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WHAT IS A SCIENCE?

ROLF SCHOCK

One of the basic problems of the philosophy of science is (or at least should be) that of exactly determining what a science is. A brief solution to this problem is proposed here on the basis of certain concepts introduced by the author in 'Contributions to syntax, semantics, and the philosophy of science' (*Notre Dame Journal of Formal Logic*, Vol. 5, 1964).

If we consider some of the sciences (for example, mathematics, physics, astronomy, geology, biology, and psychology), we find that each of them contains not just one theory, but finitely many; moreover, each of the contained theories is associated in the science with a system which determines to what extent that theory is adequate with respect to a set of objects that the science deals with. This state of affairs can be reconstructed as follows:

Definition 1. t is a theoretical system just in case s is an 8-term sequence such that

(1) $t(1)$ is a symbol sequence

(2) $t(2)$ is a theory in s

(3) $t(3)$ is an interpreter with respect to s

(4) $t(4)$ is a finite subset of $\mathsf{U}(t(3))$

(5) $t(5)$ is a finite or denumerably infinite sequence whose range is included in the set of all theorems of $t(1)$ by $t(2)$

(6) $t(6)$ is a finite or denumerably infinite sequence whose range is included in the set of all formulas of $t(1)$

(7) $t(7)$ is included in the set of all formulas of $t(1)$

(8) $t(8)$ is a finite or denumerably infinite sequence whose range is included in the set of all theorems of $t(1)$ by $t(2)$.

Definition 2. If t is a theoretical system, then

(1) the degree of confirmation of t = the degree of confirmation of $t(2)$ with respect to $t(3)$, $t(4)$, $t(5)$, and $t(1)$

(2) the explanatory power of t = the explanatory power of $t(2)$ with respect to $t(6)$ and $t(1)$

(3) the degree of deductive simplicity of t = the degree of deductive simplicity of $t(2)$ with respect to $t(7)$, $t(8)$, and $t(1)$

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(4) the degree of adequacy of t = the degree of adequacy of adequacy of $t(2)$ with respect to $t(3)$, $t(4)$, $t(5)$, $t(6)$, $t(7)$, $t(8)$, and $t(1)$.

Definition 3. If x is a set, then s is a subscience dealing with x just in case s is a non-empty finite sequence, $s(n)$ is a theoretical system for any n in the domain of s, and x = the set of all y such that, for some n in the domain of s, y is in $\bigcup((s(n))(\bar{3}))$.

Definition 4. If s is a finite sequence of real numbers, then the mean of s = the limit of the function y such that

(a) the domain of y = the domain of s

(b) for any n in the domain of y, $y(n)$ = (the sum of (s cut off at the $n + 1^{\text{th}}$ term))/n.

Definition 5. If x is a set and s is a subscience dealing with x, then

(1) the degree of confirmation of s = the mean of the function c such that the domain of c = the domain of s and $c(n)$ = the degree of confirmation of $s(n)$ for any n in the domain of c

(2) the explanatory power of s = the mean of the function e such that the domain of e = the domain of s and $e(n)$ = the explanatory power of $s(n)$ for any n in the domain of e

(3) the degree of deductive simplicity of s = the mean of the function d such that the domain of d = the domain of s and $d(n)$ = the degree of deductive simplicity of $s(n)$ for any n in the domain of d

(4) the degree of adequacy of s = the mean of the function a such that the domain of a = the domain of s and $a(n)$ = the degree of adequacy $s(n)$ for any n in the domain of a.

We call the objects of definition 3 subsciences rather than sciences because they cannot develop whereas most of the sciences have been added to and improved upon a finite number of times since their origins. We can, however, understand any science to be the non-empty finite or denumerably infinite sequence of those subsciences which are the stages of the development of that science. Thus, we arrive at the following definitions:

Definition 6. If x is a set, then s is a science dealing with x just in case s is a non-empty finite or denumerably infinite sequence and, for any n in the domain of s, $s(n)$ is a subscience dealing with x.

Definition 7. s is a science just in case, for some set x, s is a science dealing with x.

These definitions are somewhat peculiar in that many objects which are not of the structure of some known sciences are sciences according to them. Nevertheless, any possible definition of science which is more informative than one such as 's is a science just in case either s = mathematics or s = physics or ...' seems to be subject to this fate.

Definitions 6 and 7 lead to a fairly natural definition of a relation of inferiority of generality between sciences.

Definition 8. If s and t are sciences, then s is less general than t just in case, for some sets x and y, s is a science dealing with x, t is a science dealing with y, and x is a proper subset of y.

A perhaps surprising result of definition 8 is that, if mathematics is a science dealing with some set of mathematical objects and physics is a science dealing with the union of this set with the set of all physical objects, then physics is more general than mathematics.

Notice that the degree of confirmation, explanatory power, degree of deductive simplicity, and degree of adequacy of a science could be defined in more or less the same way that they were for a subscience. These definitions will not be given here.

Stockholm, Sweden