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STUDIES ON THE AXIOM OF COMPREHENSION

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In a previous paper in this journal (see [1]) I constructed a model **M** of a set theory such that the axiom

$$(Ey)\ (x)\ (x \epsilon y \leftrightarrow \phi(x))$$

is valid, where $\phi(x)$ may contain some parameters $z_1,\ldots, z_n$ and is built by conjunction and disjunction alone from atomic propositions $u \epsilon v$, where u and v are any two of $x, z_1, \ldots, z_n$. In particular it was also allowed that $\phi(x)$ is just a propositional constant *0* (false) or *1* (true). In this note I shall add a few further results concerning models of set theories for which certain axioms are given. In § 1 I first mention some general forms of the comprehension axiom and then prove a further theorem on the model in [1]. In § 2 I give a new proof of a result in [2], where a certain 3-valued logic was considered. In § 3 I show some further examples of models of set theories in ordinary 2-valued logic.

§1.

We may consider 3 forms of the axiom of comprehension. The first is that partially treated in [1], although I prefer to write it here in the form

$$(1)\quad (z_1)\ldots(z_n)\ (Ey)\ (x)\ (x \epsilon y \leftrightarrow \phi(x, z_1, \ldots, z_n)),$$

where ϕ is either a propositional constant or built from atomic expressions $u \epsilon v$ by negation, conjunction and disjunction and there are no further variables in ϕ than $x, z_1, \ldots, z_n$. The second form is

$$(2)\quad (z_1)\ldots(z_m)(Ey)(x)(x \epsilon y \leftrightarrow \prod_{u_1}\ldots\prod_{u_n}\phi(x, z_1, \ldots, z_m, u_1, \ldots, u_n)),$$

where ϕ as before is built by the connectives of the propositional calculus while each $\prod_{u_r}$ means either universal or existential quantification with regard to u_r. It may be advantagous also to consider a third form

$$(3)\quad (Ey)\ (x)\ (x \epsilon y \leftrightarrow \prod_{u_1}\ldots\prod_{u_n}\phi(x, u_1, \ldots, u_n)),$$

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where ϕ is built by the connectives from atomic terms $u \epsilon v$, where u and v are among $x, u_1, \ldots, u_n$.

Theorem 1. The model **M** *on the lower half of p. 19 in* [1] *satisfies the axiom* (2) *for any* ϕ *without negation.*

Proof. It was proved in [1] that axiom (1) is fulfilled in **M**. Therefore it will suffice to prove that in **M** every expression

$$\prod_{u_1} \ldots \prod_{u_n} A(x, z_1, \ldots, z_m, u_1, \ldots, u_n)$$

is equivalent to an expression $B(x, z_1, \ldots, z_m, c_1, \ldots, c_n)$, where $c_1, \ldots, c_n$ are some special elements of **M**. Here A and B are built from atomic propositions $u \epsilon v$ by conjunction and disjunction alone. In order to see that such an equivalence takes place it will be convenient to deal with the truthvalues so that conjunction means minimum, disjunction means maximum. Then the truthvalue of $(u)A(u)$ is $\min_u A(u)$, that is the minimum of the truthvalues of $A(u)$ when u runs through all elements of **M**, and similarly the truth value of $(Eu)A(u)$ is $\max_u A(u)$. Now looking at the ε-table for **M** we see that the values of $\varepsilon(x,y)$, that is $x \epsilon y$, constitute a steadily non-decreasing function with regard to both x and y when the elements of u are ordered by inclusion. It is then obvious that every expression $A(x, z_1, \ldots, z_m, u_1, \ldots, u_n)$ built by the operations max and min from atoms $\varepsilon(u,v)$ will possess the same property. As a consequence of this we have for example

$$\min_{u_1} A(x, z_1, \ldots, z_m, u_1, u_2, \ldots, u_n) = A(x, z_1, \ldots, z_m, O, u_2, \ldots, u_n)$$

and

$$\max_{u_1} A(x, z_1, \ldots, z_m, u_1, u_2, \ldots, u_n) = A(x, z_1, \ldots, z_m, V, u_2, \ldots, u_n)$$

where O is the null-set in **M** and V the total set. Repeating this for $u_2, \ldots, u_n$, we obtain

$$A(x, z_1, \ldots, z_m, u_1, \ldots, u_n) \quad A(x, z_1, \ldots, z_m, c_1, \ldots, c_n),$$

where each of $c_1, \ldots, c_n$ is either O or V. Thus we get an equivalence of the desired kind and our theorem is proved.

