

PROPOSITIONAL LOGIC IN PLATO'S PROTAGORAS

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1. There is no evidence in his dialogues that Plato conducted an explicit study of propositional logic. Passages in which the structure of Plato's reasoning is clear typically exhibit modes of inference which would require formalization in the context of a logic of analyzed propositions.¹ On the basis of an examination of some of these passages, Bocheński finds untenable the suggestion that the rules of inference under Plato's command were such that they hold "between propositions as wholes without any analysis of their structure."²

The problem of the first flowering of propositional logic is sufficiently important to warrant careful examination of any evidence contrary to Bocheński's thesis. There is, in fact, striking evidence in the *Protagoras* that Plato was capable of extended reasoning on the basis of connectives belonging to the field of propositional logic. Nothing in the dialogue suggests that Plato had explicit definitions of these connectives, and there is no reason to think that he would have accepted definitions exactly along the lines of those in our truth-functional logic. There is reason to claim, nonetheless, that his argument here not only fits a truth-functional interpretation quite satisfactorily, but moreover that it seems incapable of being understood in several of its more obvious aspects without such an interpretation. If this claim can be shown tenable, we will have reason for believing that Plato at least was aware of the potentialities of propositional logic.

The argument which exhibits this reasoning is part of Socrates' final attack on Protagoras' thesis that knowledge and courage are distinct qualities of the virtuous man. Contrary to interpretations which construe this attack as an example of confused reasoning or as a brilliant *ad hominem*, it can be shown that Plato's argument amounts to a complete proof that Protagoras' thesis is inconsistent with other propositions which he explicitly accepts.³ I wish to discuss the structure of this argument, and to indicate reasons for thinking that Plato's achievement of a valid argument in this form was not accidental.

2. The three propositions around which this argument is built will be designated by the symbols '(1)', '(2)', and '(3)':

- (1) Courage and knowledge are distinct parts of virtue (349^c)
- (2) Pleasure is identical with good, and pain with evil (351^c)
- (3) Knowledge of good and evil can be overcome in practical decisions by momentary pleasure and pain (352^c).

The express purpose of the discussion in which the argument occurs is to investigate (1). Under Socrates' suasion, Protagoras has relinquished his earlier claim that knowledge, temperance, justice, and holiness are distinct parts of virtue. But he insists still that courage is different from knowledge. His reason for this opinion is the observation that ignorant men often are courageous. Socrates responds by pointing out that, although courageous men are confident, and generally more confident with more knowledge of their task, confidence is found also among men who are ignorant of the dangers they face. Protagoras' insistence, subsequently, that the ignorantly confident are not courageous but mad, gives Socrates the opportunity to remark that the difference between the courageous and the mad in this case would be a matter of the knowledge possessed by one and absent in the other. This suggests that knowledge is the cause of courageous action and hence is not to be distinguished from courage itself. Protagoras will not be caught so easily, however, and distracts attention from the drift of Socrates' remark by accusing him in effect of blundering into the fallacy of false conversion.⁴

Socrates abruptly alters his tactics, asking Protagoras without warning whether he would say that pleasure itself is a good thing. In response, Protagoras suggests that they consider the identification of pleasure and good as an hypothesis and investigate its consequences.⁵ Proposition (2) thus enters as a thesis which neither discussant initially is disposed to accept, and which bears no obvious relation initially to proposition (1), the main topic of discussion.

