal' is rejected as invalid. The first premiss could be represented by $L\Pi xC\phi x\psi x$ and the second premiss by ϕa; both are true, but the conclusion, $L\psi a$, is false. The fact that a middle is added, namely ϕa, makes the conjunction of the premisses contingent, but not false, and so the definitely false conclusion cannot be implied by it. On the other hand, if the first premiss were to be understood in the divisive sense, i.e. as $\Pi xC\phi xL\psi x$, the conclusion would readily follow. Paul's reason for rejecting such inference is that it makes a descent under the term which has merely confused immobile supposition.

Another rule permits interference from an inferior to its superior with the negation added to the copula, provided that we have the "due middle"; e.g. 'Socrates is not running and Socrates is a man, therefore a man is not running'.[37] Schematically:

*4.05 If $x\epsilon a$, then if $N(x\epsilon b)$, then Oab

This rule presupposes the recognition of the important rule which permits inferences from 'ϕy' to '$\Sigma x \phi x$'. Paul does not state it explicitly, although he has the following rule in his system: "*A particulari ad suam indefinitam et e converso, tam affirmative quam negative, est bona consequentia*".[38] If we take an "indefinite" proposition to refer to any arbitrary selected individual, then Paul does have rules corresponding to the existential generalization as well as to existential instantiation. The difficulty with this interpretation is presented by the fact that medievals, including Paul, seem to consider indefinite propositions as equivalent in meaning to particular propositions and accorded to them the same analysis in terms of the descent to singulars.[39] Both 'ϕx' (considered as indefinite *proposition*) and '$\Sigma x \phi x$' would imply, and be implied by, a disjunctive set '$AA\phi a \phi b \phi c$'. The x's in both 'ϕx' and '$\Sigma x \phi x$' are genuine individual variables, whereas the x above the line in $\frac{\phi x}{\therefore\ \Sigma x \phi x}$ cannot be thought of in this way.

The requirement of the "due middle" is designed to prevent at least the following fallacies: (a) the inference from possible to impossible; (b) the inference from necessary to contingent; (c) the inference from true to false; it is also used to insure that the extension of a term which we wish to universally generalize is closed. Take, for example, the following inference: 'Socrates is not an animal, therefore a man is not an animal'. It clearly represents case (a), and it can be prevented by requiring the "middle", namely, 'Socrates is a man'. For then one would either not put forward the inference at all, seeing that the premisses are incompatible, or else put it forward as only materially valid, conforming to the rule "*ex impossibili* . . ." Take another case: 'Every man is running, therefore this (*iste*) man is running'. It appears that for Paul any subject term, even proper names and ostensive uses of general terms, may fail to refer.[40] For this reason it is possible that the above inference have true premiss and a false conclusion (case (c)). It could be validated by the addition of the "middle", 'This (man) is a man'. Of course, Paul should also make it sure that by 'this' the ostension gets at the same object in the "middle" and in the conclusion. It is interesting to observe that inferences with negative propositions do not require any middle.[41] Thus, 'No man is running, therefore this man is not running' is valid as it stands. For the conclusion is true even if the subject term should fail to refer. Again, the inference 'every man is an animal, therefore this man is an animal and this man is an animal, and so on for individuals', needs a middle, because as it stands, the antecedent is necessary and the consequent contingent (case (b)). The required middle is 'These are all the men there are', which makes the conjunctive antecedent contingent.

To prevent inferences from some to all, Paul gives this rule: "There is a valid inference from all the singulars sufficiently enumerated . . . , with the due middle, to the corresponding universal proposition''[42]:

*4.06 ϕx_1 and ϕx_2 . . . and these are all the ϕ's there are, therefore $\Pi x \phi x$

But he adds that this is invalid without the middle.

