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SOME THOMIST REFLECTIONS ON THE FOUNDATIONS OF FORMAL LOGIC

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The divergence of thought between traditional scholastic logicians and modern mathematical logicians is a matter known to all. The immediate purpose of this paper is to explain the nature and method of Thomist general formal logic, not as it always exists but as one might like it to exist. These reflections consist in a consideration of the foundations of Thomist logic in its present state of development and of a consideration of this same logic in the light of developments among the mathematical logicians. There are three parts:

I. The Nature of Thomist Logic.
II. The Method of General Formal Logic.
III. Criticism of Some Contemporary Notions of Formal Logic and Expansion to the Generalized Logic of Relations.

PART I. THE NATURE OF THOMIST LOGIC

A. THE SUBJECT OF LOGIC

I begin this analysis somewhat at a distance from logic properly so-called, proceeding by Division to narrow the area in which logic is to be found. This has the advantage of at once eliminating possible confusions and also setting the subject of logic in its proper perspective.

1. The Presence of Reality to Us. As we begin to reflect on our relation to "reality," we become aware of a diversity of modes of *presence* of reality to ourselves. By the presence of reality to us, I mean a union of some being with ourselves in such a way that we are *aware* of the being which is present. Awareness itself is a primitive term, an indefinable. We can point to beings which do not have an awareness of their surroundings or of themselves, i.e. inanimate things and plant life; but we can point to other things which we say are at least aware of their surroundings, and even to some which are aware of themselves as selves. In either case, the aware-

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ness which beings may have is ultimately understood by relation to the awareness which we ourselves have; and this latter awareness is only grasped intuitively and remains indefinable.

In human awareness, we might first distinguish between substantial awareness and awareness by activity, the former being an awareness of one's own existence arising in virtue of the very exercise of existence in a spiritual substance—the soul.[1] In awareness by activity, we may distinguish between awareness of the object of activity and the awareness of the activity itself in virtue of the very reflexivity of intellectual consciousness. This latter awareness is simultaneously an awareness of the subject-which-acts. Both this latter awareness and substantial awareness together compose what Maritain has called "concomitant consciousness"[2] of the subject as subject, which accompanies all our other intellectual awareness. We may set aside such concomitant consciousness and consider only awareness of the object of activity.

Here the first division is between *cognitive* awareness and *affective* awareness, or, we may say between awareness *by way of knowledge* and awareness *by way of affection*, or again, between *knowledge* in the proper sense and *affective connaturality*. Maritain speaks of *knowledge by way of knowledge* and *knowledge by way of inclination*,[3] to express this distinction. We shall use his expressions here.

a. Knowledge by way of knowledge. Knowledge by way of knowledge can be described as an awareness of an other as opposite the knower, as truly *other*. But the thing known is known as "there and then" and not in the immediacy of its own "here and now" (to speak metaphorically)—this is true even in reflection on our own being through knowledge by way of knowledge. Here we make ourselves "other" in order to know ourselves; we set ourselves "opposite" ourselves in order to look at ourselves. In knowledge by way of knowledge, there is a union of the knower with an other, but not with the very *otherness* of the other as having its own "here and now." The presence arising in such knowledge is *presence across a gulf*. Things here present a facet to the knower, but it is only a facet. While the real thing has its own ontological depth, its own *subjectivity*, this thing shows itself only as an *object* in knowledge by way of knowledge.

> We know those subjects, we shall never get through knowing them. We do not know them as subjects, we know them by objectising them, by achieving objective insights of them and making them our objects; for the object is nothing other than something

1. See Jacques Maritain, *Ransoming the Time* (New York, 1941), chapter X. This kind of awareness of one's own existence is in fact not ordinarily given in clear distinction from other modes of awareness. For it to be given apart from other modes would pertain to natural mysticism, in Maritain's view.
2. *loc. cit.*
3. Jacques Maritain, *Existence and the Existent*, transl. by G. Phelan and L. Galantiere (New York, 1956), p. 78.

> of the subject transferred into the state of immaterial existence of intellection in act. We know subjects not as subjects, but as objects, and therefore only in such-and-such of the intelligible aspects, or rather *inspects*, and perspectives in which they are rendered present to the mind and which we shall never get through discovering in them.[4]

It is in such knowledge that the unity of real beings is noetically broken up, according to our penetration of diverse aspects—or *formal objects*—of these beings (*material objects*), in a multiplicity of knowledges of the same reality. This is, however, necessary in knowledge by way of knowledge, inasmuch as the light of the human powers of knowledge is not sufficient to penetrate the fullness of the real being in one knowledge-act and consequently requires an ever-growing multiplicity of knowledge-acts. Indeed, for us, the reality always remains a mystery to be known yet more deeply or thoroughly.

But if one examines our knowledge by way of knowledge, it is seen to be distinguishable into two fundamentally opposed types of knowledge, which we call *sense knowledge* and *intellectual knowledge*. We are aware of two sets of objects of knowledge: one set the members of which are concrete and individual, another set the members of which are abstract and universal. For example, when there is present to us a particular group of concrete, individual, sensible appearances, we at the same time see that the thing itself which is seen through the appearances is, e.g., a dog. Or when we are confronted with a particular instance of "red", we at the same time may see what redness itself is quite independently of this particular instance. Neither of these two types of knowledge is ever found in isolation from the other; they are distinct, but never separate, knowledges of reality. But there is a subordination of the sense knowledge of concrete individuals to the abstract intellectual knowledge; for intellectual knowledge gets at what things are in their being, while sense knowledge only gets at the *manifestations* (or *phenomena*) of this being. Of course, all our intellectual knowledge is dependent on sense knowledge as on an *instrument*; for we do not see the being of things except as it is revealed to us through their sensible manifestations. Even spiritual being is known by analogy with those beings which sensibly manifest themselves to us. But this dependence of intellectual knowledge on sense knowledge for its data is like the dependence of a master on his servants and not vice-versa. The senses minister to the needs of the intellect by bringing data to it, and the intellect contemplates the being of reality which is revealed in and through these data.

b. Knowledge by way of inclination. Knowledge by way of inclination is an awareness of a *subject*—that which, properly speaking, has its own existence and activity—in the immediacy of its own "here and now" as a subject. This awareness arises in a presence of the subject to the knower through love or inclination. That such an awareness and presence actually are found can be verified by introspection. But one must be careful not to

4. *ibid.*, p. 74.

subject all the contents of consciousness to the criteria of knowledge by way of knowledge—if this were done, one could only end by rejecting knowledge by way of inclination as an illusion.

Knowledge by way of inclination can be subdivided into various types. But further discussion of this kind of knowledge and of its divisions may be left for metaphysics, ethics, and psychology. Let us note, however, that it is called "knowledge" only in an analogical sense in relation to that knowledge which is ordinarily contrasted with love. It would, perhaps, be better to say that we *live with* the subject as subject here; but this expression is also inadequate in its own way. In any case, in the rest of this paper, knowledge will be taken to mean only knowledge by way of knowledge.

2. Division of Intellectual Knowledge of Reality. Leaving aside "knowledge by way of inclination" and sense knowledge, we must now divide intellectual knowledge in the strict sense. As we reflect upon our intellectual knowledge, we note that it consists in two radically diverse types of activity.[5] The intellect makes an intuitive act (understanding and judgment) and a discursive act. The former leaves the intellect in a state of rest in a contemplated object, while the latter is the movement of the intellect from a state of achieved contemplation to a new state of contemplation. Thus discourse may be said to have intuitive knowledge as its beginning and end; the purpose of discourse is only to help to deepen our intuitive knowledge of reality. Beside intuitive knowledge we can also place *belief*. Belief, like intuitive knowledge, attains to its object without discourse; however, in belief the object itself is not directly *seen* but is grasped only by assent under the authority of one who informs us of it.

3. Discursive Intellectual Knowledge. Discourse supplies, to the extent to which it can, for the imperfection of our intellect—an intellect which must gradually move from potency to act in acquiring its intuitive knowledge of reality. As Gilson says:

> The intellect does not deduce, it intuits, it sees, and, in the light of intellectual intuition, the discursive power of reason slowly builds up from experience a determinate knowledge of concrete reality.[6]

While higher intellects than ours grasp much more of reality in a single glance, and the intellect of God embraces everything in one mental Word (identical with Himself), our intellects can only proceed in a piecemeal way. Indeed, even among men themselves there is a diversity in power of intellectual intuition, which varies according to native genius, study, and experience. But given such diversity it nevertheless remains true that all men must gradually piece together bits of knowledge in order to gain a deeper penetration of reality. This process reaches its perfection in *scientific* knowledge, knowledge through a *unified system*. Such a system renders discourse about this reality easier, in that the parts are seen in their relations to the whole and to the other parts of the system.

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5. J. Maritain, *Formal Logic*, transl. by I. Choquette (New York, 1946), p. 1.
6. Etienne Gilson, *The Unity of Philosophical Experience* (New York, 1954), pp. 313-314.

Note, however, that two sets of relations must be clearly distinguished: *real relations* between the various aspects of the reality itself which is known, and relations which exist only in the intellect—*relations of reason*—between the various knowledge-acts by which the reality is known. Of course, these relations of reason arise precisely because of the real relations—they are relations of reason with a foundation in reality. But if we consider our systematic knowledge precisely as systematic *knowledge*, it is clear that these relations of reason are the immediate unifying bonds of the system. Moreover, it will become clear as we go on that these relations of reason are not all simply copies of the real relations; for the mode of being of knowledge (abstract and universal) is quite diverse from the mode of being of reality (concrete and individual).

At this point, let us note that intellectual discourse takes place in five ways: in one way, when the intellect merely moves from the consideration of one truth to the consideration of another wholly unrelated truth (when we "change the subject" of our thoughts); in a second way, when the intellect, by free construction, makes a new notional complex out of elements it already possesses (e.g., when we construct a theory); in a third way, when the intellect moves from the consideration of some known truth to the discovery of some unknown truth as related to the first (e.g., when one, on considering a mass of data, discovers the necessary explanation for these data); in a fourth way, when the intellect moves from the consideration of one truth to the consideration of another in accordance with relations already seen between them—here the intuitive act is still immediately predominant and exercises an immediate governance over the discursive act (e.g., when one "goes through" a systematic body of knowledge already acquired about some subject); in a fifth way, when the intellect moves from the consideration of knowledge-acts as having certain relations of reason (founded in the reality which is known) to the consideration of the same knowledge-acts as having other relations of reason (also founded in the reality which is known) which were implicit in the first—here the intuitive act is only remotely predominant and exercises only a remote governance over the discursive act (i.e., when one makes any logical inference).

4. The Subject of Logic. Logic studies being as known, precisely with reference to those relations consequent upon its being known—not the relations of knowledge to the knower, but the relations of knowledge to other knowledge and to the reality which is known; and not the real relations between the real objects of intellectual acts, but the relations of reason between the intellectual acts themselves and of these acts to reality. The object of logic is constituted by the multiplicity of intellectual acts elicited in our knowledge of reality and by the discourse which results from this multiplicity. Logic may be said to study the very discourse of reason as such. But like any philosophical study of the mobile and of motion, logic seeks the immobile, necessary principles of this motion of reason.[7]

7. This does not mean that motion is said of the movement of reason and of the things in the physical universe in identically the same sense.