After renewing Protagoras' agreement to follow his lead in the discussion, Socrates asks, again with no apparent relevancy, whether Protagoras would agree with "the common man" that knowledge can be overridden by passion in our practical decisions. Protagoras rejects the suggestion that pain and pleasure are "more powerful" than knowledge in human life on the grounds merely that it is shameful. Consequently he is surprised when Socrates, who also rejects this suggestion, proposes nonetheless that they examine its consequences. Proposition (3) thus enters the discussion as an hypothesis which both discussants are committed to reject from the beginning. Socrates overcomes the other's unwillingness even to consider (3) by suggesting enigmatically that it "will help us to find out how courage is related to the other parts of virtue."⁶

Socrates then argues that persons who initially accept (3) must agree to reject (3) if they persist in maintaining (2). Such persons would claim that someone might be led to do something he knew to be evil by the distractions of momentary pleasure. But the evil to which they refer would be future pain or deprivation of future pleasure. Similarly, someone might fail to do what he knew to be good because of pain attending that action, and the good in this case would be future pleasure or the absence of future pain. In

making practical decisions, one's motivation always is to increase his lot of pleasure, whether present or in the future. Our happiness consists in choosing those courses of action which result in the most pleasure, the least pain, or the greatest overall ratio of pleasure over pain. Achieving the greatest amount of good, under (2), is a matter of the correct measurement of pleasures and pains. The man who chooses the course with greater future pain (evil) because of momentary pleasure does so because of deficient knowledge in the matter of the proper measurement of pleasure and pain, and the person who refuses the course with greatest future pleasure (good) because of present pleasure does so equally out of ignorance. It is ignorance, and not knowledge, of good and evil which is overcome by momentary pleasures and pains. One who knows how to measure pains and pleasures has knowledge of good and evil which is never overcome by passions of the moment. Proposition (3) thus is untenable under (2), since it leads to its own negation.

Ignorance in the measure of pleasure and pain, good and evil, is just what sophists like Protagoras, in fact, profess to remedy. Observing this, Plato brings Protagoras' own convictions back into the discussion, and with considerable blandishment obtains his explicit agreement with proposition (2).⁷ Perhaps Protagoras is more comfortable with (2) than at first, having seen that it can be used towards the rejection of (3). At any rate, (2) for Protagoras is no longer an hypothesis to be tested, but has become a thesis which he is committed to uphold.

On the basis of (2) and the negation of (3), Protagoras now is called upon to defend his assertion of (1). He agrees that no one chooses an action in expectation of evil (358^d). The expectation of evil is defined, to the satisfaction of all concerned, as fear. It follows that no one will choose what he fears when it is possible for him to avoid this (358^e), and that everyone, courageous or not, will choose the course of action which inspires confidence. Protagoras insists, however, that what the coward approaches (*ἔρχονται*: 359^e) must be just the opposite of what the brave man approaches, since it is a fact that the brave man willingly meets battle while the coward tends to retreat. This inclination to battle on the part of the brave is honorable, while the opposite tendency of the cowardly is disgraceful. But what is honorable is good, and by hypothesis what is good is pleasant. Thus the coward acts out of ignorance when he refuses to enter battle, since in that direction lies the greatest overall pleasure. The brave man enters battle, on the other hand, out of expectation of pleasure in that action.

Thus it turns out that the difference between the coward and the brave man is the difference between ignorance and knowledge of pleasure and pain. Courage is just knowledge of what is and what is not to be feared. Faced with this consequence, Protagoras refuses to answer the question whether he still believes that the ignorant can be courageous and retires from the argument.

3. Socrates has shown, in a fashion satisfactory at least to Protagoras, that if one accepts (2) then he must admit both that the assertion of (3) entails its own negation and that the negation of (3) entails the negation of (1).

That is, if Protagoras accepts (2), then he can accept (3) only if he accepts the negation of (3), and if he accepts the negation of (3) then he must accept the negation of (1). Moreover, Socrates has elicited from the incautious Protagoras an explicit acceptance of (2). Protagoras therefore is committed to accepting the following complex proposition:

$$(A) \quad (2) \supset [(3) \supset \neg(3). \neg(3) \supset \neg(1)]. \quad (2)$$

It is obvious, however, that (A) entails the negation of (1), being in fact equivalent to the conjunction $\neg(1). (2). \neg(3)$. Protagoras had explicitly accepted the latter two members of this conjunction, but had not bargained for the former. On the basis of his agreement with (2), Protagoras' original assertion (1) has been shown untenable.