It was shown in [1] that no finite model exists for which (1) is valid for ϕ without negation. It may therefore be of some interest to see that finite models exist for which (3) is valid for negation free ϕ. Let us for example take a model **M** with the 5 elements *0, 1, 2, 3, 4* while the relation ε is given by the table

ε	*0*	*1*	*2*	*3*	*4*
0	*0*	*0*	*0*	*0*	*1*
1	*0*	*0*	*0*	*1*	*1*
2	*0*	*0*	*1*	*1*	*1*
3	*0*	*0*	*1*	*1*	*1*
4	*0*	*1*	*1*	*1*	*1*

By the way this model was obtained in [1] on p. 18 as a preliminary step in the construction of a model satisfying (1). Of course (1) is not valid for this finite model **M** but (3) is. In order to show this it is again advantageous to notice that $\varepsilon(x,y)$ is a steadily non-decreasing function with regard to x and y in the ordering $0 < 1 < 2 < 3 < 4$ and consequently every expression $A(x,u_1,.., u_n)$ built up by max and min from atomic terms $\varepsilon(u,v)$ will have the same property with regard to all of the variables $x,u_1,..., u_n$. Hence for example $\min_{u_1} A(x,u_1,.., u_n)$ where u_1 runs through $0,1,2,3,4$ or more explicitly written

$$\min\,(A(x,0,u_2,..,u_n), A(x,1,u_2,..,u_n), \ldots, A(x,4,u_2,..,u_n))$$

must be equal to

$$A(x,0,u_2,\ldots,u_n)$$

and similarly

$$\max_{u_1} A(x,u_1,u_2,\ldots,u_n) = A(x,4,u_2,\ldots,u_n).$$

Repeating this one obtains

$$\prod_{u_1} \ldots \prod_{u_n} A(x,u_1,\ldots,u_n) = B(x)\ ,$$

where $B(x)$ contains no variable except x. Indeed $B(x)$ is built by max and min from atoms $\varepsilon(u,v)$, where u and v are either both x or one of them 0 or 4. However $\varepsilon(u,0) = 0$ for all u, $\varepsilon(u,4) = 1$ for all u and for all v $\varepsilon(0,v) = \varepsilon(v,1)$, $\varepsilon(4,v) = \varepsilon(v,3)$. Therefore $B(x)$ can in any case be built by max and min from $\varepsilon(x,x)$ which is $= \varepsilon(x,2)$ and $\varepsilon(x,1)$, $\varepsilon(x,3)$. However the truthvalues $0,1$ together with $\varepsilon(x,1)$, $\varepsilon(x,2)$ and $\varepsilon(x,3)$ yield by max or min again one of the same values. Therefore $B(x)$ is either one of the truthvalues or it is $\varepsilon(x,1)$ or $\varepsilon(x,2)$ or $\varepsilon(x,3)$. But that means that the value of $B(x)$ is equal to one of $\varepsilon(x,0)$, $\varepsilon(x,1)$, $\varepsilon(x,2)$, $\varepsilon(x,3)$, $\varepsilon(x,4)$ so that (1) is valid.

I shall now show the existence of a set theoretic domain for which axiom (1) is valid for an arbitrary ϕ in which also negation may occur, provided that no atomic term of the forms $u \epsilon u$ or $u \bar{\epsilon} u$ occurs. In the first instance it can be shown that (1) will be valid for a domain **M**, if and only if **M** contains a null set and a total set and the sets in **M** are reproduced in the following way:

1) To every a in **M** there is an individual $\breve{a}$ such that for all x in **M**

$$a \epsilon x \leftrightarrow x \epsilon a$$

I will call $\breve{a}$ the convers of a.

2) To every a in **M** there is an $\bar{a}$ such that for all x in **M**

$$x \epsilon \bar{a} \leftrightarrow x \bar{\epsilon} a.$$

I call $\bar{a}$ the complement of a.