But it should be clear that, of the five ways of intellectual discourse mentioned in the preceding section, only the last three are of direct concern to logic; for only in these can there be a genuine intrinsic order (and consequently intrinsic intelligible principles). Logic will study the general modes of ordering our knowledge in itself and the modes of discoursing from a given order to other orders implicit in the first. One can see then that there will be a priority, in logic, of the study of logical relations to the study of logical reasoning. This is a fundamental consideration in logical method, as we shall soon see.

Logic, as I have indicated, studies not only the relations of knowledge-acts to other knowledge-acts but also the relation of the knowledge-acts to the reality which they represent. This is necessary since discourse is not an idle thought-game, a meaningless though entertaining gymnastic. Discourse, just as much as intellectual intuition, although in its own proper way, essentially tends toward reality itself. Its full intelligibility can only be grasped if we also consider the relationship of our intellectual acts to reality itself. Moreover, if logic is to study such *relations* as we have described, it is necessary that it also study the *relata*—the elements which are related. Thus there is necessity for a logical consideration of the *content* of our intellectual acts.

We have just prepared the ground for a fundamental division of logic which we shall see later on in this paper.

B. THE DISTINCTION OF LOGIC FROM OTHER SCIENCES

1. The Distinction of Logic from Psychology. The subject of logic is the very activity of the intellect. But philosophical psychology, in its study of the nature of living things, must consider the activities of human beings, even including the activity of the intellect. However, logic and philosophical psychology approach this activity from diverse points of view. As we have pointed out above, logic considers this activity in its inner relations and in its relation to reality; logic discusses the contents of such activity only in view of these relations. But psychology studies this same activity *in relation to the being who is acting*, in so far as it proceeds from, and manifests something of the nature of, man.

2. The Distinction of Logic from Mathematics. Especially in modern logic there is a tendency to confuse, if not to identify, logic and mathematics. This tendency has roots in the very nature of logic and of mathematics but has been accentuated by the symbolization of logic itself (in mathematical logic) and by the attempt to remove determinate interpretation from the symbolism of mathematics (in formalist mathematics). Such an identification of logic and mathematics presupposes that logic is confined to a study of relations without a consideration of the determinate character of the relata (this logic corresponds to what we will call *general formal logic*), and that mathematics treats of relationships between uninterpreted symbols. If we restrict ourselves to a consideration of such formalist mathematics and purely formal mathematical logic, then it seems impossible to distinguish

the two studies. What has happened is that mathematics, by becoming formalist, has really become formal logic. Leaving out the *content* of mathematical notions (manifolds, quantitative or at least analogically quantitative), there remains only a set of abstract relations of reason between indeterminate contents. This in turn is expressed by a set of uninterpreted material symbols with similar relationships. Whether logic ought to proceed by such an abstraction of relations from mathematical content is a question to be discussed elsewhere.

In any event, once the proper object of mathematics is seen to be the quantitative manifold, we can clearly distinguish mathematics from logic; for logic does not have a real content, even if real is taken in the diminished sense in which it is applied to mathematical entities. The content of logic is precisely the network of relata and relations arising in our knowledge of reality *considered as a knowledge*. Mathematics ultimately arises from an insight into reality (though the contents of this insight are considered by the mathematician apart from their real context); and even in its most artificial constructions mathematics always retains a remote ordination toward real manifolds. Logic, on the other hand, arises from reflection on our mode of knowing reality.

NOTE. If a modern formalist mathematician were to object that he is after all a mathematician, we might say that he can of course call himself what he wishes. One might even concede that "mathematics," in its original etymological sense of "discipline," is a very appropriate name for several sciences. Nor can it be denied that, among many moderns without roots in tradition, the term "mathematics", as well as many other terms, has lost its traditional significance. But if we are to retain the name of mathematics in its traditional meaning, then we must say that he has ceased to be a mathematician in moving into an equally valuable, but quite distinct, type of knowledge.

3. The Distinction of Logic from Metaphysics and from All Other Studies of the Real World. What we have just said about logic in relation to mathematics can now be generalized. Logic is distinguished from all studies of reality itself in that logic is a study of our very knowledge of reality. Logic studies all being, just as does metaphysics. But while metaphysics studies being as it really is, with respect to what makes it to be at all, logic studies being as it exists in knowledge, with respect to the peculiar relationships which arise precisely in virtue of its existence as known by the intellect.

C. THE DIVISIONS OF LOGIC

1. Divisions of Logic as Taken in a Wider Sense. We shall now for a moment consider logic in a wider sense than above. Logic is concerned, as we have seen, to know the order which the intellect makes and should make among its acts. If this order is a genuinely *rational* order, one made by the reason as such, then this order is always made under the direction of

knowledge. Therefore, even prior to the logic that we have been speaking of, there must be a spontaneous logic according to which the intellect naturally orders its acts, a comparatively unreflective logic on the level of "common sense" knowledge. Moreover, if our *reflective* logic is concerned first with the study of this spontaneously achieved order and then with all possible orders of intellectual acts, it is possible to look to this reflectively constituted logic for direction in ordering more easily and correctly all thought. Accordingly, it is necessary to distinguish three "states" of logic in the wide sense: logic as the "natural art" of reason, logic as the "reflective science" of reason, and logic as the "scientific art" of reason. It is logic as the reflective science of reason which concerns us in this paper, but it will be instructive to consider its relations to the other two "states" of logic.

a. Logic as the natural art of reason. Art is the "right ordering of makeables." Art always refers to something to be made; and making always takes place under the direction of an intellect, which alone can *order*. Art, according to the imposition of the name, has first of all to do with making *material* artifacts out of parts of the natural world. But we also speak of arts which make a product in the intellect itself, a construction of knowledge. Such arts we call liberal arts.

All scientific knowledge[8] in the human intellect requires the work of a liberal art, the art of logic. We understand reality in each of its domains through a multiplicity of knowledge-acts. In order to achieve as perfect or as scientific a knowledge as possible of any domain of reality, this multiplicity of knowledge-acts must be ordered into a systematic structure.

This structure can be viewed from two aspects: it has a *content* and an *order*. By a set of (logical) relations, the content is set into an order. The content itself is either abstracted immediately from reality or constructed from elements ultimately derived from reality. But in formal logic we are not concerned with the abstraction or construction of content in various sciences, such as physics, chemistry, philosophy of nature, or ethics. Such a concern would pertain to material logic or to psychology. Our concern is with the content as ordered and related, whether this content in itself be simply abstracted or constructed.

The order of a science is not simply imposed upon the intellect by reality. It is as such an attribute of our knowledge, made by the intellect itself, which seeks to relate, to unify, the manifold of content coming to it from reality. Reality itself strikes us in an often very haphazard way; this is so much the case that even when we attempt to make our pursuit of knowledge as methodical as possible, reality still "takes us by surprise." Consider, for example, the many accidental discoveries in the field of the positive sciences. The same can also be said for philosophical "intuitions". But this haphazard influx of content to the intellect is organized by the intellect

8. The use of the term "science" here is much broader than the Aristotelian use. The term will here apply to *any systematized knowledge*.

and also referred to the "apperceptive mass," so to speak, already present in the intellect.

Thus there is some validity in the Kantian notion of the intellect as unifying and relating a manifold, even when we consider the overall life of the intellect and not merely its behaviour in the phenomenal sciences. But it is not a sense manifold that we speak of; it is a manifold of intelligible content. Furthermore, the relations employed in this unification are not the natural apparatus of the intellect; for relations do not exist without relata. They are the fruit of the spontaneous activity of the unifying intellect in response to a multiplicity of intelligible content. Note, however, that this response is always conditioned by the character of the intelligible multiplicity. The relations formed in the intellect are formed in accordance with the exigencies of the subject-matter. For example, the relation of class-inclusion is set up between "man" and "animal" and not between "man" and "ox". Thus we observe the genesis of logical relations as "beings of the reason with a foundation in reality," *entia rationis cum fundamento in re*.

If logic as an art is concerned with the construction of such order in knowledge, then it is clear that we are here confronted with a "natural logical art" of the intellect. We are always relating our knowledge in this way, in virtue of the very spontaneity of the intellect itself. We do not suddently resolve to begin this unification; we have always been working at it. We only one day note that it is going on, and on that day the reflective science of logic is born in us. Note, moreover, that this is true not only as regards thought-processes in general but also as regards the thought-processes of particular sciences. We do not come to study the logic of physics until after physics is already a going concern; to attempt to reflect on the special logic of physics before physics exists would be impossible. Moreover, we have no *a priori* guarantee that in unveiling new aspects of reality we shall employ the same logical relations as in some other sciences, since the relations employed are always made in accordance with the demands of the content.

We have already seen historical examples of the evolution of entirely new logical structures in new sciences. The logical relations characteristic of the modern sciences of phenomena are not those of the *Posterior Analytics* of Aristotle. The structure of metaphysical science is not at all that of the philosophy of nature.

Finally, logic, as the "natural art" of reason, also manifests itself in the discourse from the already related actual intelligible multiplicity to what is virtually contained in it. This discourse takes place precisely in virtue of those relations which the intellect has already established. The most commonly cited example of such discourse is the categorical syllogism. In such discourse, we are not moving to the absolutely new; that could only take place by new "intuition" or "belief". Rather, we are simply drawing out the "implications" of what we already know.

b. Logic as the "reflective science" of reason. Once the reason, by its natural spontaneity, has begun to relate truth to truth, and to discourse from one set of relations between knowledge-acts to another set—that is,

once reason has begun to make an order among its acts—the reflective science of logic may begin. The object of this science is, as we have said, precisely "the order which the intellect makes among its intentions."

c. Logic as the "scientific art" of reason. When logic has been constituted as a science, it perfects the spontaneous art of the reason. This perfection is of two kinds. First, scientific logic habitually possessed stands as a constant critic of the spontaneous movement of the reason of the possessor and of others. Second, its cultivation renders more intense the habit of discoursing well. It should be clear that the intellect, without the "scientific art" of logic, would always lack the fullest perfection of science in whatever area it should discourse. Science might be, and often is, achieved without this "scientific art"; but the perfection of science is not achieved without it.[9]

2. Divisions of Logic Taken as the Reflective Science of Reason.

a. Formal and material logic. Scientific logic is immediately subdivided into formal logic and material logic. We have already seen that the subject of logic is the very act of the intellect itself. We have also pointed out that the act of the intellect may be here considered from three points of view: in its relations to other intellectual acts, in its relation to reality itself, and in its content. The first consideration is that of *formal* logic, while the second and third are the consideration of *material* logic.

Formal logic as such is not concerned with "truth", if by truth is meant the conformity of knowledge to reality. But formal logic is concerned with validity, the necessary connection between one structure of knowledge-acts and another structure of knowledge-acts, in virtue of which we infer the second from the first. Validity is sometimes referred to as "formal truth". To ask whether a law of formal logic is true is really to ask whether it is valid. Hence we can dispense with the word "truth" in formal logic.

b. General and special formal logic. Formal logic is subdivided into general and special logic. General formal logic treats of logical relations which do not depend in any way on the specific type of content of the relata, e.g. extension, inclusion, identity, material implication. Special formal logic treats of logical relations which do depend on the specific type of content of the relata, e.g. genus, species, property, accident, and the relations of modal logic.

Of course, even much "general" formal logic may require distinctions between certain general types of relata. But it will remain general in that it does not descend to a consideration of the diversity within the general types. However, we could develop a *generalized logic of relations*, which would be so general as not to presuppose even the aforementioned general distinctions

9. The material of sections 1 a, b, c has been reproduced with little change from my "The Art and Science of Formal Logic in Thomistic Philosophy" in *The Thomist*, XXII, 4 (October, 1959), pp. 534-537, 540, with the kind permission of the editors of *The Thomist*.

of relata (see Part III). The distinction between general and special formal logic is in fact only a matter of degree, until we finally adopt the standpoint of the generalized logic of relations.