In connection with several rules, Paul gives examples which involve dyadic predicates, although he does not seem to attribute any importance to this fact. One of the rules is the following: ''There is a valid inference from a superior to its inferior affirmatively and with distribution signs, and with the due middle''.[43] An example to which it applies is: 'You are different from donkey and Brunellus is a donkey, therefore you are different from Brunellus'. If we treat the propositions in which proper terms and personal pronouns occur in the subject position as universal propositions, the syllogism that results when we add the middle would presumably be in the mood Cesare (*Eab, Acb* ∴ *Eac*), since 'differs' or 'is different from' has the same effect on the supposition of terms as 'not'.[44] However if we treat it so, its descriptive character is lost; for a syllogism employing the very same categorematics in the same arrangement but with a different relational sign, e.g. 'is other than', 'is not the same as', 'is in need of', 'is without', etc., would be indistinguishable from one another as well as from a syllogism which employs 'not' instead of the above dyadic predicates. On the other hand, if we should treat 'different from donkey' and 'different from Brunellus' themselves as categorematics of the above syllogism, there would be five terms and consequently no valid inference. Yet, since Paul did recognize the peculiar confounding force of such dyadic predicates, he may have visualized *Cesare* not in its standard form given above but as

*4.06 If $\Pi x C\phi x\psi ax$ and ϕb, then ψab

Another rule involving a relational term is the following: ''There is a valid inference from a proposition in the active voice to the corresponding proposition in the passive voice, and conversely.''[45] Example: 'I love God, therefore God is loved by me'. Both the antecedent and the consequent may be represented by the schema ϕxy, since the inference is permitted in both directions.

Still another rule, an example of which contains a relative term is the following: ''From a term with merely confused or determinate supposition to the same term with distributive supposition is invalid''.[46] E.g. 'You are different from every man, therefore you are different from man'. The term 'man' in the premiss is preceded by the sign of distribution 'every' which makes it have distributive supposition. However, since 'every' is preceded by the verb 'differs from' which has logical properties similar to those of 'not', the term 'man' does not keep distributive supposition but acquires rather determinate supposition. In the conclusion, on the other hand, this same term has distributive supposition, since only the verb 'differs from' with its distributive force without any other sign which would modify it, occurs in it. The rule thus attempts to prevent inferences from '$\Sigma xK\phi x\psi ax$' to '$\Pi xC\phi x\psi ax$'.

Paul also rejects inferences from a term with merely confused supposition to the same term having determinate supposition with respect to the same categorematic sign, and again his example employs a dyadic predicate: 'Every man has a head, therefore a head has every man'.[47] In the premiss, 'head', rather 'has a head' is taken to be a term, and since it occurs in the predicate position of an *A* proposition, it has merely confused supposition. In the conclusion, on the other hand, the same term is the subject of a particular proposition and as such has determinate supposition. While the logical status of the word 'has' is not clear - Paul seems to treat it as a copula - , the rule makes quite effectively the important distinction between '$\Pi x\Sigma y\phi xy$' and '$\Sigma y\Pi x\phi xy$'. Note also that Paul is careful to qualify this rule by the phrase "with respect to the same categorematic sign"; he is not trying to deny the validity of '$Aab \therefore Iba$' (where '*b*' has merely confused supposition in its first occurrence and determinate supposition in its second occurrence); nor of '$\Pi x\Sigma y\phi xy \therefore \Sigma y\Sigma x\phi xy$'; - both of which inferences presuppose that universal propositions have existential import (cf. below **4.07*).

Unlike in modern systems, Paul admits the inference from universal propositions to their subalterns,[48] i.e.

*4.07 If *Aab*, then *Iab*
*4.071 If *Eab*, then *Oab*

An affirmative exclusive proposition is recognized as equivalent to an *A* proposition with the same terms transposed.[49] Let *T=tantum* (only, then:

*4.08 If *Tab*, then *Aba;* If *Aab*, then *Tba*

Analyses of the exclusive proposition are given elsewhere.[50] One rule permits us to draw '*Tab*' from '*Iab*' and '*Eab*', and conversely. But whereas we would permit the inference to '*Tab*' from '*Eab*' alone (i.e. $\Pi xCN\phi xN\psi x \therefore \Pi xC\psi x\phi x$), he does require in addition the statement which secures the existence of *a*'s; only then can we infer an *A* proposition which has existential import and is not a mere denial of certain particular (viz. *O*) proposition.

