TOPIC AND CONSEQUENCE IN OCKHAM'S LOGIC

OTTO BIRD

How Ockham Recast the Topical Tradition

The late Fr. Boehner once suggested that the mediaeval theory of Consequences had its origin in the study of the Topics. For this he cited Ockham as "a witness and a contemporary" (PB. 54).* I have shown in a previous paper that this development was already well under way in Abelard's analysis of the Topics (BI). In this paper I propose to analyze that part of Ockham's theory of Consequences which can be shown to be based on the Topical tradition that runs back through Peter of Spain and Abelard to Boethius (in his work *De Differentiis Topicis*). Although this work was ultimately based on the *Topica* of Aristotle (through the commentary of Themistius, which Boethius summarizes), it provided the standard *auctoritas* for the mediaeval Topical tradition, not only during the centuries in which the Aristotelian text was unavailable, but also even after its recovery in the 12th century. This I shall refer to as the Boethian Topical tradition, as distinguished from the Aristotelian, by which I shall mean Topical analysis based immediately upon the text of Aristotle. Ockham is something of a rarity among mediaeval logicians in that he deals, in separate parts, with both the Boethian and the Aristotelian tradition of the Topics. The treatment of the Topics in such representative works as those of William of Shyreswood, Peter of Spain, Albert of Saxony, John Buridan seems to be confined entirely to the Boethian tradition.

In this paper I shall be dealing primarily with Ockham's treatment of the Boethian tradition. I shall show how he recast that tradition so as to obtain distinctions for his theory of consequences. It will then appear that Boehner was at least over-hasty and inaccurate in saying that Ockham has only a "loose arrangement" and "wisely omitted" a systematic ordering of the Topical consequences. (PB. 55). Furthermore, previous study of Ockham's theory of Consequences–Salamucha, Boehner, Moody–has paid practically no attention to those Consequences which derive immediately from Topical analysis. Yet these in quantity alone far outweigh all the others. Thus Boehner in his exposition draws upon only one of the forty-

*Such citations are to the table of references at the end of the paper.

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five chapters of Ockham's treatise on Consequences (*Summa totius logicae*, III-III), and leaves to one side the thirty chapters dealing directly with the Topics (PB. 55-69). The same is true of Salamucha (S. 104-116).

The Traditional Analysis of a Topical Argument

To show what Ockham does to the Topics it is necessary to know the traditional analysis. For this I shall use Peter of Spain. His treatise on the Topics is one of the most concise, and the work in which it occurs, Tract V of the *Summulae Logicales*, had become the standard elementary textbook by the 14th century.

The traditional analysis is perhaps most clearly seen from an example. Thus as an argument from the Topic of Genus we have:

T1. An animal is not running, therefore a man is not running. (PH. 5.16)

If we ask, what validates or warrants such an argument, we are told that it consists of two things. First, of what is called the Topical Difference (*Locus differentia maximae* - PH. 5.07), which in this case consists of:

'Animal' is the genus of the species 'man'.

Secondly, of the appropriate Topical Maxim (*Locus maxima* - PH. 5.07), in this case the following:

M1. Whatever is removed from the genus is also removed from the species (*Quicquid removetur a genere, et a specie* – PH. 5.16)

In our example, T1, the predicate 'running' is removed from 'animal', which is the genus of 'man', and therefore it can also be removed from 'man'.

A Topical argument, i.e. T1, may thus be looked upon as an incomplete argument which is completed by appeal to the Topical Difference, and the Topical Maxim, M1. The complete lay-out of the argument, consisting of all three, is thus an inference-scheme.

In the Boethian tradition the treatise on the Topics consists of an enumeration of the Topical Differences with their appropriate Maxims. Peter of Spain enumerates some 21 Topical Differences with 81 Maxims. He divides them according as they are Intrinsic, Extrinsic, or Mixed. In this he is following Boethius, who distinguishes Topical Maxims according as they are composed from terms appearing in the question, i.e. in the Topical argument–in which case they are Intrinsic–or are taken from outside those terms–they are Extrinsic–or both–i.e. Mediate or Mixed (BDT. 1186D).