3. Division of General Formal Logic. The subject of this paper is general formal logic. We shall now lay down some divisions of the content of general formal logic.

a. Relations, laws, and rules. Formal logic treats of the structure of human intellectual knowledge. This structure, as we have seen, is formally constituted by logical relations. An appropriate division of general formal logic would be according to the distinction of logical *relations*, logical *laws*, and logical *rules*. We may regard this as a division according to the formal principles of logical structure.

In order to understand this division, we must first set out some definitions. *Logical relations* are relations between intellectual knowledge-acts, as opposed to those between realities. *Logical elements* are the knowledge-acts which are related through logical relations.[10] *Logical operations* are activities of the intellect by which logical elements are made or set into some logical relation with each other. *Logical variables* are "indeterminate logical elements," or elements in abstraction from determinate content. These are divided into *individual-variables*, *term-variables*, *proposition-variables*, etc. (To each such "logical variable" can be correlated some appropriate material symbol or "symbolic variable," but one must be careful so as not to confuse the one with the other.) *Logical laws* are structures in the intellect composed of logical variables and relations in which certain necessary relations (of implication or equivalence), ordinarily between similar sub-structures, are expressed. *Logical rules* are statements concerning the proper application of logical laws in intellectual discourse.

It is clear, as we said above, that the primary formal principles of the structure of human intellectual knowledge are the logical relations themselves. Logical laws, on the other hand, are, in a way, the very structure itself, considered from an abstract viewpoint. And yet, because of the abstract character of these laws, they may also be regarded as (secondary) formal principles of the structure. Logical rules are extrinsic formal principles of the structure, inasmuch as they direct the intellect in its construction of determinate structures of knowledge in accordance with the necessities of logical laws.

The logical *relations* presuppose, and are even in a way constituted by, logical *operations*. These logical operations reduce to three: *simple apprehension*, *judgment*, and *reasoning*. These operations will be considered below, when we consider the next division of general formal logic.

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10. Among such elements are included even the knowledge-acts by which we understand *individuals*. Such knowledge is not purely intellectual. It is a composite of sense and intellectual knowledge. But it is the intellectual component which is of direct interest to logic.

It should be at once clear that logical laws, and consequently logical rules, are not "given" to the intellect in any natural *a priori* manner. They are always a function of the special logical relations that we happen to find appropriate in various fields of knowledge. From this viewpoint, if in one field of knowledge we were to employ the logical relation of "strict" implication and in another the relation of "material implication", we might speak of using two "different logics," in so far as we would have two different sets of logical laws. However, if we regard general formal logic as the general study of logical relations and the laws which contain them, we may speak of one logic, unified by its formal object—human intellectual knowledge in so far as its multiplicity of elements is unified in structures of knowledge through logical relations.

Frequently, in traditional logic, the discussion is entirely on the level of the logical rule. The rules (e.g., the "eight rules of the syllogism") are laid out very carefully with the intention that the relations and laws should be seen as underlying these rules. This was not the method of Aristotle. For him, the logical law and relation are of greatest importance. See the first chapter of the *Prior Analytics*, where the relations consequent upon predication are described in terms of the *de omni et de nullo* principle. The meaning of implication in general is contained implicitly in the definition of "syllogism" in this same chapter. In the *Prior Analytics*, Aristotle presents the moods of the categorical syllogism in terms of both logical laws and logical rules; there he utilizes logical relations of inclusion, which are ultimately reducible, from the viewpoint of general formal logic, to logical relations of extensions to other extensions. He first states the rules and then states the laws which underlie and justify these rules.[11]

The "logical law" may sometimes be mistakenly regarded as a statement about reality, since in fact we can often substitute determinate knowledge-acts referring to determinate real things for the indeterminate variables. To so regard the logical law would be a very gross error; for we are in the logical law considering these real things only according to the relations which they have precisely in virtue of their being known and only in so far as they are known. The "things" that we substitute are only the *objects of intellectual acts as in the intellect*—these also refer to reality itself, but this is not the concern of formal logic. We see here the necessity, for the understanding of logic, of distinguishing the object *in the intellect* from the object *in the thing*, in the manner of Cajetan and John of St. Thomas.[12]

According to this division of general formal logic in terms of relation, law, and rule, we would first consider various logical relations, then the logical laws constructed out of these relations and the related variables, and

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11. *Analytica Priora*, Alpha 3-6.
12. Cajetan, *Commentaria in Summam Theologicam S. Thomae Aquinatis*, in editioni Leonina (Rome, 1888), Vol. IV, in I, q. 1, a. 3, comm. III. John of St. Thomas, *Cursus Philosophicus*, edit. Reiser (Taurini, 1930), *Ars Logica*, Part II, q. XXVII, a. 1.

finally the logical rules for observing these logical laws in human thought. But it should be clear that this division is, in the last analysis, inadequate. For relations can neither exist nor be discussed in complete abstraction from the relata. Hence we are led to consider the general division of the knowledge-acts (the relata) themselves—a division of the *material principles of logical structure.*

b. Terms, propositions, and inferences. Intellectual knowledge is the result of the activity of the intellect. This activity is of three kinds: *simple apprehension, judgment,* and *reasoning.* Simple apprehension is the act by which the intellect apprehends what something is without any affirmation or negation being made. For example, I simply apprehend what a man or a dog is, when I merely grasp, with more or less clarity, "human nature" or "canine nature," without saying anything about this nature or about the things which might have it. Judgment is the act by which the intellect affirms or denies something of something. For example, I judge when I think "This man is sick," or "Some dogs are brown." Reasoning is the act by which the intellect moves from one or more judgments having been made to another judgment made in virtue of the first judgment or judgments. For example, one is reasoning if he moves from thinking that "All men are mortal," to "Some mortal things are men," or from thinking that "All Americans are practical, and all astronomers are Americans," to thinking that "All astronomers are practical."

These three activities of the intellect result in three respective products, or knowledge-acts. Simple apprehension results in the formation of the *mental term.* The mental term gives knowledge of what things are, of their "essences." Judgment results in the formation of the *proposition* formally considered.[13] Such a proposition yields knowledge that something is, or has this or that characteristic—it includes, above and beyond its constituent term or terms in relationship, a reference to some mode of existence. Reasoning results in the formation of a concrete *syllogism* or *inference.*[14] Such an inference expresses a relation of some particular structures of knowledge to each other—every (concrete) inference derives its formal structure as an inference from some logical law (abstract inference). Reasoning always proceeds in accordance with some logical law (abstract inference) to produce some concrete inference.

The human intellectual knowledge of individuals, as we have said above, is not simply an act of the intellect; for such knowledge requires reflection upon sense-data. And yet this knowledge still remains truly intellectual. As intellectual, this knowledge is of interest to logic. Only in so far as there is knowledge of individuals (at least possible individuals) can the fundamental logical relation of extension be formed; only with such

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13. The distinction between the proposition formally considered and the proposition materially considered is the same as that between the enuntiated and the enuntiable.
14. Aristotle defined "syllogism" as a "discourse in which, some things having been posited, something else follows of necessity from the fact that the first have been posited" (*Anal. Priora,* Alpha 1, 24b 18-20).

knowledge can a singular proposition be formed. The intellectual knowledge of individuals, however, does not represent a distinct fourth act of the intellect—it arises either directly from our viewing the real world of sensible things under the light of the mental term or the proposition, or from some analogy based on this experience of the sensible real.

Thus we have three distinct knowledge-acts of the intellect: (1) through the mental term we know essences; (2) through the proposition we know existence and the relations between essences; (3) through the inference we know consequence or the necessary relation of one structure of intellectual knowledge to another, i.e. that from the fact that one such structure is posited the other must necessarily be posited.

It would be possible for general formal logic to be divided into treatises on the mental term, the proposition, and the inference. In this division, certain relations would be discussed in the first two parts, while relations, laws and rules would be discussed in the third.

PART II. THE METHOD OF GENERAL FORMAL LOGIC

We have already seen that formal logic considers the multiplicity of acts of the human intellect in their relational structure. General formal logic should therefore begin with an inventory and general discussion of relata and relations in human intellectual knowledge. This inventory should be quite general and without any pretense of completeness. It should be concerned with our thought in general as opposed to our thought in the special sciences. The initial inventory and discussion have been partially carried out above, in section C 3.

After this beginning, seven steps must be taken to develop the science:

(1) Detailed abstractive and intuitive consideration of fundamental relations and laws.
(2) Symbolization of elements and relations.
(3) Discovery of hitherto unknown relations and laws implicit in the known relations and laws.
(4) Construction of more complex relations and laws.
(5) Discovery of other relations and laws implicit in these constructions.
(6) Axiomatization.
(7) Statement of logical rules.

We shall now briefly discuss each of these steps in turn.

1. Detailed Abstractive and Intuitive Consideration of Fundamental Relations and Laws. Our general inventory can only be achieved through an *abstractive* consideration. How could we meaningfully speak of the term, the proposition, implication, etc., unless we had achieved an abstractive *intuition* or *visualization* in which, all determinate content of individual instances being set aside, we could see something *common to many instances*? How can the complex multiplicity and flux of human intellectual knowledge become ordered in our consideration under general categories and common relations unless we make such an abstraction?

There is the same necessity for abstraction in logic as in any scientific knowledge of the concrete real itself. We must not, however, emphasize too greatly what is being *left out* by abstraction; for abstraction is not ordered toward negation and ignorance—rather it is ordered toward affirmation and knowledge. Abstraction takes place in order that we may know common characteristics precisely as common and as not restricted to this moment or individual object--only thus can we seek and achieve *scientific* knowledge of these characteristics.

But the abstractive consideration, the abstractive visualization of the very dynamism of thought itself, does not cease with the general inventory. It is necessary further to clarify what we have seen. This clarification cannot take place through mere *construction* of other elements and relations. We are saying that the way of abstractive intuition, or visualization, of human discourse should be still further followed before any construction is attempted with the aid of this abstracted knowledge. Such a construction is not so much a means of clarification as a further development presupposing clarification already achieved. We must examine carefully, again abstractively and intuitively, the data already uncovered. This abstractive and intuitive examination results in a clearer and more detailed knowledge of a number of relations and laws—a knowledge which is on the properly scientific level. In the light of this clearer, more detailed intuitive knowledge, we can proceed discursively in two directions: to the *deduction* of new logical laws contained implicitly in what we have already seen, or to the *construction* of more complex relations and laws. We shall consider these two steps below. But first it is necessary to note and describe another step which should immediately follow the detailed abstractive and intuitive consideration of fundamental relations and laws, and which will greatly facilitate all the succeeding steps.

2. Symbolization of Elements and Relations. Every human intellectual act is accompanied by some sense-image. Since man himself is not a pure spirit, all his activity must, in one way or another, be *incarnate*. His immaterial activities of thought and will must therefore be "symbolized" in some material medium—his intellectual act must be symbolized in a sense-image. It is partly because of this necessity that we have language, music, painting, ritual, mythology, sacraments, etc. In order to appreciate the necessity of such symbols, one must also here take into account the requirements of human intercommunication. Again because the operation of our intellects requires the operation of our senses, human intercommunication can take place only through the development of *material symbols* suitable to express whatever "message" we have to communicate. These two motives for the making of symbols—the *incarnation* and *communication* of intellections and volitions—interact in a complex manner in their government of our symbolization. Because of this necessary concomitance of sense-imagery and human thought,[15] our thought will be as subtle and precise as the

15. But this concomitance never reduces to an *intrinsic* dependence of thought on this sense-imagery—for the act of thought is immaterial, as philosophical psychology notes; and the immaterial is intrinsically independent of matter.

concomitant sense-imagery. It is because of this extrinsic limitation of our intelligence that we evolve our complex spoken and written language as well as the various complex art forms.