3) To any individuals a and b in **M** there is an individual $a \cup b$ such that for all x in **M**

$$x \epsilon a \cup b \leftrightarrow x \epsilon a \vee x \epsilon b.$$

We call $a \cup b$ the union of a and b.

4) To any a and b in **M** there is an individual $a \cap b$ such that for all x in **M**

$$x \epsilon a \cap b \leftrightarrow x \epsilon a \ \& \ x \epsilon b.$$

The set $a \cap b$ is called the intersection of a and b.

It is evident that if (1) is valid for all expressions ϕ, where no term $u \epsilon u$ or $u \bar{\epsilon} u$ occurs, then **M** possesses the 4 properties of reproduction and there is a set O and a set V in **M**. Now let us inversely assume that **M** has these properties. We may write any given ϕ as a disjunction $K_1 \vee K_2 \vee \ldots \vee K_\ell$, where each K_r is a conjunction of terms $u \epsilon v$ and $u \bar{\epsilon} v$, where u and v are any two different ones of $x, z_1, .., z_n$. Now every $z_i \epsilon z_j$ and $z_i \bar{\epsilon} z_j$ will simply be one of the truth values for arbitrarily given $z_1, .., z_n$. Further $z_i \epsilon x$ can be replaced by $x \epsilon z_i$ and $x \epsilon u \ \& \ x \epsilon v$ can be replaced by $x \epsilon u \cap v$ besides $x \epsilon u \vee x \epsilon v$ by $x \epsilon u \cup v$. By repeated use of these transformations every K_r can be written in the form $x \epsilon y_r$, where y_r is composed from $z_1, .., z_n$ by conversion, building of the complement, building of intersections. Finally the whole expression ϕ is equivalent $x \epsilon y_1 \cup y_2 \cup \ldots \cup y_\ell$, so that (1) is correct.

Theorem 2. We may construct a set theoretic domain **M** *for which the axiom* (1) *is valid for any ϕ in which no atomic term $u \epsilon u$ or $u \bar{\epsilon} u$ occurs.*

Proof. We let **M** be the series of non negative integers. A suitable relation ε can be defined as follows. The number 0 shall be the null-set, that is for every integer x we shall have $x \bar{\epsilon} 0$. Then 1 shall be the total set, that is for every integer we have $x \epsilon 1$. Further the truth values of $x \epsilon y$ is defined recursively. Letting $p(x,y)$ be a function such as for example $\binom{x+y+1}{2} + x$ which yields a one to one correspondence between the pairs of integers and the integers themselves I take

1) $2n$ as the complement of $2n+1$
2) $6n+7$ as the converse of n
3) $6p(m,n)+3$ as the intersection of m and n
4) $6p(m,n)+5$ as the union of m and n.

It is easy to see that this is a recursive definition of the relation ε. Indeed we have already determined the truthvalue of $x \epsilon y$ for $y = 0$ and $y = 1$. Let a pair (x,y) be declared smaller than (z,u) if $x+y < z+u$. Because of 1) it is sufficient to be able to determine the truthvalue for an odd number ℓ provided that we know the value for all numbers $y \leqq \ell - 2$ in smaller pairs. Now ℓ will be of just one of the three forms $6h+7$, $6h+3$, $6h+5$. In the first case $x \epsilon \ell$ has the same truthvalue as $h \epsilon x$. In the second case $x \epsilon \ell$ has the same value as $x \epsilon m \ \& \ x \epsilon n$, where $p(m,n) = h$ and similarly in the third case. Thus the value of $x \epsilon y$ is determined by referring it to values for smaller pairs.

