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A NOTE ON PRIOR'S SYSTEMS IN "THE THEORY OF DEDUCTION"

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In [3] Prior investigates two modal systems, say P1 and P2, which are related to S5 and S4 respectively and which can be described as follows:
1) Their primitive functors are $\mathfrak{C}$ (denoted in [3] by "F"), C and O (a constant impossible proposition).
2) They have the rules of procedure:

RI *If* $\vdash\alpha$ *and* $\vdash\mathfrak{C}\alpha\beta$, *then* $\vdash\beta$
RII *If* $\vdash C\alpha\beta$, *then* $\vdash\mathfrak{C}\alpha\beta$
RIII Substitution for variables and C for $\mathfrak{C}$ throughout any thesis.

3) The functors L, N and M are defined in the following way:

Df.1 $Lp = \mathfrak{C}\mathfrak{C}ppp$; *Df.2* $Np = COp$; *Df.3* $Mp = NLNp$

4) In P1 the following axioms are accepted:

A1 $\mathfrak{C}\mathfrak{C}\mathfrak{C}\mathfrak{C}pqrs\mathfrak{C}\mathfrak{C}qs\mathfrak{C}ps$
A2 $\mathfrak{C}pCqp$
A3 $\mathfrak{C}\mathfrak{C}pCpq\mathfrak{C}pq$
A4 $\mathfrak{C}\mathfrak{C}pqCpq$
A5 $\mathfrak{C}Op$

5) In P2 Prior adopts

A1' $\mathfrak{C}\mathfrak{C}pq\mathfrak{C}s\mathfrak{C}\mathfrak{C}qr\mathfrak{C}pr$
A2' $\mathfrak{C}CCpqpp$

and the axioms *A3*, *A4* and *A5*.

Prior has proved that, if we add to S5 and S4 axiomatized in the well-known manner of Gödel, *cf.* [2] and [1], a new primitive functor O and a new axiom, viz.

COp

then S5 and S4 strengthened in such a way are equivalent to P1 and P2 respectively. Besides, Prior presented a proof that in both these systems the following two theses

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N1 *ℭLCpqCpq* [Prior's formula *17*]
N2 *ℭℭpqLCpq* [Prior's formula *19*]

are provable. It seems that in these interesting systems Prior formulated rule **RIII** in too strong a way, because its unrestricted application reduces P1 and P2 to the classical propositional calculus. Namely, let us accept systems P1 and P2, thesis *N2* and, additionally, the theses

N3 *ℭLpp*
N4 *ℭℭpqℭℭqrℭpr*
N5 *ℭpCqp*
N6 *ℭLCNppLp*

We note that theses *N2-N6* are provable in both systems P1 and P2, although *N5* is not an axiom in P2. Now, we can procede as follows:

N7 *CCpqLCpq* [*N2*;**RIII**]
N8 *ℭCpqLCpq* [*N7*;**RII**]
N9 *ℭqLCpq* [*N4,p/q,q/Cqp,r/LCpq;N5,p/q,q/p,N8*;**RI**]
N10 *ℭpLp* [*N4,q/LCNpp,r/Lp;N9,p/Np,q/p;N6*;**RI**]

Since we have *N3* and *N10* in P1 and P2, both these systems are reducible to the classical propositional calculus. But, an inspection of the deductions presented in [3] shows clearly that we can easily improve this situation and, therefore, save both these systems. And, it can be accomplished even without a reduction of their deductive powers. Namely, to this end we should merely reformulate Prior's rule **RIII** as follows:

RIII* Substitution for variables throughout any thesis and substitution of *C* for ℭ throughout any thesis in which every constant is ℭ.

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