Notre Dame Journal of Formal Logic
Volume IV, Number 3, July 1963

A FINAL NOTE ON S1° AND THE BROUWERIAN AXIOMS

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[1] ended with the question whether the addition of any Brouwerian-axiom B_{2n} : $LCpL^{2n}Mp$ $(n \geq 1)$ to S1° would yield S5. It is shown here that no single axiom B_n taken with the T° of [2], i.e. S1° and the Gödel-rule to infer LP from P, is sufficient to contain even T. We interpret in the property calculus as is done for T, S4, S5 in [3] where $(Np)a = N(pa)$, $(Cpq)a = C(pa)(qa)$, $(Lp)a = \Pi bCUabpb$, $(Mp)a = (NLNp)a = \Sigma bKUabpb$. It is easy to see that the transcriptions of the axioms of S1° are obtainable without special axioms for U, as are those of the Gödel-rule and the Lewis-rules except detachment and replacement, which need (1) $\Pi b\Sigma aUab$ which we adopt as an axiom. Having the Gödel-rule, we need only consider B_n in its material form CpL^nMp. To get the transcription of this we add the axiom:

(2) $CUa_0a_1CUa_1a_2 \ldots CUa_{n-1}a_nUa_na_0$ whence the required

(3) $Cpa_0\Pi a_1CUa_0a_1\Pi a_2CUa_1a_2 \ldots \Pi a_nCUa_{n-1}a_n\Sigma bKUa_nbpb$ is readily proved.

Interpreting U in a set of n + 1 elements, $0, 1, 2, \ldots, n$, and putting $Ujk = 1$ except for $k = j + 1$ and $j = n$, $k = 0$ when $Ujk = 0$; adopting also the usual $0 - 1$ matrix for C, K, N etc. with 0 designated, and putting $\Pi aFa = 0$ $(\Sigma aFa = 0)$ if $Fa = 0$ for all (some) values of a and otherwise $= 1$, we see that (1) takes the value 0, as does (2) in all cases. But the transcription of $CpMp$, viz. $Cpa\Sigma bKUabpb$, gets the value 1 if we take px as Unx and give a the value 0. Thus T° augmented by any one B_n does not contain T.

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Received June 14, 1963

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