However there is still one thing that ought to be proved namely that the axiom of extensionality is fulfilled. One observes that every set will be repeated infinitely often. For example the integer 3 will be the intersection of 0 and 0, that is again 0. Now let $a < b$ but having the same elements. It is then possible to prove by induction that always $a \epsilon y \leftrightarrow b \epsilon y$. Indeed we have at once that this is so for $y \leqq 1$. Let us therefore assume the correctness of this for all $y \leqq n$. If $a \epsilon n$ and $b \epsilon n$ have different truthvalues, so have $a \epsilon n + 1$ and $b \epsilon n + 1$ if n is even. Therefore, letting ν be the odd one of n and $n + 1$, we have $a \epsilon \nu$ inequivalent $b \epsilon \nu$. Here ν may have the form $6h + 7$. Then we get that $h \epsilon a$ and $h \epsilon b$ should have different truthvalues contrary to the supposition that a and b have the same elements. Then ν may have the form $6h + 3$. Let c and d be the numbers such that $p(c,d) = h$. Then it follows that $a \epsilon c$ & $a \epsilon d$ is inequivalent $b \epsilon c$ & $b \epsilon d$, whence either $a \epsilon c$ inequivalent $b \epsilon c$ or $a \epsilon d$ inequivalent $b \epsilon d$. However this is contrary to the hypothesis of induction because c and d will be $< \nu - 1 \leqq n$. The same reasoning applies if ν has the form $6h + 5$. Thus our domain is a correct model.

§ 2.

Two years ago I published a paper (see [2]) in which I studied an axiom of comprehension in a logic with 3 truthvalues say $0, \frac{1}{2}, 1$ and the 3 connectives conjunction, disjunction, negation with values given as minimum, maximum, 1-p. I constructed in [2] a model for which (1) was valid. I will here carry out this construction in a simpler way which is analogous to that in § 1.

We may first notice that (1) is valid in a domain **M** of objects if and only if **M** has the following 5 properties:

1) There is in **M** a nullset O, a total set V, a set H and a set W such that for all x in **M** the value of $x \epsilon H$ is $\frac{1}{2}$ and

$$x \epsilon W \leftrightarrow x \epsilon x.$$

2) Every set a in **M** has a converse $\breve{a}$ in **M**.
3) " " complement $\bar{a}$ **M**.
4) Any sets a and b in **M** has in **M** an intersection $a \cap b$
5) " " " a union $a \cup b$.

It is evident that **M** must possess these properties when (1) is valid. Let us inversely assume that **M** has all 5 properties. We may also here write a propositional function ϕ built by the connectives from atomic terms $u \epsilon v$ as a disjunction $K_1 \vee K_2 \vee \ldots \vee K_{\ell}$, where every K_r is a conjunction of atoms $u \epsilon v$ and $u \bar{\epsilon} v$. Here u and v are any two of $x, z_1, .., z_n$. For given individuals $z_1, .., z_n$ every term $z_i \epsilon z_j$ is just one of $0, \frac{1}{2}, 1$. Further $z_i \epsilon x$ is $\leftrightarrow x \epsilon \breve{z}_i$, $x \epsilon \breve{z}_i \leftrightarrow x \epsilon z_i$, $x \epsilon z_i$ & $x \bar{\epsilon} z_j \leftrightarrow x \epsilon \bar{z}_i \cap z_j$, $x \epsilon x \leftrightarrow x \epsilon W$. Therefore every K_r can be written in the form $x \epsilon y_r$, where y_r is built up by the operations $\smile$, $-$, $\cap$, $\cup$ from $z_1, .. z_n$ and W. Finally we get $\phi \leftrightarrow x \epsilon y_1 \cup y_2 \cup \ldots \cup y_{\ell}$ because of the property 5) which shows that (1) is true for **M**. Then I assert:

Theorem 3. It is possible to construct a model **M** *for which* (1) *is valid.*

Proof. We may again let **M** be the sequence of non negative integers defining a suitable ε-relation between them. We let the integers $0, 1, 2, 4$ be respectively the sets O, V, W, H. Further I put $3 = \bar{2}$ and for every $n > 1$

$$2n + 2 = \overline{2n + 1}, \quad 6n + 1 = \breve{n} \text{ for } n > 0,$$

$6p(m,n) + 3 = m \cap n$ for $m + n > 0$, $6p(m,n) + 5 = m \cup n$ for all m, n.