4. Reductions to absurdity are not uncommon in the Platonic dialogues. In the *Phaedo*, where Socrates is made to give an explicit description of his dialectical method, it is stated (101^d) that the first task in the examination of an hypothesis under debate is to test its consequences for consistency. This procedure is illustrated copiously in the *Phaedo* itself, as well as in the *Euthyphro* and the *Meno* among the earlier and the *Theaetetus* and the *Sophist* among the later dialogues. What makes this encounter with Protagoras worthy of particular interest is Plato's competent use of a system of logical relations for which presumably he had neither explicit notation nor proof procedure, and the deliberate care with which he introduces into the argument just those assertions which are needed to achieve a formally sound proof of inconsistency within propositional logic.

Both (2) and (3), in fact, are introduced into the argument in a rather peculiar fashion, which in either case has been misinterpreted by commentators who have not grasped the overall structure of the argument. Proposition (3) was interjected, over Protagoras' protestations, as representing the view of "the common man" which neither discussant accepts. The elaborate argument which follows is interpreted by Taylor as Plato's response to an obligation he feels to show that the common acceptance of (3) is based on an improper analysis of the facts of practical decision.⁸ It is true that Socrates does refer frequently in these passages to "the opinion of the common man." But this opinion is not represented by any character in the dialogue, and it is not characteristic of Plato to portray an elaborate discussion of a view which is not represented explicitly either by a well-known philosopher or by a member of the discussion. Taylor's interpretation, moreover, allows no suggestion as to how we should construe Socrates' remark that (3) "will help us find out how courage is related to the other parts of virtue," which is his justification for examining (3) in spite of the fact that no one in the discussion is willing to defend it.

It is more satisfactory to conceive that Plato was quite aware of the structure of the logical net in which Protagoras unwittingly was becoming enmeshed, and that he realized the net would be unsound without (3) and its consequences. It is easily seen, in fact, that if this examination of the view of "the common man" were eliminated from the argument, Protagoras still could consistently maintain (1). The proposition:

(B) $(2) \supset [- (3) \supset - (1)]$. (2)

is equivalent to $'(2)(3) \vee - (1)(2) - (3)'$. Protagoras' reasons for rejecting (3) in the first place are obviously uncritical (he thought it shameful to suggest that knowledge could be overruled by passion). If he had been faced with the alternative of accepting (3) or of rejecting (1), which (B) allows, presumably he would have preferred merely to have changed his mind about (3). The proposition (B) in itself is insufficient to constrain Protagoras to relinquish (1), which is the main purpose of Socrates' argument.

The introduction of (2) into the argument is equally forced, and equally necessary for Socrates' purpose. Protagoras just previously had accused Socrates of an elementary logical blunder. Instead of replying to this charge, Socrates without warning asks Protagoras' opinion on (2), a proposition which seems irrelevant to the discussion at that point. Proposition (2), like (3), at first is defended by neither discussant, and Taylor is right in correcting those critics who have charged Plato with hedonism on the basis of this passage.⁹ But Taylor is wrong in remarking that Protagoras himself does not accept (2).¹⁰ Although at first (2) is treated as an hypothesis to be examined, subsequently not only Protagoras but also his colleagues Hippias and Prodicus are portrayed as deliberately accepting it.⁷ The reason for this again seems clear, if we allow that Plato was aware of the logical structure of his argument. The proposition:

(C) $(2) \supset [(3) \supset - (3)]. - (3) \supset - (1)$

which is merely (A) without the conjunction of (2), is equivalent to $'-(2) \vee - (1)(2) - (3)'$. Since Protagoras initially had agreed to consider (2) only as an hypothesis, he could readily have rejected it by accepting the left member of this disjunction, thus preserving his thesis (1).