It would be most interesting to pursue a detailed philosophical inquiry into the origin and development of the diverse modes of symbolization which we employ.[16] But such an inquiry would be beyond the scope of this paper. The formal logician, it is true, must note the presence of material symbols for thought, especially "words"; for he must himself use words, and, even more significantly, he can only examine the act of thought itself through the "instrumentality" of the symbolic expressions of thought. It is also necessary that he distinguish diverse types of signs and symbols from each other, that he clearly understand the diverse modes of signification. But, in a discussion strictly confined to *logic*, there should be no treatment of signs and symbols as to their origin and development. Such a treatment pertains to philosophical psychology and to anthropology.

a. The sign and types of sign. As John of St. Thomas tells us,[17] "A sign is what represents something other than itself to a knowing power." A sign leads a knower beyond the sign itself to a knowledge of the *signified*. This property of leading the knower to the knowledge of the signified is called the *signification* of the sign. Sometimes, the usage of the term *signification* is also extended to designate the signified itself. An example of a sign might be smoke, the signified in this instance being fire.

Signs may be divided into *natural* signs and *artificial* signs. This distinction is founded on diversity in the *origin* of signification. A natural sign is such because of some natural relation to the signified; e.g., smoke is naturally caused by fire. An artificial sign is such because of some relation to the signified which is artificially caused by some intelligence; e.g., the red light has no *real* relation to stopping the passing traffic but a relation made by human reason. In the light of this distinction between natural and artificial signs, we sharply distinguish between the modes of signification of words (language) and of intellectual acts (thought). While thought is a natural sign of the reality which is known through thought, language is only an artificial sign both of the thought which it expresses and of the reality to which the thought refers. This is evidenced by the wide variety of human languages which can be used to express the same thought. Nor is the fact that in diverse languages many corresponding words do not have precisely the same meaning (at least with respect to their "connotations") a serious objection. For we can without difficulty arbitrarily substitute any neologism we wish for any existing word and give to this neologism precisely the same meaning by convention. For example, we could substitute "gurk" for

16. Suzanne Langer's *Philosophy in a New Key* (New York, 1948) is a fine example of such an inquiry, although there is much light that the Thomist tradition could shed on the findings of this work.

17. *Ars Logica*, Part II, q. 21, a. 1: "Signum est id quod repraesentat aliud a se potentiae cognoscenti."

"mud", giving to it exactly the same signification as had been previously given to "mud".

Signs may also be distinguished into *instrumental* and *formal* signs. This distinction is founded on the diversity in the *mode* of signification. The instrumental sign is a sign which, being explicitly and directly first known in itself, then leads the knower to the knowledge of something distinct from itself. The formal sign is a sign which, without being explicitly and directly first known in itself, leads the knower to the knowledge of something distinct from itself. Smoke is an instrumental sign of fire; the sense-image in the eye[18] is a formal sign of what is being seen. We may say that an instrumental sign possesses more than a merely signifcative being, while a formal sign is a *pure* sign—its whole essence is to signify.[19] While thought is a formal sign of the reality which is known, language is only an instrumental sign both of the thought which it expresses and of the reality which is known through the thought. This is evidenced by the fact that the meaning of such language could remain unknown even as we see the language before us (we might not know the language), while this is not possible with the thought itself—thought is wholly and essentially a reference beyond itself; indeed we know what is thought about more immediately than we know the thought itself.

But even a formal sign may itself become the object of a reflective knowledge, in virtue of the very reflexivity of spirit itself. This is the case in formal logic, where we are studying formal signs, not *as* formal signs (i.e. precisely in their formal signification) but rather in their formal relations with each other.[20]

b. The symbol and types of symbol. The reader who expects nothing but logic in this paper must now be asked to permit a brief digression into philosophical psychology and other areas, a digression which could perhaps be omitted—but only at the price of passing over valuable light shed on the tendency of modern mathematical logic toward a quasi-mathematical symbolism.

We shall call all artificial signs symbols.[21] As we have already noted, every act of human thought must be "symbolized" in a material medium, a

18. Not the sense-image considered as a mere physical modification of the retina, but the sense-image considered as the intentional modification of the power of sight—this point is elucidated in philosophical psychology.
19. The reader may consult John of St. Thomas, *Ars Logica*, Part II, q. 22, or Maritain's *Les Degrés du Savoir*, c. III and Annexe I, for further discussions of the formal sign.
20. The diverse modes of signification of formal signs are studied in *material logic*, under the theory of supposition. The understanding of the ontological ground for formal signification pertains to *metaphysics*, in its treatise on truth, and to *philosophical psychology*, in its treatise on knowledge.
21. Maritain uses the term "symbol" to apply to a special type of artificial sign, i.e., the sign which is also an "image"—the "sign-image." See his *Quatre Essais sur l'Esprit dans sa Condition Charnelle* (Paris, 1939), p. 70.

sense-image. Philosophical psychology teaches us that no act of human thought would take place at all, were it not for the abstraction of intelligible species from concomitantly present sensible phantasms.[22] The initial phantasms, it is true, have the status of *natural instrumental signs* with respect to the ideas as they are first formed. But if we are to retain these ideas, we must make *artificial instrumental signs* (symbols); for we can neither retain firmly the entirety of the original complex phantasms (it would be too great a burden on the imagination) nor retain an idea without some associated phantasm. Moreover, only if these ideas are firmly retained can we hope to develop a more complex intellectual life. Hence the necessity for artificially organized sense-images as material symbols for our intellectual life.

"Human intelligence begins with conception, the prime mental activity; the process of conception always culminates in symbolic expression. A conception is fixed and held only when it has been embodied in a symbol."[23]

(i) Poetic and economical symbols of our intellectual acts. In the formation of these symbols, we proceed in two directions: toward complexity and toward economy and simplicity. Sometimes we find the sense-image constructed with a wealth of detail, making a much more apt instrument for clearly signifying a given intelligibility—this ordinarily takes place when the sense-image is externalized (in a multitude of possible ways, such as painting, sculpture, literature, etc. etc.) through a work of art. It may be a mere diagram or it may be a work of fine art. (Not that the work of fine art is to be fully understood from this viewpoint—there is more to it than this.) More often we find the sense-image constructed in an economical manner. It is only through the relations of this image to other images buried deeper in our unconscious that this image serves as an adequate sign of a given intelligibility. But economy of explicit symbolism gives us the ability to deal with more complex intelligibilities and chains of reasoning. It also enables us to communicate more complex meanings to each other and with greater ease—provided that we can work out a system of economical symbols reproducible in external matter in a *language* common to our social group.[24] Such economical symbols usually take the form of marks and sounds[25]—unless perhaps there is question of communicating with persons who have lost the use of both sight and hearing, such as Helen Keller. Then a system of tactile symbols must be invented, as has been done. We shall here restrict our attention to symbolic *marks* and *sounds*.

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22. St. Thomas Aquinas, *Summa Theologiae*, Part I, q. 85, a. 1.
23. Suzanne Langer, Introduction to *Language and Myth* by Ernst Cassirer, transl. by S. Langer (New York, 1946), p. ix.
24. From this point on, we will not distinguish between the symbolic sense-image and its reproduction in exterior matter.
25. But one must not ignore the role of *gestures*. Also, with sound, the pitch and rate of utterance, etc. may contribute as much to a symbol as the "shape" of the sound.

(ii) Simple and complex symbolic marks and sounds. The most common instances of symbolic marks and sounds are those of written and spoken language, which is ultimately composed of words. Words themselves are composed (at least in all non-ideographic languages[26]) of elements called letters, which by themselves do not constitute symbolic marks and sounds. Accordingly, words will be called complex symbolic marks and sounds. Of course, even in ideographic language, the written and spoken words are complex structures--but the complexity is not a complexity of "alphabetical" marks and sounds. Written and spoken language can then be described as a system of complex, economical, symbolic marks and sounds to facilitate thought and communication.

We find the same structure in all non-ideographic written languages: there is an alphabet of letters which can be combined in countless ways to form words. The words themselves are economical symbols, but a vast number of words are needed to correspond with the variety of knowledges needing symbolization. An indication of the scope of this variety can be seen in the fact that we so often give one word many shades of meaning—language never catches up with the increasing wealth of thought. To make our work of symbolization more economical, we need a comparatively small alphabet of primitive non-symbolic marks and sounds which can be used to construct the more complex symbolic marks and sounds.

Do we have instances of truly *simple* symbolic marks and sounds—that is, symbolic marks and sounds which are not composed of more primitive non-symbolic marks and sounds? One finds such simple symbols occasionally even in ordinary language, as in the case of the English article "a" or the interjection "O" or in punctuation marks. But these are rare exceptions in ordinary language. To find such symbolism at its best, one must go to modern mathematics, mathematical physics, and mathematical logic. Simple symbolism has so well succeeded in insuring the rapid progress of mathematics, and also of mathematical physics, that simple symbolic marks and sounds are generally called "mathematical"—and thus we speak of "mathematical" logic. But perhaps it would be better to call such symbols "quasi-mathematical," since considerable diversities in meaning may underlie the superficial similarity of all simple symbolism.

Of course, not even mathematics has achieved a thoroughly simple symbolism, in the sense defined at the beginning of the last paragraph. But we certainly can describe it as a *relatively* simple symbolism.[27]

(iii) The use of simple economical symbols. The value of simple symbolism consists in the fact that it can be held in the imagination more easily and in greater multiplicity than could a corresponding complex symbolism; for the complex symbolism already possesses a certain

26. The Chinese and Japanese have ideographic languages.

27. For example, the sign of differentiation d/dt is a complex symbol; but it is often replaced by a dot over the quantity to be differentiated. Even in the first case, of course, the symbol is still relatively simple.

multiplicity which cannot be compounded too greatly. Long chains of mathematical symbolism can still be comprehended relatively easily; but these would be incomprehensible if they were expressed in ordinary non-mathematical language—we would have forgotten the beginning by the time we reached the end. Of course, there are differences in imaginative power among men; but there is, nevertheless, a clear advantage in the possession of relatively simple symbolism for scientific purposes.

Carnap says, in this regard:

> A further advantage of using artificial symbols in place of words lies in the brevity and perspicuity of the symbolic formulas. Frequently a sentence that requires many lines in a word-language (and whose perspicuity is consequently slight) can be represented symbolically in a line or less. Brevity and perspicuity facilitate manipulation and comparison and inference to an extraordinary degree.[28]

The achievements made possible by such symbolism in mathematics, physics, and logic have been noted. But what makes such a symbolism especially possible in these sciences? We must say that it is their great degree of abstraction from determinate content. We have already seen that the immense diversity of content known calls for the complex symbolism of words and ordinary language. To resort to a simple symbolism in the face of this diversity of content is to attempt to make a simple ideographic language, which would present a great burden to the imagination, to say the least. Thus complex symbolism seems to be a desideratum with respect to the great diversity which we know in the world of daily experience, as well as in certain sciences. But in sciences in which diversity is greatly ignored and attention given to a relatively few common characteristics, there is the opportunity for simple symbolism. Thus it is with the study of quantitative manifolds, the study of the measurable appearances of matter, and—most pertinent to our inquiry—the study of the act of the intellect as to its internal relations considered in abstraction from the determinate content of the relata.[29]

c. Symbolization in formal logic. From the preceding, it should be clear that formal logic should aim at simple symbolism, at a quasi-mathematical symbolism. This is a necessity for its proper growth and entirely in keeping with the abstract nature of formal logic. Both elements and relations (and consequently laws as well) should be symbolized.