This yields a recursive definition of the relation ε. Indeed $x \epsilon 2n + 2$ is defined as having the values $0, \frac{1}{2}, 1$ according as $x \epsilon 2n + 1$ has the values $1, \frac{1}{2}, 0$. Further if y is an odd number, it is either of the form $6n + 1$ for some n, or it is of the form $6p(m,n) + 3$ for some m,n or the form $6p(m,n) + 5$. Then $x \epsilon 6n + 1$ is defined by saying that it shall have the same truthvalue as $n \epsilon x$, further $x \epsilon 6p(m,n) + 3$ the same value as $x \epsilon m$ & $x \epsilon n$ and $x \epsilon 6p(m,n) + 5$ the same value as $x \epsilon m \vee x \epsilon n$. In all cases $x \epsilon y$ is determined by reference to the truthvalues of the ε-relation for smaller pairs. Since these values are given for $y = 0, 1, 4$ we can determine every $x \epsilon y$ either by $x \epsilon 0, 1$ or 4 so that it is found or by $x \epsilon 2$. This last truthvalue will depend on x, let it be called $\xi(x)$. Then in particular $x \epsilon x$ will be a function $f(\xi(x))$, where f is built by the operations min, max and $1-$ from the argument $\xi(x)$. However it was proved in [2] that such a function always has a fixpoint. Therefore we can always find a value $0, \frac{1}{2}, 1$ of $\xi(x)$ such that

$$\xi(x) = f(\xi(x))$$

or in other words

$$x \epsilon x \leftrightarrow x \epsilon W.$$

According to the preceeding considerations the series of non negative integers supplied with the defined ε-relation constitute a domain for which (1) is true. That the axiom of extensionality is valid as well is easily proved in the same way as we did in §1.

In an earlier paper (see [3]) I treated the comprehension axiom using a logic set forth by J. Łukasiewicz with infinitely many truthvalues. I proved that also for this logic the axiom (1) could be satisfied without contradiction. A modified proof of this similar to those given above is certainly possible. If I have understood correctly what Prof. C. C. Chang at Princeton wrote to me in a letter, he is able to show the consistency even of axiom (2). I shall not here pursue any further the problems connected with many valued logics, but give some further examples of models satisfying axioms in ordinary 2-valued logic. These are of course of greater interest for mathematicians.

§ 3.

One has often the impression that mathematicians talk about set theory as something unique. They appear to mean the Zermelo-Fraenkel theory. Of course the assumed uniqueness is illusory. Already the axiom of choice has been a subject of discussion. One might say however that the set theoretic reasoning in ordinary mathematics mostly follows the axioms of the Z+F- theory except that some authors will not accept the axiom of choice. By the way, the axiom of choice is quite different from the other axioms which are or at least can be formulated (see [4]) as cases of the axiom (2). Some examples of systems of axioms which are cases of (2) shall here be shown to be consistent.

First of all it may be noticed that the Z+ F-axioms with the only exception of the axiom of infinity are valid in certain domains consisting of finite sets exclusively. The simplest domain of this kind is obtained by defining an ε-relation between the non negative integers thus: We say that $x \epsilon y$ shall mean that x is an exponent of a power of 2 occurring in the decomposition of y as a sum of different powers of 2. Since for example $7 = 2^2 + 2 + 1$ the integers $0, 1, 2$ are the elements of 7. Every number y will then be conceived as a finite set of numbers. Clearly all Z+ F-axioms are valid except the axiom of infinity.

Making this domain slightly more complicated we can get infinite sets but in order to be sure that the other axioms remain valid we would have to introduce such complications that I cannot carry out. I must be content with the following remark. Let us say that $x \epsilon 2y$ shall mean that x is an exponent of 2 in the decomposition of y as a sum of powers of 2, whereas $x \epsilon 2y + 1$ shall mean that x is not an exponent of 2 in this decomposition of y. Then the series of integers constitute a domain N of sets with the following properties:

1) To every m in N there is an individual $\{m\}$ in N such that $x \epsilon \{m\} \leftrightarrow x = m$.