The inferential force of exclusive propositions is indicated by the following rule "There is a valid inference from an inferior to its superior on part of the subject term preceded by the sign of exclusion":[51]

*4.09 If *Aab*, then if *Tac*, then *Tbc*

But Paul rejects the inference from '*Abc*' and '*Tab*' to '*Tac*' (e.g. 'Only man is running, therefore only man is moving') on the ground that it procedes from the inferior to the corresponding superior with terms having distributive supposition. For by **4.08* '*Tab*' is equivalent to '*Aba*' and '*Tac*' to '*Aca*' and the premisses '*Abc*' and '*Aba*' clearly do not yield '*Aca*'. On the other hand, **4.09* is valid because it procedes from an inferior to the corresponding superior with terms having merely confused mobile supposition (being subjects of exclusive propositions which by **4.08* become predicates of A propositions).

A set of five rules governs inferences involving disparate, convertible, correlative, privative, and infinite terms.

*4.09 If $NK\phi x\psi x$, then if ϕx, then $N\psi x$ ("The inference from the affirmation of one of a pair of disparates to the negation of the other is valid")[52]

The converse inference is rejected on the ground that from a negative proposition never follows an affirmative one:

4.091 If $NK\phi x\psi x$, then if $N\phi x$, then ψx.

Here, again, Paul explicitly admits the possibility that a personal pronoun may fail to refer. The inference in question is: *'Tu non es albus, ergo tu es niger'* on which he comments: *"Stat enim te non esse et sic antecedens esset verum et consequens falsum"*.[53]

*4.10 If $E\phi x\psi x$ then if $\Sigma x K\phi x\theta x$, then $\Sigma x K\psi x\theta x$, and conversely ("From one convertible term to the other the argument is valid")[54]

*4.11 If $\Pi x\Pi y C\phi xy\psi yx$, then if $E!\phi$, then $E!\psi$.

There is a member of a domain iff there is a member of the converse domain or, if there is a referent, there is a relatum. "There is a valid consequence from one correlative to the other with *de secundo adiacente* propositions".[55] The converse inference also holds. Example: 'Double is, therefore half is', Paul rejects similar inferences in which correlatives are employed as predicates in *de tertio adiacente* propositions, as 'The world is double, therefore the workd is half'.

If we render privative terms by ϕ' and infinite terms by $\overline{\phi}$, we can state the remaining rules of this group:

*4.12 If $\Pi x C\phi' x\overline{\psi}x$, then if $\Sigma x K\theta x\phi' x$, then $\Sigma x K\theta x\overline{\psi}x$ ("There is a valid inference from a privative term to the infinite term")[56]

The converse inference is rejected because the applicability of a negative term to a thing does not imply that that thing is in a state of privation. If, for example, a man does not see, it does not follow that he is blind.

*4.13 If $\Pi x C\phi' x N\psi x$ then if $\phi' a$, then $N\psi a$

*4.131 If $\Pi x C\overline{\phi}x N\psi x$, then if $\overline{\phi}a$, then $N\psi a$ ("There is a valid inference from an affirmative proposition with a privative or infinite predicate to a negative proposition with the corresponding positive predicate")[57]

Examples: 'You are blind, therefore you do not see': 'You are non-seeing, therefore you are not seeing'. **1.131* provides the means of translating propositions in which occur infinite terms into propositions in which 'not' occurs only in its syntactic capacity as a propositional functor. The converse inferences are rejected because a negative proposition does not imply an affirmative one. Paul makes an exception to this rule. He says, that with the appropriate middle securing the existence of the subject, the inference from a negative proposition to an affirmative proposition with the infinite predicate does hold; but never without the middle.

As a final example of Paul's concern with logical analysis, the following rule might be cited ('T' for 'true' and 's' for 'adequately signifies'):

*4.14 If *T'p'* and *'p'sp,* then *p*

Example: "This proposition is true: 'God exists', which adequately signifies that God exists, therefore it is true that God exists".[58] The consequence satisfies the rule that one may validly pass from officiating propositions to the officiated one (but not conversely). The use of the accusative-with-infinitive construction (*Deum esse*), as opposed to that of the proposition *'Deus est',* may be taken to indicate that Paul distinguished between proposition and propositional contents (*complexum significabile*), both of which must be distinguished from the corresponding fact. The talk about the objective contents appears to have persisted since Gregor of Rimini (d. 1358) as a rival to other views such as that a (true) proposition refers to a real fact, or that it refers to a mental act. But whichever view was in fact held by Pergola, it plays no significant role in his logic.

NOTES

1. This tract forms the first part of Paul's manual *Logica,* edited by S. M. Anthony Brown, OSF, St. Bonaventure, New York, 1961. All the references to Paul of Pergola are to this work in its modern edition.