Some have concluded, from observing the value of quasi-mathematical symbolism in logic, that formal logic ought to aim at producing a material image of thought processes by application of the "principle of formalism"—

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28. Rudolf Carnap, *Introduction to Symbolic Logic and Its Applications*, transl. by W. H. Meyer and J. Wilkinson (New York, 1958), p. 2.
29. One might suspect that metaphysics is also amenable to expression in simple symbolism, inasmuch as it does not consider explicitly the implicit diversity of being—this suspicion seems to be justified, as I hope to show elsewhere.

that every nuance in thought should have a corresponding nuance in explicit symbolic expression. This in turn would dispense us from the need of considering thought in itself altogether. We could have a logic of the symbol instead of a logic of the symbolized. We must insist that the principle of formalism sets up an unattainable ideal; and that such a logic, even if it were achieved, could only be a secondary logic dependent for its intelligibility on the primary logic of human thought considered in itself. But the fuller discussion of these points is reserved for Part III.

3. Discovery of Hitherto Unknown Relations and Laws, Implicit in the Known Relations and Laws. Once the relations and laws which have been abstractively discovered and clarified are properly symbolized, the next step is to move from the knowledge of given relations and laws to the knowledge of other hitherto unknown relations and laws. For there are relations implicit in other relations and laws implicit in other laws. Relations implicit in other relations are discovered by mere abstractive visualization of the same relation-context from a new perspective. In virtue of the correlativity of relations, we are always able to move from the proposition that a has the relation R to b to the proposition that b has the relation S to a, where S is simply the reverse-correlate of R (e.g., if a is the father of b, then b is the son of a). This is really just a new abstractive visualization of the same context from a distinct perspective; there is no deduction here. Following such a movement of thought with respect to *logical relations* R and S, we might go on to formulate a *logical law*, "If a has the relation R to b, then b has the relation S to a" (e.g., if a includes b, then b is included in a). This is again an abstractive visualization of an elementary kind. Such logical laws are the simplest of all logical laws. We might call them "laws of correlativity."

Laws, except for the just mentioned "laws of correlativity," implicit in known laws are discovered by a true deduction from these known laws. The already known laws serve two functions in such a deduction. Some constitute the premises of the deduction, while at least one constitutes the law according to which this deduction takes place.

4. Construction of More Complex Relations and Laws. Step three may continue *ad infinitum*, for one may deduce an unlimited multitude of logical laws from a few given laws. But one may also move to another step in logical thought—where we construct[30] new complex logical relations and laws which are neither given in abstractive intuition nor deducible from what is given in abstractive intuition. One must distinguish two modes of constructing complex logical relations: (1) through a complex series of logical operations, (2) through the combination of various properties of logical relations.

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30. All logical relations are constructs with respect to their *being*; they are due to the natural or scientific art of reason, which is logic (see section C 1 above). But we are here distinguishing logical relations which are reflectively known by abstractive intuition from those which become known precisely through their construction.

The first mode of constructing logical relations proceeds as follows. As we saw in section C 3 a, logical relations are generated through logical operations. Accordingly, by a complex series of affirmations, negations, and reasonings, new complex relations may be generated which would not otherwise appear in logical discourse.[31] Such a deliberate generation of relations could not, of course, take place without a prior abstractive intuition of the simple logical operations and the relations consequent upon these operations.

The second mode of constructing complex relations is as follows. Relations have many properties. When we isolate these properties, it will be possible to combine them in various ways to obtain many new types of relations. Once again, however, such isolation and combination of properties presupposes a prior abstractive intuition of some relations having such properties. This second method of constructing relations will be of greatest importance in the generalized logic of relations. Once complex logical relations have been constructed, new logical laws containing these relations will be seen—these laws themselves can be called *constructed* laws, in so far as they only arise through the construction of some of their constituent relations.

5. Discovery of Other Relations and Laws Implicit in These Constructions. Step five follows the pattern of step three; we will omit the repetition of the details.

6. Axiomatization. Every human science seeks to achieve a unity out of some multiplicity of knowledge, to make a system of knowledge. Confronted with a flux of multiplicity, the human intellect seeks an underlying stable unity from which the multiplicity can be seen to flow. In some sciences, the underlying unity is the object of progressively clearer *abstractive intuition* and *demonstration*, e.g., in metaphysics and the philosophy of nature. In other sciences, the underlying unity is a *constructed* unity, e.g., a chemical or physical theory. In those sciences of the first type, the unity is an ontological unity, one that is seen in the things themselves; in the sciences of the second type, the unity is a non-ontological unity, a logical construction. But in each case, the achieved unity becomes a principle of deduction of the multiplicity.

In the science of logic, we have an almost unique case.[32] The unities underlying logical discourse—the primary logical relations and consequent laws—are themselves in a way *constructed* by the art of logic, although necessarily, as we have explained in section C 1 a of this chapter, in dependence on the nature of the subject-matter of the discourse. But these

31. An example of such a complex relation is that of the obverse to the obvertend; another is that of the conclusion of a traditional hypothetical syllogism to the premises.
32. This case is partially duplicated in mathematics. But the first principles of mathematical knowledge are radically non-constructed aspects of quantitative manifolds known in abstractive intuition; the first principles of logic, on the other hand, are radically constructed (through the activity of the intellect described in section C 1 a of the first part), although they may be known through reflective abstractive intuition.

primary logical relations and laws are also *known* in reflective abstractive intuition. The entire structure of abstract logical laws radically *originates* from the logical art of constructing logical relations, but these laws are *known* in abstractive intuition. Logic as a science must come first to know by abstractive intuition (and *re*-construction, as we shall see in Part III) the fundamental relations and laws which order the acts of human reason, and then to know and relate secondary relations and laws to the primary relations and laws in a constructive and deductive manner. The development of such a hierarchy of logical relations and laws is called *axiomatization*. The wealth of detail of logical relations and laws is by axiomatization reduced to the unity of a few primitive axioms and elements.

7. Statement of Logical Rules. Merely abstract logical relations and laws do not yet give us concrete order in the acts of human thought. It is necessary that appropriate logical relations be made between determinate acts of the intellect and that the logical laws be actually observed in any consequent discourse of reason. Propositions governing such concrete applications of logical laws are called logical rules.[33] If logical laws may be said to constitute an intrinsic formal structure of human thought, logical rules may be considered as the proximate extrinsic norms governing the imposition of such formal structure on human thought.

Thus, logical rules do not pertain so much to the *science* of logic as to the *art*. Therefore, the statement of logical rules might be left out of a strict development of the science of logic. Some have maintained that the logical rules, since they are propositions about logical propositions, should properly be studied in a logical science distinct from the logic we have been speaking of—in metalogic. And this is to be followed by a meta-metalogic, etc., etc. But if logic already treats of all relations between any acts of the intellect, taken indeterminately, and if the propositions stating logical rules are simply particular determinate acts of the intellect, then these rules are already studied from the logical viewpoint in what we have been calling logic. There is then no need for a metalogic at all, from this point of view. However, the distinction between propositions and propositions about propositions will be found to be a valuable one in logic. The requirement of having a metalogic in addition to logic itself is closely bound up with the view that logic confines itself to the study of language and is not concerned with the act of the intellect as such. This view will be discussed in Part III.

PART III. CRITICISM OF SOME CONTEMPORARY NOTIONS OF FORMAL LOGIC AND EXPANSION TO THE GENERALIZED LOGIC OF RELATIONS

In parts I and II, we discussed the nature and method of the science of logic from a Thomist viewpoint; but in contemporary philosophy the term "logic" is very often employed to name something other than what we have called logic. There are also serious differences as to what the method of

33. For example, the classical "eight rules of the syllogism" are logical rules.

logic should be. Of course, the term "logic" is only a word, a conventional sign; one may use such a sign to designate anything he wishes. From this viewpoint, logic may be any subject one wishes, and discussions and controversies as to the nature of logic are thereby rendered useless. But we hope to show here that the notions of formal logic about to be considered do in fact ultimately demand, or are but partial or distorted notions of, the formal logic which we described in the first two parts. Thus we will be able to conclude to a kind of priority for the use of the term "logic" to designate that subject. At the same time, the "generalized logic of relations" will be seen to be much broader in scope than Thomist formal logic as it is usually considered.

A. LOGIC AS "ARISTOTELIAN" LOGIC

For some, logic is only Aristotelian logic. "To think is to think like Aristotle." Immanuel Kant has concisely summarized this position, in the *Critique of Pure Reason*, as follows:

> That logic has already, from the earliest times, proceeded upon this sure path is evidenced by the fact that since Aristotle it has not required to retrace a single step, unless, indeed, we care to count as improvements the removal of certain needless subtleties or the clearer exposition of its recognized teaching, features which concern the elegance rather than the certainty of the science. It is remarkable also that to the present day this logic has not been able to advance a single step and is thus to all appearance a closed and completed body of doctrine.[34]

This conviction is fairly widespread among traditional[35] logicians. In general formal logic, it especially involves insistence that absolutely all forms of argumentation must be reducible to the Aristotelian logic of terms. Once we set aside immediate inference, this means that all general forms of argumentation must be reduced to the Aristotelian syllogism.

Oddly enough, Aristotle himself made no attempt at such a reduction. Rather, he did not explicitly treat of other general modes of argumentation. At approximately the same time as Aristotle in the history of logic, the Stoics were studying the logic of unanalyzed propositions; but Aristotle himself restricted his discussion to the logic of terms. Hence the attempt at reduction must be attributed to some of the "Aristotelians" rather than to Aristotle. Strangely, these "traditionalists" also often misrepresent the "Aristotelian" categorical syllogism itself, as Łukasiewicz has pointed out in his noteworthy study of *Aristotle's Syllogistic*.[36] But here we shall con-

34. Preface to the Second Edition, transl. by N. K. Smith (London, 1953), p. 17.
35. We use the term "traditional" here to designate logicians in the line passing from Aristotle through the medieval scholastics, on through John of St. Thomas, into modern times. It is contrasted with "mathematical," which designates logicians in the line passing from Boole and De Morgan through the early twentieth century mathmaticians seeking the foundations of mathematics in logic on to the contemporary "mathematical logicians." We shall consider some notions of these mathematical logicians about logic below.
36. Jan Łukasiewicz, *Aristotle's Syllogistic*, 2nd rev. ed. (Oxford, 1957), pp. 1-3.

fine our consideration to the attempts at reduction, without pointing out these discrepancies between the Aristotelian categorical syllogism and some of its "traditional" representations. These discrepancies do not seriously affect this question of reduction.

We shall here discuss three traditionalist positions concerning the reducibility of all modes of argumentation to that of the categorical syllogism. First, we shall consider the most naive position, which simply affirms reducibility without further discussion of any of the difficulties. Then we shall move to the consideration of a more philosophical reductionist position, but one which still insists that all arguments derive their force from some categorical syllogism or syllogisms. Then we shall consider the position which maintains that there are other types of argumentation which possess an intrinsic validity apart from any such reduction. We shall refer to such a view as logical pluralism. This position, once developed, can be seen to point toward a more general logic of which the Aristotelian syllogistic is only a part.