2) To m and n in N there is an individual $\{m,n\}$ in N such that $x \epsilon \{m, n\} \leftrightarrow x = m \vee x = n$

3) To every m in N there is an individual $\overline{m}$ in N such that $x \epsilon \overline{m} \leftrightarrow x \bar{\epsilon} m$

4) To every m in N there is a $\mathbf{S}m$ in N such that $x \epsilon \mathbf{S}m \leftrightarrow (Ey(x \epsilon y \ \& \ y \epsilon m)$

5) " $\mathbf{D}m$ " $x \epsilon \mathbf{D}m \leftrightarrow (y)(x \epsilon y \vee y \bar{\epsilon} m)$

6) " m and n " intersection $m \cap n$

7) " " " union $m \cup n$

8) There are sets m in N, for example N itself, such that $0 \epsilon m$ & $(x)(x \epsilon m \rightarrow \{x\} \epsilon m)$

The proofs of these properties are very simple add are therefore omitted. By the way some of the properties are trivial consequences of the other ones. Perhaps it ought to be noticed that $\mathbf{S}m = N$ and $\mathbf{D}m = O$ for all infinite sets m. The property 8) is just Zermelo's axiom of infinity. The existence of a total set N and of complements is not in accordance with the Z+ F-theory, but in Quine's theory it is so. It would be of great interest to try to modify the domain so that the mentioned properties remain valid while further the existence of a powerset $\mathbf{U}m$ for every m was realized which means that for all x in the domain

$$x \epsilon \mathbf{U} m \leftrightarrow (y)(y \bar{\epsilon} x \vee y \epsilon m).$$

I have not made any serious attempt to accomplish this.

I shall now show an example of a domain N with the properties:

1) There is a nullset O. To every m there is a set $\{m\}$.
2) To every m there is a set $\mathbf{Z}m$ consisting of the sets m, $\{m\}$, $\{\{m\}\}$, ...
3) For every set m there is a powerset $\mathbf{U}m$.

We let again N be the sequence of non negative integers. The null set may be the integer 0. Further 1 can be $\{0\}$. For $m > 0$ we let $\{m\}$ be $3m$. Further $\mathbf{Z}m$ and $\mathbf{U}m$ can be $3m+1$ and $3m+2$ respectively. It is then clear that $3m$ has just the single element m, every $3m+1$ has the elements $m, 3m, 3^2m, \ldots$ so that it only remains to find the elements of $3m+2$. In order to see that $x \epsilon y$ can be defined by a kind of recursion we may define first an auxiliary notion "degree." Every number can be derived from 0 by use of the three operations

$$m \rightarrow 3m, \; m \rightarrow 3m + 1, \; m \rightarrow 3m + 2,$$

and this can only be done in one way if we ignore applications of the first operation on 0 leaving 0 unchanged. By the degree of a number y I will then understand the number of times the third operation has been applied in the formation of y from 0. Now $x \epsilon 3m+2$ may be defined by $x \epsilon z$ for z of lower degree than $3m+2$. Indeed for all x of the same or higher degree than y we shall always have $x \epsilon y$. Further for x of lower degree than $3m+2$ we have

$$x \epsilon 3m + 2 \leftrightarrow (y)(y \bar{\epsilon} x \vee y \epsilon m).$$

Since x and m are of lower degree than $3m+2$ we observe that this is a definition of $x \epsilon 3m+2$ provided $x \epsilon y$ is known for y of lower degree than $3m+2$. This is a recursive definition of the same kind as that mentioned for the predicate u(a,k) in [5], p. 287. By the way it is in the case considered here rather easy to find effectively all elements of $3m+2$ for given m but I omit an exposition of that. I shall only mention as an example that the elements of 8 are $0, 3^n, 3^n+1, 3^{n+1}+2$ for $n = 0,1,2, \ldots$ Further it may be added that every m in our domain has a union $\mathbf{S}m$. Indeed $\mathbf{S}0 = 0$ and every set $\neq 0$ is of one of the forms $\{m\}$, $\mathbf{Z}m$, $\mathbf{U}m$, while $\mathbf{S}\{m\} = m$, $\mathbf{SZ}m = \mathbf{Z}\{m\}$, $\mathbf{SU}m = m$.

It will not be difficult in an analogous manner to construct a model which not only has the properties of the just treated model but also together with any two of its members m and n contains a member $\{m,n\}$.

I have not yet had the opportunity to extend these investigations to systems of axioms, where also the "Aussonderungsaxiom" or cases of it are taken into account, but I don't think there will be any essential difficulties.

According to a theorem of Löwenheim the use of the natural number series as a model is not a limitation in any case.

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