   By the subjects of the *logica vetus* are meant such topics as are discussed in *On Interpretation, Categories, Isagoge,* and in Boethius' logical works; the subjects of *logica nova* contain, in addition to the above, the remaining portions of the Organon, and Gilbert de la Porre's *Liber de sex principiis.* The collective name for both, the *logica vetus* and the *logica nova* was *logica antiqua,* which was opposed to *logica moderna;* the latter includes the typically medieval logical topics, in particular the syncategoremata, properties of terms, consequences, obligations, and sophismata and insolubilia.

2. "Modi sunt sex: Possibile, Impossibile, Necessarium, Contigens, Verum, Falsum". *Logica,* 11.

3-6 *Logica,* p. 17: "Conditionalis est oratio in qua coniunguntur plures propositiones per notam conditionis . . . . Conditionalis vera est quando contradictorium consequentis repugnat antecedenti . . . Ad veritatem conditionalis requiritur quod oppositum consequentis non stet cum antecedente . . . Omnis conditionalis vera est possibilis et necessaria, et omnis falsa est impossibilis et nulla est quae sit contingens."

7. *Ibid.,* p. 17.

8. *Ibid.,* p. 18.

9. *Ibid.,* p. 18.

10. *Ibid.,* p. 18. "Ad veritatem disiunctivae sufficit unam partem esse veram."

11. *Ibid.*, p. 19. "Ad contingentiam requiritur quam libit partem esse contingentem et alteram non repugnantem nec contradictoriam illarum."

12. *Ibid.*, p. 19.

13. An early medieval division of propositions according to matter is found in Peter of Spain's Summulae logicales: "Propositionum triplex est materia, scilicet naturalis, contingens, et remota. *Naturalis* est in qua praedicatum est de essentia subiecti vel proprium eius, ut 'homo est animal' vel 'homo est risibilis'. *Contingens* materia est illa, in qua praedicatum potest adesse et abesse subiecto, ut 'homo est albus'. *Remota* materia est in qua praedicatum non convenit subiecto, ut 'homo est asinus'." (*1.15* in I. M. Bocheński's edition, Torino: Marietti, 1946, p. 6.)

    St. Thomas, too, adds this fifth division of proposition to the four of Aristotle: "Potest autem accipi quinta divisio enuntiationum secundum materiam, quae quidem divisio attenditur secundum habitudinem praedicati ad subiectum; nam si praedicatum *per se* insit subiecto, dicetur esse enuntiatio in *materia necessaria* vel *naturali;* ut cum dicitur, 'Homo est animal', vel 'Homo est risibile'. Si vero praedicatum per se repugnet subiecto quasi excludens rationem ipsius, dicetur esse in *Materia impossibili* sive *remota;* ut cum dicitur, 'Homo est asinus'. Si vero medio modo se habeat praedicatum ad subiectum, ut scilicet nec per se repugnet subiecto, nec per se insit, dicetur enuntiatio esse in *materia possibili* sive *contingenti.*" (*In Perihermenias* I, lect. 13.)

    Albert of Saxony, in the following century, mirrors the view of his contemporaries, the the division is still made in much the same way: "Those propositions are said to be of natural matter, which are such that the predicate signifies the same that the subject signifies, and cannot be truly denied of that subject; or they are propositions in which the more universal is predicated of a less universal term included under it, or a definition of its definiendum, or a part of the definition is predicated of the term defined, or in which a term is predicated of itself. Other propositions are said to be of *contingent matter,* whose predicate can be predicated of its subject either affirmatively or negatively, in contingent manner. But a proposition is said to be of *remote matter,* whose predicate cannot be (truly) predicated of its subject at all. An example of the first type is, 'Man is an animal'; an example of the second is, 'Man runs'; an example of the third is, 'Man is an ass'." Translated from *Perutilis Logica* III, c. 10 (1522 Venice edition) by E. A. Moody and quoted in his *Truth and Consequence in Mediaeval Logic,* Amsterdam 1953, p. 61.