1. Naive Reductionism. Some, perhaps too greatly impressed by the beauty of the categorical syllogism, but in any case unable to reach a wider perspective, simply affirm without further discussion the reducibility of all forms of argumentation to the categorical syllogism. This attitude is still to be found among some traditionalists. But it really seems impossible to maintain such a simple position if the developments of the last century be at all considered. We will pass on to a considerably more philosophical reductionism.

2. Philosophical Reductionism. A far more serious reductionist treatment is to be found in the work of Maritain. Maritain is, of course, aware of the existence of hypothetical syllogisms and of their distinction from categorical syllogisms. He is also aware of the existence of the so-called "logic of relations." Let us follow him in his discussion of the relations of these modes of argumentation to the categorical syllogism. First, Maritain points out that hypothetical syllogisms differ from categorical syllogisms in their very structure.[37] Then he says that all hypothetical syllogisms can be reduced to the conditional syllogism.[38] Therefore his discussion focuses on the relation of the conditional syllogism to the categorical syllogism. Maritain says that the conditional syllogism is not properly *reducible* to the categorical syllogism, since these are different in kind.[39] But nevertheless he claims that the conditional can be *translated*, or *resolved*, into a categorical syllogism.[40] Such resolution, he says, in fact destroys the conditional syllogism;[41] but it is possible, since "the conditional syllogism...

37. Jacques Maritain, *Formal Logic*, p. 236.
38. *loc. cit.*
39. *ibid.*, p. 241.
40. *ibid.*, p. 242.
41. *loc. cit.*: "True, in proceeding thus, we have destroyed both the unity of the conditional syllogism, and that which constituted its proper nature. But this is so whenever a whole is resolved into its parts; that which constitutes the unity of the whole as such is by that very act dissolved."

contains virtually in its unity either one or two categorical syllogisms."[42]

The mode of resolution employed by Maritain is indicated in the following passage:

> ... The conditional syllogism is not a primary process of the reason; this manner of inferring a proposition, not from another proposition, but from a previously affirmed sequence between two propositions is, so to speak, a reasoning in the second degree, grafted upon the categorical syllogism which is the sole truly primary form of rational *discursus*: ...In order to give the *reason* for a conclusion we must always definitively attain to an essence or a universal nature, that is, to a categorical syllogism (or to an induction in the experimental sciences).[43]

From this passage, it should be quite clear that Maritain can only be referring to the logic of analyzed propositions. He does not seem to see the distinction between the logic of analyzed propositions and the logic of unanalyzed propositions. One wonders what Maritain might say concerning the logic of unanalyzed propositions. Perhaps some hint of this can be gained as we consider what he has to say concerning the general "logic of relations." First, he insists that propositions asserting relations other than the inherence of a predicate in a subject must be reduced to propositions asserting such a relation of inherence, if they are to contribute in any way to logical inference.[44] Then he asserts that syllogisms which seem to conclude in virtue of some relation other than that of a predicate to a subject (inherence, as he calls it) are not formally valid, but rather true in virtue of their matter. However, such syllogisms, he says, imply or suppose some formally valid categorical syllogisms and resolve into them.[45] He uses the example of the relation, "is greater than," and reduces the syllogism

A is greater than B.
B is greater than C.
Therefore A is greater than C.

to the categorical syllogism

Everything greater than B is greater than C.
But A is greater than B.
Therefore A is greater than C.

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42. *loc. cit.*
43. *ibid.*, pp. 244-245.
44. *ibid.*, pp. 95-96: "It may very well be that a certain confusion on this point has accidentally strengthened the theorists of the "logic of relation," particularly Bertrand Russell, in their opposition to the logic of inherence or predication. For against this logic they cite the irreducibility of an "affirmation of number," such as "Behold three men," to an affirmation of inherence.

    As a matter of fact, the speech with which we are concerned is either something other than an enunciation and is to that extent outside the domain of Logic, or else it is nothing other than an enunciation and is thus always reducible to the affirmation or negation of the presence of a predicate in a subject—or, in other words, of the identity *in re* of this Pr. and S."
45. *ibid.*, pp. 251-253.

From what we have just seen, it should be clear that Maritain is a rather thoroughgoing reductionist, although not to be located among the "naive" reductionists. The significant differences are: (1) an explicit treatment of the modes of resolution of the hypothetical to the categorical syllogism, after an explicit acknowledgement of their essential differences of structure; (2) an explicit discussion of the logic of relations and reduction of this to the logic of the categorical syllogism.

3. Logical Pluralism. But not all traditional logicians are reductionists. Many are "logical pluralists," maintaining that modes of reasoning other than the categorical syllogism can have a validity independently of the categorical syllogism. Some of the logical pluralists will still insist that a reduction to the categorical syllogism remains possible for all forms of argumentation, even if such reduction is not always necessary to establish the validity of such arguments. Others will insist that such a reduction does violence to such arguments, that in fact these other forms of argumentation not only have independent validity but also are simply irreducible to the categorical syllogism. To the former group belongs, for example, Esser,[46] while to the latter group belongs Joseph.[47] But this dispute is perhaps more a dispute over what to call reduction than over any real difference of logical doctrine. The agreement of all the logical pluralists that there are independently valid syllogisms other than categorical syllogisms is of much greater importance than this question. A concise statement of logical pluralism by a traditional scholastic logician can be seen in the following texts from Coffey's *Science of Logic*:

> Now, it is conceivable at all events, that the mind may establish, between its concepts, relations other than that of "*subject* to *attribute*." And we have in fact encountered some such already, e.g. *ground* and *consequent* in the hypothetical judgment. These relations may reveal others, by way of mediate inference, in virtue of certain principles distinct from, but just as immediately and intuitively evident as, the axioms that govern the various figures of the categorical syllogism.[48]

> Neither can it be objected that the *Dictum* is the only axiom that is conceptual in the sense that it alone regards the concept as a "*secunda intentio mentis*." The "extension" aspect of the concept belongs, of course, to the latter only in so far as the latter is considered to be a "*secunda intentio mentis*." The abstract concept is universalized only in and by the mind: the "*intentio universalitatis*" results from the mental comparison of the abstract type with the individuals from which it was abstracted, and thus makes the abstract type a "*second intention*." But precisely the same is true of the other relations, superadded to the direct concept by the mind, in the case of the other axioms. The mental act of relating one objective concept to others as an *equal*, *greater*, or *less*, magnitude than others, or as related in *time* or *space* to others, also makes such concept a *secunda intentio mentis*. And it is because there is a *foundation in reality* for *all* those mental relations, which we establish between our concepts, that our *conceptual* processes of inference are also *real*—that the application of the *secunda intentio* to the *prima intentio* is legitimate.[49]

46. Gerard Esser, S.V.D., *Logica* (Techny, Illinois, 1942), pp. 8, 150-159.
47. H. W. B. Joseph, *An Introduction to Logic*, 2nd rev. ed. (Oxford, 1916), pp. 339-340.
48. P. Coffey, *The Science of Logic* (New York, 1938), Vol. I, p. 386.
49. *ibid.*, p. 390.

Logical pluralism compels one to move toward a generalized logic of relations, such as we have spoken of above and will speak of below.[50] It also compels one to develop a notion of logical law (abstract syllogism) taking into account the fact that something like the Aristotelian "middle term" is still truly essential in all our non-immediate inferences. The generalized logic of relations still requires a term (but not necessarily a *mental term* in the strict sense—it could just as well be a *proposition* or a *set of intellectual acts*), to which two other terms are related in such a way that these latter two must themselves be related in such and such a way—or combinations of such triads—in order to constitute logical (syllogistic) laws.[51] Thus, logical pluralism, in leading us beyond the limitations of Aristotle's investigation, reveals the hitherto unsuspected analogicity of the "middle term." From this viewpoint, all valid syllogisms are *analogically* similar to the categorical syllogism; but this is a long way from reductionism.

B. LOGIC AS THE SYNTAX OF LANGUAGE

For many modern logicians, logic should concern itself not with the very act of thought but rather with *language*. For them, thought is an object too difficult to attain in itself; but, since thought finds its expression in language, these logicians believe that an analysis of the syntax of language can take the place of logic in the traditional sense. For example, consider the words of Reichenbach:

> ... we connect logical analysis, not with actual thinking, but with thinking in the form of its rational reconstruction. There can be no doubt that this reconstruction is bound to linguistic form; this is the reason that logic is so closely connected with language. Only after thinking processes have been cast into linguistic form do they attain the precision that makes them accessible to logical tests; logical validity is therefore a predicate of linguistic forms. Considerations of this kind have led to the contention that logic is *analysis of language*, and that the term 'logical laws' should be replaced by the term 'rules of language.'[52]

It must not be thought that these logicians all reject the very existence of an "act of thought." What they assert is that logic should confine its investigations and discussions to the "appearance" of thought in language, much as the same men often say that the natural scientist should confine his investigations and discussions to the "appearance" of the natural world in observable phenomena. About the existence of a translinguistic thought, logicians may feel as they choose, so this view holds, so long as the object

50. J. N. Keynes, *Formal Logic*, 3rd ed. rev. and enlarged (London, 1894), pp. 344-345: "What is required to fill the logical gap which is created by the admission that the syllogism is *not* the norm of all valid formal inference has been called the *logic of relatives*. The function of the logic of relatives is to "take account of relations generally, instead of those merely which are indicated by the ordinary logical copula *is*."

51. E. A. Burtt, *Principles and Problems of Right Thinking* (New York, 1928), pp. 130-226.

52. H. Reichenbach, *Elements of Symbolic Logic* (New York, 1947), p. 2. See also Carnap, *The Logical Syntax of Language* (London, 1937), pp. 1-2.

of logic itself is acknowledged to be restricted to the phenomenon of language.

Indeed, the "principle of formalism," as it is put forth, for example, by Łukasiewicz, seems to presuppose the existence of thought even while attempting to eliminate any need for its consideration in logic.

> Formalism requires that the same thought should always be expressed by means of exactly the same series of words ordered in exactly the same manner. When a proof is formed according to this principle, we are able to control its validity on the basis of its external form only, without referring to the meaning of the terms used in the proof.[53]

If the aim of this principle of formalism could ever be achieved, then "logic" could become a "grammar"; indeed, it could be a grammar in a much more strict sense than is traditional grammar. For traditional grammar has usually retained an intrinsic reference to the 'act of thought', even though its focus of interest has been in the 'expression of thought in language;' in the same way, traditional *logic* has felt the necessity of examining linguistic forms, even though its primary focus of interest is in the 'act of thought itself.' But for many this new logic considers linguistic expressions in themselves apart from any reference to thought at all.[54]

The science the object of which is constituted by this thorough application of the principle of formalism is neither logic nor grammar in the traditional senses of these terms. This so-called "logic" is directed only toward the construction of chains of uninterpreted signs in determinate, mutually consistent relationships. These "abstract" signs may then be "interpreted" by substituting concrete signs invested with determinate meanings. Thus "logic" here becomes a kind of "abstract language,"[55] or

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53. Łukasiewicz, *op. cit.*, p. 16.

54. But some would still retain a reference to the act of thought in their analysis of language or even make the analysis of language only a *means* to facilitate a better analysis of thought.

