Notre Dame Journal of Formal Logic
Volume III, Number 2, April 1962

ON THE INFINITY OF POSITIVE LOGIC

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No direct proof seems to have been published of the theorem that no finite matrix can be adequate to the positive logic of implication, which is here proved.

In the alphabet $p_1, p_2, \ldots, p_n$, $(n > 1)$, form $(p_i \supset p_j) \supset p_j$ for all i, j: $1 \leqq i < j \leqq n$, and $p_i \supset p_1$ for all i: $1 < i < n$. A_n is to have all these expressions as antecedents, p_1 as consequent. Then A_n is not a positive thesis but becomes so if any two variables are identified. Hence any n–1 valued matrix that validates the positive system, validates A_n. Hence no finite matrix is adequate to the positive system.

Proof: That for no n is A_n positive is shown by the fact that if the variables are valued by their subscripts A_n has the value 1 in the infinite matrix of Dummett's **LC** for which cf. [1]. While if any two variables are identified, there results either an antecedent equivalent in the positive system to p_1, or a pair of antecedents p_i, $p_i \supset p_1$, the consequent being always p_1.

REFERENCE

[1] M. A. E. Dummett: A propositional calculus with denumerable matrix, *The Journal of Symbolic Logic*, vol. 24 (1959), pp. 97-106.

Received February 24, 1962