Mysterious as the intrusion of (2) and (3) into the argument appears at first reading, it is clear that Protagoras could not have been convicted of inconsistency in his assertion of (1) without them as the argument stands. On the basis of the discussion above, it seems reasonable to believe that Plato was aware of this logical fact. Yet if Plato had neither a notation nor a proof procedure for some form of propositional logic, we must assume that in some sense he "intuited" the rather complex relations exhibited in a comparison of (A), (B), and (C).¹¹ This, even for Plato, would be an admirable "intuition."

NOTES and REFERENCES

1. See *A History of Formal Logic*, I. M. Bocheński, translated and edited by Ivo Thomas, University of Notre Dame Press, 1961, p. 31.
2. *Ibid.*
3. For these interpretations respectively see Grote's *Plato*, Vol. 2, London, 1875, third edition, p. 86, and Taylor's *Plato: The Man and His Work*, Meridian Edition, pp. 257-261. Plato's argument occurs in the passages

349^b through 360^c. References to the *Protagoras* henceforth will be indicated in the text. All English quotations are from the translation of W. K. C. Guthrie, reprinted in *Plato: The Collected Dialogues*, edited by Edith Hamilton and Huntington Cairns. All references in Greek are from *The Protagoras of Plato* of Harper's Classical Series, 1881.

4. Protagoras misconstrues Socrates as having argued that, since the courageous are confident and those with knowledge are confident, courage and confidence are identical. Taylor suggests that the purpose of this brief episode is merely to bring out the distinction between 'all A is B' and 'all B is A'. Bocheński remarks, apropos of this passage, that Plato found questions like this difficult, and that "to show the invalidity of the foregoing rule of conversion, he betakes himself to complicated extra-logical discourse—about bodily strength, for instance (p. 35)." The interpretations of Protagoras, Bocheński, and Taylor are equally wrong, for Plato does not make Socrates commit a logical blunder in this passage. The reasoning here presages Mill's Method of Difference: Οὐκοῦν οὗτοι, ἧν δ' ἐγώ, οἱ οὕτω θαρραλέοι ὄντες οὐκ ἀνδρεῖοι ἀλλὰ μαινόμενοι φαίνονται; καὶ ἐκεῖ αὖ οἱ σοφώτατοι οὗτοι καὶ θαρραλεώτατοί εἰσιν, θαρραλεώτατοι δὲ ὄντες ἀνδρεϊότατοι; καὶ κατὰ τοῦτον τὸν λόγον ἢ σοφία ἢ ἀνδρεία εἶη; (Well, those who are thus ignorantly confident show themselves not courageous but mad, and conversely, in the other case it is [sic] the wisest that are also most confident, and therefore most courageous? On this argument it is their knowledge that must be courage: 350^c). One of the dramatic functions of this passage is to give a preview of the final phase of the longer argument which follows, and which Protagoras in the end finds inescapable.
5. The "hedonistic hypothesis," according to which pleasure and good are identified, is introduced by Protagoras, not by Socrates. This should be born in mind in considering the opinion that Plato is erecting an hedonistic ethics in this dialogue. (see Grote, *op. cit.*, pp. 87-89). Socrates has merely suggested that all pleasure is good, not that all good is pleasure. After mistakenly censoring Socrates for improper conversion, Protagoras should have been alert to the distinction. Note also that in proposing this hypothesis for consideration, Protagoras calls attention to Socrates' dialectical method by which hypotheses are accepted or rejected according to their consequences. This should alert the reader to the presence of a similar procedure within the dialogue itself at this point.
6. Τί δέ, ὦ Σώκρατες, δεῖ ἡμᾶς σκοπεῖσθαι τὴν τῶν πολλῶν δόξαν ἀνθρώπων. οἷ ὅ τι ἂν τύχῃσι τοῦτο λέγουσιν; Οἶμαι, ἧν δ' ἐγώ, εἰναί τι ἡμῖν τοῦτο πρὸς τὸ ἐξευρεῖν περὶ ἀνδρείας, πρὸς τὰλλα μῦρια τὰ τῆς ἀρετῆς πῶς ποτ' ἔχει. εἰ οὖν σοι δοκεῖ ἐμμένειν οὕς ἄρτι ἔδοξεν ἡμῖν, ἐμὲ ἠγήσασθαι, ἧ οἶμαι ἂν ἔγωγε κάλλιστα φανερόν γενέσθαι, ἔπου· εἰ δὲ μὴ βούλει, εἴ σοι φίλον, ἐῷ χαίρειν. 'Ἄλλ', ἔφη, ὀρθῶς