> "The purpose of the symbolic language in mathematical logic is to achieve in logic what it has achieved in mathematics, namely, an exact scientific treatment of its subject-matter. The logical relations which hold with regard to judgments, concepts, etc., are represented by formulas whose interpretation is free from the ambiguities so common in ordinary language. The transition from statements to their logical consequences, as occurs in the drawing of conclusions, is analysed into its primitive elements, and appears as a formal transformation of the initial formulas in accordance with certain rules, similar to the rules of algebra; logical thinking is reflected in a logical calculus. This calculus makes possible a successful attack on problems whose nature precludes their solution by purely intuitive logical thinking. Among these, for instance, is the problem of characterizing those statements which can be deduced from given premises." D. Hilbert and W. Ackermann, *Principles of Mathematical Logic*, transl. by L. M. Hammond, G. G. Leckie, and F. Steinhardt (New York, 1950), p. 1.

55. Carnap, *Introduction to Symbolic Logic and Its Applications*, transl. by W. H. Meyer and J. Wilkinson (New York, 1958), p. 1: "Symbolic logic (also called mathematical logic or logistic) is the modern form of logic developed in the last hundred years. This book presents a system of symbolic logic, together with illustrations of its use. Such a system is not a theory (i.e. a system of assertions about objects), but a *language* (i.e. a system of signs and of rules for their use). We will so construct this

a "logical syntax of language." Such a science does in fact find a place as a *part of* grammar in the traditional sense, when this *syntax* is taken together with *semantics* and *pragmatics* as constituting the science of *semiotic*. Indeed semiotic as a whole may be said to coincide with grammar in the traditional sense.[56] But when logical syntax is studied by itself, then it constitutes a science bearing much the same relation to traditional logic as the sciences of observable phenomena bear to the philosophy of nature. But if such a logical syntax is concerned only with the material symbols which are the "phenomena" of thought, and if these "phenomena" are merely marks or sounds without any necessary interrelationships, whence arises the necessity of logical laws? Whence does it come that such symbols present to us something as necessary as a tautology? Here appears a second analogy between the logical syntax of language and the sciences of observable natural phenomena.

The necessity of syntactical laws is imposed on language by convention, by the free construction of the human intellect. For the relations between linguistic symbols are imposed on these symbols by the human intellect, and the syntactical laws hold solely in virtue of such relations. The whole structure of relations and laws is very much like the constructed *a priori* level of physical science which is imposed on the phenomenal data by the theoretical physicist. However, one may ask how these relations arise in the first place, before they are imposed on the linguistic symbols. Clearly, they cannot be said simply to appear by themselves in the intellect. That would be to postulate a Kantian *a priori* for the formation of language. (Even if they were a natural apparatus of the human intellect, they would still of necessity be *thought*; this would still suffice to destroy the position of those who would exclude thought from logic altogether.) Rather, these relations arise in our *thought about reality* (see Part I, section C 1 a) and become reflected in the structure of language--the expression of this thought. Not that the structure of language is ever a perfect mirror of the

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symbolic language that into it can be translated the sentences of any given theory about any objects whatever, provided only that some signs of the language have received determinate interpretations such that the signs serve to designate the basic concepts of the theory in question. So long as we remain in the domain of pure logic (i.e. so long as we are concerned with building this language, and not with its application and interpretation respecting a given theory), the signs of our language remain uninterpreted. Strictly speaking, what we construct is not a language but a schema or skeleton of a language: out of this schema we can produce at need a proper language (conceived as an instrument of communication) by interpretation of certain signs."

56. Carnap, *Introduction to Semantics* (Cambridge, 1942), p. 8: "In an application of language, we may distinguish three chief factors: the speaker, the expression uttered, and the designatum of the expression, i.e. that to which the speaker intends to refer by the expression. In *semiotic*, the general theory of signs and languages, three fields are distinguished. An investigation of a language belongs to *pragmatics* if explicit reference to a speaker is made; it belongs to *semantics* if designata but not speakers are referred to; it belongs to *syntax* if neither speakers nor designata but only expressions are dealt with."

structure of thought.[57] But one can try to achieve an ever more perfect mirror of thought by the evolution of more and more technical expression. Such is the proper aim of the principle of formalism. The logical syntax of language can be actually constituted only in dependence on a "logical syntax of thought," even if it can be studied without explicit reference to such a logical syntax of thought. Since the whole order of language is derived from this order of thought, it is clear that the term "logic" should be applied primarily to the study of the order of thought and only secondarily to the distinct study of the order of language.

But since we need material symbols for our thought, it is desirable to evolve an ever more perfect symbolism, in accordance with the principle of formalism, in logic (the study of thought) in order to achieve ever more perfect logical analysis. Moreover, since even our *reflection* on our thought requires concomitant sense-images, the study of linguistic forms already made is indispensable in the cultivation of the science of logic (the reflective study of thought). But these two modes of dependence of logic, considered as the study of thought, on language do not in any way break down the essential distinction between this logic and the logical syntax of language.[58]

C. LOGIC AS THE STUDY OF THE STRUCTURE OF REALITY ITSELF

In *An Introduction to Logic and Scientific Method*, by Cohen and Nagel, we read that logic is "concerned with the question of the adequacy or probative value of different kinds of evidence. Traditionally, however, it has devoted itself in the main to the study of what constitutes proof, that is, complete or conclusive evidence."[59] This description is further amplified later when we are told that "The essential purpose of logic is attained if we can analyze the various forms of inference and arrive at a systematic way of discriminating the valid from the invalid forms."[60]

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57. Maritain, *Formal Logic*, p. 58: "Nor is it the purpose of language to furnish such a facsimile of thought: its object is to permit the intellect of the hearer to think, by an active, repetitive effort, what the intellect of the speaker is thinking."
    See also *ibid.*, pp. 75-76.
58. *ibid.*, p. 222: "We shall see that Logistics *differs essentially* from Logic. For whereas the latter bears upon the act of reason itself in its progress towards the true, and thus upon the order of concepts themselves and of thought, Logistics is concerned with the *relations between* ideographic *signs* and therefore with these signs themselves which, once determined, are taken as sufficient."
    ibid., p. 223: "In any event, aside from the question of whether or not the former is a legitimate and viable method, Logistics and Logic remain separate disciplines, entirely foreign to one another. Correctly understood they cannot contradict each other, since in reality they do not have the same object."
59. M. R. Cohen and E. Nagel, *An Introduction to Logic and Scientific Method* (London, 1934), p. 5.
60. *ibid.*, p. 20.

While these descriptions might both be criticized as descriptions of logic in general, it is still clear that they are at least rough descriptions of the subject we have been discussing in this paper. But when the authors go on to explain this subject further, it becomes evident that they see this subject in an entirely different perspective than we do. For they regard logic as revealing to us the most general characteristics of the real world itself.

> We turn now to the first formulation of the three so-called laws of thought. This formulation is an obvious counterpart of the propositional formulation. And it expresses, perhaps even more clearly, that their subject matter is certain *general or generic traits of all things whatsoever*. And the same may be said of *all* the principles of logic. From this point of view, logic may be regarded as the study of the most general, the most pervasive characters of both whatever is and whatever may be.[61]

Indeed, the authors even go so far as to attribute such a view to Aristotle himself!

> The insight that logical principles express the most general nature of things was first clearly expressed by Aristotle. At the same time he recognized that since the general nature of things is the ground for the correctness or incorrectness of reasoning, that general nature is also expressed in the principles of logic or inference. According to him, therefore, logic studies the nature of anything that is; "it investigates being as being." It is differentiated from other sciences because while the other disciplines examine the properties which distinguish one subject matter from another, logic studies those truths which hold for everything that is, and not for some special subdivision of what is apart from the others.[62]

These characteristics which the authors have ascribed to logic in the name of Aristotle—"it investigates being as being;" "logic studies those truths which hold for everything that is, and not for some special subdivision of what is apart from the others"—are ascribed by Aristotle himself not to logic but to metaphysics![63]

But let us set aside this historical error in the interpretation of Aristotle. The question still remains as to the actual subject of the science of logic. Does this science which we have been discussing reveal the nature of the real itself or not? Others also have claimed that it tells us of the nature of reality. Read, for example, the words of Bernard Bosanquet.

> ... The work of intellectually constituting that totality which we call the real world is the work of knowledge. The work of analyzing the process of this constitution or determination is the work of logic, which might be described as the self-consciousness of knowledge, or the reflection of knowledge upon itself.[64]

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61. *ibid.*, pp. 185-186.
62. *loc. cit.*
63. Aristotle, *Gamma Metaphysicorum* 1, 1003a 21-26: "There is a science which investigates being as being and the attributes which belong to this in virtue of its own nature. Now this is not the same as any of the so-called special sciences; for none of these others treats universally of being as being. They cut off a part of being and investigate the attribute of this part; this is what the mathematical sciences for instance do." (Ross translation)
64. Bernard Bosanquet, *Logic* (Oxford, 1888), Vol. I, p. 3.

In this text of Bosanquet, one can easily see the radically idealist presuppositions entailed by such a view of the nature of logic. If logic is the study of thought, and yet in its study of thought reveals to us the general characteristics of the real itself, this can only be because thought and reality are ultimately identical.

Indeed, the history of philosophy is full of examples of philosophies which have projected the attributes of thought considered in logic, onto the face of the real world itself. But they have done this in so far as they have confused the being of thought with the being of reality.[65] Consider, for example, the Platonic ideas--they are only hypostatized universal ideas. Certainly for Plato our logical analysis of universal ideas should seem to reveal something of the structure of the real itself. But this is only because he has made the initial mistake of projecting these ideas into the real world. Consider also the *real* emanation of beings, in the philosophy of Spinoza, by *logical* necessity. Or consider the dialectical becoming of reality, in the philosophy of Hegel, which is simply a projection of logical processes into the real itself. Reality tends to show the structure which is revealed in the logical analysis of thought, to the extent to which we have conceived reality after the manner of thought in the first place.

But while thought does in a way correspond to the real (with respect to *that which* the intellectual knowledge-act presents to us), nevertheless it also differs from the real (with respect to the *manner in which* intellectual knowledge-acts present their objects to us). Being exists in the intellect in a new way distinct from the way in which it exists in reality. This is true both in the concept and in the judgment. In the concept, the now immaterialized (abstract) content is accompanied by a relation to an actual or possible multitude of individuals in which this content could be realised. This relation of universality could not be found in reality, where only concrete individuals exist. In the judgment, notes which exist *in the intellect* in distinction from each other may be intellectually joined together by reason of their *real* identity in the thing itself. Affirmative judgment presupposes a distinction in thought, of contents which are judged to be identical in reality.

Of course, many relations between knowledge-acts do parallel many real relations. But, as we have just seen, some at least do not. This latter class is quite sufficient to force us to sharply distinguish between the world of reality and the world of thought. And once this distinction has been made, the respective provinces of metaphysics and logic are clearly distinguished. Leaving to metaphysics the investigation of "being as being" and "those truths which hold for everything that is, and not for some special subdivision of what is apart from the others," logic confines itself to the study of being as known, of those truths which hold for everything that is known in so far as it is known. From this viewpoint, if logic should happen, in some of its

65. See Maritain, *A Preface to Metaphysics* (New York, 1948), pp. 33-38, and *Reflexions sur l'Intelligence* (Paris, 1930), pp. 14-16.

content, to reflect the structure of reality itself in some way, this is accidental and not pertaining to the real purpose and interest of logic.[66]

And yet, one may ask, how then can logical inference be employed in sciences of reality? If the "laws of thought" are not, as such, the "laws of being," how can we securely discourse about reality itself in accordance with these "laws of thought"? Here it is necessary to recall that logical relations are, after all, relations between knowledge-acts which themselves refer to *reality* (actual or possible). These relations, "logical" with respect to their place and mode of existence, are nevertheless "real" in their ultimate origin—they have a foundation in reality (actual or possible). It is this foundation in reality which enables us to use such relations, and the laws consequent upon them, in discoursing about reality itself.