    Paul of Pergola makes the same distinctions in matter, subdividing propositions of contingent matter into two classes, with the help of modal notions: "Propositio possibilis est illa cuius primarium significatum et adaequatum est possible ut: Homo est papa. Propositio impossibilis est cuius primarium significatum et adaequatum est impossibile ut: Homo est capra. Propositio contingens est cuius primarium significatum et adaequatum est contingens ut: Homo est albus. Propositio necessaria est cuius primarium significatum et adaequatum est necessarium ut: Deus est." *Logica,* pp. 10f. This division should not be confused with Paul's division of modal propositions (ibid., p. 11f). While 'Man is white' is a proposition of contingent matter, it is not a modal proposition.

14. For a distinction between law-statements and the historically closed descriptive Generalizations see my paper "The Logical Structure of Medieval Law-Statements", *Proceedings of the American Catholic Philosophical Association* 38 (1964), pp. 86-95.

15. Supposition for Paul is very definitely a property which a term has when it is used in a proposition. The requirement that a term be used in a proposition in order to have any type of supposition is made by most medieval logicians. The only known exception is that of Peter of Spain who holds that a term considered absolutely has

natural supposition. This was pointed out by J. P. Mullally, The *Summulae logicales of Peter of Spain,* Notre Dame, 1945, p. xlvii. Peter of Spain puts the matter as follows: "Supposition is the interpretation of a substantive term for something. Supposition differs from signification, because signification arises through imposing on a vocal sound the function of signifying something, whereas supposition is the interpretation of the already significant term, for something, Thus when we say, 'A man runs', this term 'man' stands for Socrates or Plato, and so on. Hence signification is a property of a vocal sound, whereas supposition is a property of the term already constituted from a vocal sound and a signification." (*6.03* of his *Summulae logicales,* Bocheński's edition) and translated by E. A. Moody, *Truth and Consequence in Mediaeval Logic,* p. 20. Moody objects to Mullally's interpretation on three grounds: (a) that Peter explicitly claims supposition to be a property of a *term* and not any vocal sound (the term was defined by him as "that into which a proposition is resolved." cf. *op. cit., 4.01*); (b) that the various types of supposition which a term may have depend on its occurrence in a proposition; and (c) that the distinction between signification and supposition, insisted upon by Peter, would be made trivial: "to say that a name, taken alone, stands for its objects, is equivalent to the trivial statement that a name is the name of whatever it is the name". (cf. Moody *op. cit.,* p. 22). In my opinion, neither of these reasons supports Moody in his contention that supposition was held even by Peter to be a property of term only when used in a proposition. *Ad* (a): Note that the quotation from Peter's *Summulae* given above reads: "Supposition is the interpretation of a *substantive* term for something . . . " (my italics!) This suggests that Peter is using here 'term' for any word, including the syncategoremata, as we often do in English; he claims that only those terms which are substantive can be interpreted for something other than themselves. Natural supposition would thus seem to be for him what was more recently called the extension of a term. *Ad* (b): Since in propositions not all terms are distributed, but stand for various extension-regions, we have a ground for various types of "accidental" supposition. *Ad* (c): "To say that a name, taken alone, stands for its objects, is equivalent to the trivial statement that a name is the name of whatever it is the name" is indeed true within the framework of Ockham's ontology, but it is not trivial in an ontology such as that of Peter which does not claim that the only existents (or subsistents) are individuals. While Peter's ontology may be objectionable, his recognition of natural supposition in the sense of Mullally seems to me to be not only not objectionable but even demanded.

Paul of Pergola, of course, does have a notion of supposition such as Moody attributes to Ockham and Albert of Saxony (*ibid.,* pp. 20f.): "Suppositio est acceptio termini in propositione pro aliquo vel pro aliquis" (Logica, p. 24) But this does not prove that supposition might not be held to be a property of term even when not used in a proposition within a different ontology.

Not even the claim of George of Brussels that the theory of supposition has been developed for an inquiry into the truth and falsity which characterizes only propositions and not terms taken alone entails, of itself, the impossibility for a term to have this property even when not used in a proposition. Truth is a complex property of propositions which might need for its explication notions such as signification, supposition, categorematics, syncategorematics, all of which may be, and usually are, discussed within propositional contexts, but need not be. (For George of Brussels' view see P. Boehner, "A Medieval Theory of Supposition", *Franciscan Studies* 18 (1958), p. 251, *n.* 5

16. Cf. *Logica,* p. 26. It must be stressed however that Paul requires, in addition to '*Aab*', an assertion to the effect that these are all the *a*'s there are. E.g. "Omnis homo currit *et isti sunt omnes homines,* ergo ille homo currit et ille homo currit, et sic de singulis" (Logica, p. 26, p. 29; italics are mine). Paul calls the propositions of the type I italicized "the necessary middle" (debitum medium).

