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A NOTE ON A WEAKENED GOLDBACH-LIKE CONJECTURE

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Introduction. This paper presents a constructive existence proof for a certain maximal doubly mutant set [1] in the algebraic system determined by the additive and multiplicative monoids of all nonnegative integers. It shows that if a weakened version of Goldbach's conjecture is a theorem then that constructed maximal property can be extended to be relative to *all* nonnegative integers.

1. *Results.*

Lemma: In the additive and multiplicative monoids of nonnegative integers there exists, *in a constructive sense*, a doubly mutant set that is a maximal one relative to the set of all positive odd integers.

Proof: Put E_o equal to the set of all odd prime numbers, i.e., $E_o = \{3, 5, \ldots\}$. Put $E_1 = \{P_{i_1} \cdot P_{i_2} \cdot P_{i_3} : P_{i_j} \epsilon E_o\}, \ldots, E_n = \{P_{i_1} \cdot P_{i_2} \cdots P_{i_{2n+1}} : P_{i_j} \epsilon E_o\}, \ldots.$ Put $\mathbf{S} = \bigcup_{i=0}^{\infty} E_i$. Clearly $\mathbf{S}$ is a doubly mutant set. Specifically $\mathbf{S}$ is an additive mutant set since, e.g., the sum of two odd integers is an even integer. It is a multiplicative mutant as is easily verified by using the fact that the set of all positive odd integers is an additive mutant and by using the *Fundamental Theorem of Arithmetic.* Clearly $\mathbf{S}$ is a doubly mutant set that is a maximal one relative to the set of all odd positive integers since by the *Fundamental Theorem of Arithmetic* $\mathbf{S} \cdot \mathbf{S}$ equals the complement of $\mathbf{S}$ relative to the set of all positive odd integers greater than the multiplicative idempotent element *1*, which cannot be an element of a multiplicative mutant set.

Now suppose that $\mathbf{S} + \mathbf{S}$ equals the set of all positive even integers greater than four (a weakened form of Goldbach's conjecture [2], [3]). Then $\mathbf{S}$ is a doubly mutant set that is a maximal one relative to the set of *all* nonnegative integers and, *a fortiori*, if Goldbach's conjecture is a theorem, then, among numerous other things, $\mathbf{S}$ is a doubly mutant set that is a maximal one relative to the set of all nonnegative integers.

REFERENCES

[1] A. A. Mullin, "On Multiply Mutant Sets," *Notices Amer. Math. Soc.*, vol. 8, No. 4, August, 1961, p. 357.

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[2] R. G. Archibald, "Goldbach's Theorem," *Scripta Math.*, vol. 3, 1935, pp. 44-50 and pp. 153-161; further references to Goldbach's conjecture may be found there.

[3] A. A. Mullin, "An Abstract Formulation of a Problem Related to Goldbach's Conjecture," *Amer. Math. Monthly,* vol. 68, No. 5, May, 1961, pp. 487-488; *erratum*: in line 3 of definition 1.3, for "$\bigcap_{i \in I} M_i$" read "M_i".

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