For example, consider the logical relation of *contradiction*. It arises from affirmation and negation. But the affirmation and negation themselves refer to the *real* opposition of being to non-being. The logical relation of contradiction exists only between knowledge-acts, but it is founded on the real opposition of being to non-being. Accordingly, the "law of contradiction" of logic $-p \rightarrow \sim(\sim p)$—can be employed in discourse about all real being which is opposed to non-being, i.e. about all real being whatsoever. The principle of contradiction is the "logical correlate" of the ontological "principle of identity."

Similarly, the logical relation of *extension* arises from the real possibility or actuality of the same nature being realized in diverse individuals. We may say that it is "founded on" such a real possibility or actuality. Because of this real foundation, relations of extensions to each other may be employed, in the logic of terms, in rational discourse about reality itself.

We must be careful to separate logical relations from reality, lest we fall into an idealist metaphysics. But, on the other hand, we must also be careful to bear in mind that these relations have their ultimate foundations

66. Note that it is one thing to identify logic and metaphysics and quite another to *use* logical constructions in the elaboration of a metaphysics. Such use is especially possible and fruitful when logic is envisaged as the study of abstract relations between indeterminate relata, as will be seen in the next section of this paper. Examples of such a metaphysical use of logic even among modern symbolic logicians, from rather diverse "metaphysical" points of view, may be seen in Carnap's *Der Logische Aufbau der Welt* (Berlin, 1928) and in Goodman's *The Structure of Appearance* (Cambridge, 1951), and is in fact becoming common. I put "metaphysical" between quotation marks out of respect for the positivist rejection of metaphysics among many symbolic logicians. Yet even in rejecting metaphysics, they make some alternative metaphysics of their own. For any attempt to use the constructions of symbolic logic to speak about general structures of the world can be called metaphysical in some sense, since ontology is indeed the first part of metaphysics. Such metaphysical use of logic was and is common in traditional Thomist metaphysics, although frequently enough without sufficient reflex awareness of the procedure. But, again, the *use* of logical constructions as means for a general understanding and ordering of the diversity, whether phenomenal or trans-phenomenal, of the real is quite another thing from the claim that logical constructions *of themselves* represent to us the ultimate metaphysical structure of the world. It is the latter view which is in this section subjected to criticism.

in reality, lest we place in doubt the relevance of logic to intellectual discourse *about reality*.

And yet, the view that logic studies the structure of reality itself is not without a foundation in fact. When formal logic is conceived simply as the study of abstract relations between indeterminate relata, as we shall consider it in the next section, *what* is studied indeed pertains to the order of thought, hence to logic in the Thomist sense; but these abstract relations, as will be noted, can be employed as *means* to the understanding not only of concrete logical relations but also of concrete real relations—that is to say, they are studied in logic, but they can be used by both logic and metaphysics.

D. LOGIC AS THE STUDY OF ABSTRACT RELATIONS BETWEEN INDETERMINATE RELATA

In the view of Suzanne Langer, logic is the study of forms, independently of all determinate content.[67] Since "the logical form of a thing is the way that the thing is *constructed*,"[68] and since this structure, composed of elements,[69] in relations to each other,[70] is especially characterized by these relations,[71] logic here must be conceived as the study of abstract relations between indeterminate relata. This notion of logic is expressed by Miss Langer in the following texts.

> But if we would hold aloof awhile from any special science and really gain insight into the great storehouse of forms which may be interpretable physically, or psychically, or for any realm of experience whatever, we must consider abstracted patterns as such—the orders in which any things whatever may be arranged, the modes under which anything whatever may present itself to our understanding.[72]

> ... Many things which look utterly unlike in experience—far more unlike than the motions of skyscrapers and of violin-strings, or tops and planets—are really made up in very similar ways, only it requires a good deal of practice to see this.
>
> The tracing of such types and relations among abstracted forms, or concepts, is the business of logic.[73]

Note that no distinction has been here recognized between real relations and relations of reason. Hence, it would seem that the subject here described is not logic, in our sense of the term. But this is not quite the case; for the subject here described does in a way include logic, at least partially. For if we examine what it means to consider a relation in abstraction from determinate relata, we shall see that this can only be a "second intentional," or logical, consideration.

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67. Suzanne Langer, *An Introduction to Symbolic Logic*, 2nd ed. rev., (New York, 1953), pp. 40, 240.
68. *ibid.*, p. 24.
69. *ibid.*, p. 47.
70. *ibid.*, p. 48.
71. *ibid.*, p. 45.
72. *ibid.*, p. 39.
73. *loc. cit.*

There is a dispute among scholastic philosophers as to whether real relations possess a being distinct from the being of the relata (the *subject* and the *term* of the relation considered as *founding* the relation). This dispute pertains to metaphysics and will not be discussed here. But whether or not any real relations possess a being distinct from the being of the relata, it is evident that the being of every relation is entirely dependent on the being of the relata. A relation presupposes the "related". Even those relations which arise from the consideration of the intellect (relations of reason) can arise only if there exist "relatables" in knowledge, and only in so far as they are relatable. Every relation is constituted as a relation through its relata. Since relations are thus dependent on the relata, no real relation can be considered as a real relation without a reference to its real and determinate relata. The same holds for any relation at all. Therefore, any consideration of a relation in abstraction from all determinate relata is a consideration of a *mental fiction* precisely as such.

There is left, in such a fiction, no relation at all, in the proper sense of the term "relation", since the relation had been "constituted" by its determinate relata which we have now abstracted from. We now have certain abstract "properties of relativity" rather than a relation in the proper sense. And yet this collection of "properties of relativity" still deserves, in some way, the name of "relation" inasmuch as it is of its very nature *relative*—this is a most peculiar situation! What can we call it except a "*relation-fiction*"? The consideration of such a fiction in itself is "second intentional," pertaining to the second intentional science of logic.

Metaphysics and the other sciences of the real are concerned with relations, but it is more proper to say that they are concerned with *real things as related.* Indeed, even logic itself, taken in its totality as composed of both formal and material logic, is concerned more with *knowledge-acts as related* than with the relations by themselves. But formal logic is focused directly on these very relations. For formal logic, it is more proper to say that it is concerned with the relations of knowledge-acts than that it is concerned with knowledge-acts as related. The removal of every reference to a determinate "absolute" in the consideration of "relation-fictions" enables us to focus our attention directly on the very properties of "relativity" themselves—the properties which are of direct importance to formal logic.

It is legitimate, and even necessary, for the full development of formal logic, for it to consider these "fictions" of relations in abstraction from all determinate relata—but on the condition that they be understood to be fictions. This understanding is not manifested by some of the proponents of the "logic of relations." Accordingly, they confuse real and logical relations, and they also confuse concrete logical relations (constituted by their relata) with abstract logical relations (fictions created by the logician). Could one perhaps object that such fictions are as usable for the analysis of concrete real relations as for the analysis of concrete logical relations? After all, is not such a fiction just as much removed from the concrete reality of thought as from the concrete reality of reality? Is it not just as

"dialectical" (i.e., proceeding through principles extraneous to the matter studied) to employ these fictions in either case? And if this is so, then this consideration of abstract relations between indeterminate relata might seem to pertain just as much to metaphysics as to logic.

We must answer that the two cases are not exactly parallel to each other. For although no relations can *exist* apart from determinate relata, nevertheless the relativity of a relation may be *conceived* (by a fiction) in distinction from, and without explicit reference to, determinate relata. Hence, in thought, "relations" can be found in abstraction from determinate relata; and therefore the reflective science of logic can consider them as part of its proper object, albeit at another level than that of concrete discourse itself. It is true, however, that the *use* of such relation-fictions is just as possible in the analysis of real relations as it is in the logical analysis of logical relations.

It is valuable at this point to consider just how these relation-fictions enter into the structure of formal logic. This is simply to go over what we have outlined earlier, in Part II. First, we make the relata to be completely indeterminate (in the generalized logic of relations) or relatively indeterminate (in the logic of terms or other such less general logics). Then we use these indeterminate relata together with the relation-fictions (which we have earlier simply referred to as *logical relations*) to express the logical laws.

In Part II, section 1, we described some of these abstract relation-fictions as being known "abstractively and intuitively" through an analysis of concrete discourse. But now we can understand that what are properly known abstractively and intuitively are only the concrete relations as having certain "properties of relativity"—these "properties of relativity" are then themselves abstracted from the concrete relations and combined in the relation-fictions to constitute the "abstract" logical relations. Hence, even these so-called "abstractively and intuitively known" logical relations presuppose a *constructive* combination of various properties of relativity for their constitution as abstract logical relations. But this constructive combination is not a distinct step in our understanding of such "abstractively and intuitively known" relations; the construction here takes place spontaneously and simultaneously with the abstractive intuition, and we are reflectively confronted with the "finished product"—"abstractively and intuitively known," so to speak. Later on, other abstract logical relations are constructed *reflectively* through a similar combination of properties of relativity. We referred to this step in logic in Part II, section 4.

Thus the logical law (abstract inference) can now be seen to be radically different in kind from the concrete inference. For the concrete inference includes logical relations between determinate relata—these relations are not the mere "relation-fictions" we have been speaking of here. When we substitute determinate relata for the variables of a logical law, the relation-fictions themselves become changed into determinate relations, but having the same properties of relativity as the relation-fictions had. It is only because of the identity of the properties of relativity in both

the abstract relation-fictions and the concrete determinate relations that we can persist in saying that the logical law (with its abstract relation-fictions) is an intrinsic formal principle of concrete inference (with its concrete determinate relations).

Thus we may distinguish two roles for the abstract relation-fictions in logic. First, they may be studied in themselves and as constituents of the abstract logical laws. Then secondly, they may be employed as *means* for the understanding of concrete logical relations and inferences. As such means, as we have indicated, they must be understood to be distinct from the concrete relations and inferences, and yet to coincide with these concrete relations and inferences with respect to the "properties of relativity" themselves.

Note that the first consideration of relation-fictions just mentioned is restricted to logic, since it takes place only through a reflection on thought itself, in which these fictions exist. It is from this point of view that we say that the *consideration* of abstract relations is "second intentional," pertaining to the second intentional science of logic. But the second role of relation-fictions is not restricted to logic. The abstract collections of properties of relativity may be employed as *means* for the understanding of concrete *real* relations as well as of concrete *logical* relations. It is from this point of view that we say that the *use* of abstract relations is not confined to logic.

From the perspective which we have adopted in this section, it is clear that our presentation of logical relations and laws, and of their relation to concrete inference, in Parts I and II, was a great simplification. For it did not at all take into account the differences between the abstract relation-fictions of abstract inference (logical law) and the concrete relations of concrete inference.

Finally, in accepting the notion of logic—in the light of the above analysis—as including the study of abstract relations between indeterminate relata, we have in fact moved from the only relatively general formal logic of Aristotle and the traditional logicians to the completely generalized formal logic of relations. The generalized logic of relations, taken in the light of the principles here enunciated, is in no way inimical to the spirit of the traditional formal logic, but rather can be considered simply as a natural development along the same line.

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