17. The "middle" is not needed for making a descent from a distributed term in negative propositions (cf. *Logica,* p. 94: "Ab universali negativa ad quamlibit suarum singularium, sive cum medio sive absque medio est bonum argumentum".)

18. "Ab exclusiva affirmativa ad universalem affirmativam de terminis transpositis est bona consequentia et e converso" (p. 95).

19. Cf. *n.* 14 above.

20. *Logica,* p. 26.

21. *Ibid.,* p. 30.

22. *Ibid.,* pp. 31f.

23. The role of the notion of supposition in the analysis of truth of categorical propositions is stressed especially in P. Boehner's article "Ockham's Theory of Supposition and the Notion of Truth", *Franciscan Studies* 6 (1946), pp. 261-292. But insofar as any inference requires for its validity the impossibility of the conjunction of true premisses and a false conclusion, the supposition theory may be equally legitimately employed in the analysis of inference-conditions.

    According to W. Kneale, "three different interests were served by this part of medieval logic (i.e. by supposition theory). The first was an interest in the making of a general theory of language and the elucidation of such notions as meaning, application, and reference. The second was an interest in the precise description of various idioms of the natural language used for philosophizing. And the third was an interest in the elaboration of rules for valid inference in general logic, or quantification theory, as it is sometimes called." *The Development of Logic,* Oxford, 1962, pp. 273 f.

24. *Logica,* pp. 31f.

25. "Ex impossibili sequitur quodlibet . . . Necessarium sequitur ad quodlibet". *Ibid.,* p. 88.

26. *Ibid.,* p. 60.

27. The statements of the rules *3.01 - *3.082 are to be found in the same order on pp. 89-92 of *Logica.*

28. *Logica,* p. 99.

29. *Ibid.,* pp. 100f.

30. *Ibid.,* p. 101.

31. *Ibid.,* p. 88; see *n.* 25 above.

32. *Ibid.,* p. 101.

33. *Ibid.,* p. 92.

34. *Ibid.,* p. 92.

35. *Ibid.,* p. 92.

36. *Ibid.,* p. 99. For a fuller discussion of the composite and the divided sense see Paul's last tract in his *Logica,* pp. 149-158.

37. *Ibid.,* p. 92.

38. *Ibid.,* p. 95.

39. Cf. Moody, *op. cit.,* p. 46.

40. *Logica,* p. 94.

41. *Ibid.*, p. 94. On the medieval square of opposition with negative propositions analyzed as not having existential import see Moody, *op. cit.*, p. 52.

42. *Logica*, p. 94.

43. *Ibid.*, p. 93.

44. "Comparativus gradus et superlativus et isti termini, ita, sicut differt, aliud, non idem, egeo, careo, indigeo, sive, absque, confundunt confuse distributive mobiliter terminum sequentem non impeditum et rectum a parte post." *Logica*, p. 33.

45. *Ibid.*, p. 99.

46. *Ibid.*, p. 96.

47. *Ibid.*, p. 96.

48. *Ibid.*, p. 93.

49. *Ibid.*, p. 95.

50. *Viz.* in his third tract of the *Logica*, "De probationibus terminorum" (cf. pp. 57-60).

51. *Logica*, p. 96.

52. *Ibid.*, p. 96.

53. *Ibid.*, p. 97. (rule V.) and p. 99 (rule X.) Of course, one could argue that since *'tu'* is a token-reflexive word, this example is not a conclusive proof that Paul admitted the possibility of proper names to fail to refer; that *'tu'* is used in much the same way as 'Socrates' in our examples of a syllogism with singular conclusion. Whatever weight such an argument would have in the case of *'tu'*, it would also have in the case of *'iste'* as used in the example referred to in *n.* 40 above.

54. *Ibid.*, p. 97.

55. *Ibid.*, p. 97.

56. *Ibid.*, p. 97.

57. *Ibid.*, p. 97.

58. *Ibid.*, p. 98.

Clarke College
Dubuque, Iowa