λέγεις· καὶ πέραινεν ὡςπερ ἤρξω (But why must we look into the opinions of the common man, who says whatever comes into his head? I believe, I replied, that it will help us to find out how courage is related to the other parts of virtue. So if you are content to keep to our decision, that I should lead the way in whatever direction I think we shall best see the light, then follow me. Otherwise, if you wish, I shall give it up. No, you are right, he said. Carry on as you have begun: 353^b).

7. ὑμᾶς δὲ δὴ μετὰ Πρωταγόρου ἐρωτῶ, [ὦ] Ἱππία τε καὶ Πρόδικε—κοινὸς γὰρ δὴ ἔστω ὑμῖν ὁ λόγος—πότερον δοκῶ ὑμῖν ἀληθῆ λέγειν ἢ ψεῦδεσθαι. Ὑπερφύως ἐδόκει ἅπασιν ἀληθῆ εἶναι τὰ εἰρημένα. Ὁμολογεῖτε ἄρα, ἦν δ' ἐγώ, τὸ μὲν ἡδὺ ἀγαθὸν εἶναι, τὸ δὲ ἀνιαρὸν κακόν. τῆν δὲ Προδίκου τοῦδε διαίρεσιν τῶν ὀνομάτων παραιτοῦμαι. εἴτε γὰρ ἡδὺ εἴτε τερπνὸν λέγεις εἴτε χαρτόν, εἴτε ὀπόθεν καὶ ὅπως χαίρεις τὰ τοιαῦτα ὀνομάζων, ὃ βέλτιστε Πρόδικε, τοῦτό μοι πρὸς ὃ βούλομαι ἀποκρίναι. Γελάσας οὖν ὁ Πρόδικος συνωμολόγησε, καὶ οἱ ἄλλοι. (That then is the answer we should make to the ordinary run of people, and I ask you—Hippias and Prodicus as well as Protagoras, for I want you to share our discussion—whether you think what I say is true. They all agreed most emphatically that it was true. You agree then, said I, that the pleasant is good and the painful bad. I ask exemption from Prodicus' precise verbal distinctions. Whether you call it pleasant, agreeable, or enjoyable, my dear Prodicus, or whatever name you like to apply to it, please answer in the sense of my request. Prodicus laughed and assented, and so did the others: 358^{a, b}).
8. See *Plato: The Man and His Work*, p. 259. Taylor, more than other commentators, is aware of a sound logical structure in the argument of 351^b-360^e, but does not indicate awareness of the full impact of the argument.
9. Taylor, *op. cit.*, p. 260.
10. *Ibid.*
11. Another passage in which Plato exploits relations of propositional logic which would be easier to "intuit" occurs at *Republic*, 327^c. Polemarchus warns Socrates that either (4) he is more than a match for the company, or (5) he must stay to converse. Socrates proposes an alternative, (6) that he persuade Polemarchus and the others to let him go. Polemarchus points out that they cannot be persuaded if (7) they refuse to listen, and states that they will indeed refuse. It follows, by *modus ponens* and disjunctive syllogism, that either (4) or (5), as Polemarchus originally had insisted. The structure of the argument is: (4) ∨ (5) ∨ (6) . (7) ⊃ -(6) . (7) ∴ (4) ∨ (5). I am indebted to Milton Fisk for this example.