The following table presents by name the enumeration of the Topics in the orders given by Peter of Spain (PH. 5.09-5.47). I have numbered them for ready reference as follows:

TABLE I

Enumeration of the Topics in Peter of Spain

Intrinsic Topics

1. Definition
2. Description
3. Nominal Meaning
4. Whole:
4.1 Universal or generic
4.2 Integral
4.3 Quantitative
4.4 In Mode
4.5 In Place
4.6 In Time
5. Part:
5.1 Subjective or specific
5.2 Integral
5.3 Quantitative
5.4 In Mode
5.5 In Place
5.6 In Time
6. Cause: Efficient, Material, Formal, Final
7. Effect: Efficient, Material, Formal
8. Generation
9. Corruption
10. Use or operation
11. Common accidents or concomitants

Extrinsic Topics

12. Opposites:
12.1 Relative
12.2 Contrary
12.3 Privative
12.4 Contradictory
13. The More
14. The Less
15. The Similar
16. Proportion
17. Transumption
18. Authority

Mixed Topics

19. Conjugates
20. Case or Adverb
21. Division

Ockham's Analysis of a Topical Argument

When we turn to Ockham, we fail to find in his work a distinct treatise entitled *De Locis*. Instead we find that the familiar rules regarding inferences from Definition, Description, Nominal Meaning, etc., are included in the third of the third part of the *Summa*, which is said to be concerned with "arguments and consequences which are not syllogistic in form, such as Enthymemes...or with non demonstrative syllogisms" (O. 3-3.1,383). There is no mention by name either of Topic or of Topical Maxim or Difference. However, earlier in enumerating the kinds of syllogism he does note that a non-demonstrative probable syllogism is a Topical syllogism (O. 3-1.1,327). Earlier also, in treating the kinds of propositions, he had postponed consideration of conditionals to this part on the ground that "a conditional is equivalent to a consequence and hence can be deferred to the treatise on Consequences" (O. 2.31,315). Thus like Abelard he looks upon the study of the Topics as a way of studying the properties of conditionals (A. 253).

However, this third treatise contains much more than is included in the traditional treatises on the Topics. In addition to the Boethian tradition, there is a section dealing with Modal Consequences (chs. 10-16), another covering the Aristotelian tradition of the Topics (chs. 17-30), a section on induction (chs. 31-35), a chapter on equivocation (ch. 36), another on general rules of consequences (ch. 37), which has been interpreted by Salamucha and Boehner as the Basis of Ockham's propositional logic, and sections devoted to subjects that form standard parts of late Mediaeval logic—one on Obligations dealing with the rules of formal disputation (chs. 38-44), and another on Insolubilia or logical paradoxes (ch. 45).

It is to the first section, dealing with the Boethian Topical tradition that I shall devote my attention.* This too has been greatly changed. Although, as I shall show, Ockham is going over the same ground, he does so in a very different way. He makes different distinctions, uses old ones for different purposes and re-organizes the enumeration of the Topics on an entirely different basis. Perhaps the briefest and clearest way of showing something of Ockham's accomplishment is to compare his exposition of a Topic with the corresponding one in Peter of Spain. For that purpose let us look again at the Topic of Genus. Following is a literal translation of the paragraph on it in Peter of Spain (PH. 5.16):

> "The *Topic from a universal whole*, as taken here, is that of the superior and substantial to its inferior, as animal to man and man to Socrates.... The Topic from a universal whole or from Genus is the habitude of it to its part and is always destructive. It contains two arguments and two Maxims: First, negatively subjecting the universal whole, as in:

*Since the semi-critical edition begun by Boehner has not yet reached this part of the *Summa totius logicae*, I shall quote from the last printed edition—that of Oxford, 1675.

T1. 'An animal is not running, therefore a man is not running'.

M1. Maxim: Whatever is removed from the genus is also removed from the species. Secondly, predicating it negatively, as in:

T2. 'A stone is not an animal, therefore a stone is not a man'.

M2. Maxim: Removing the genus also removes the species."

Here it should be noted that this Topic can be called indifferently that of Universal Whole, of Superior, or of Genus. It is said to be always "destructive", i.e. negative, as opposed to others, which may be "constructive", i.e. affirmative, only, or both. For it two Maxims are cited, both expressed in terms of Genus-Species.

Ockham's treatment of this inference-relation occurs distributed among four different chapters. For it he cites four rules (*regulae*); he never speaks of Maxims. Of these only one is expressed in terms of Genus-Species, the other three being expressed as that of Superior-Inferior, as follows:

R1. From a distributed superior to a distributed inferior there is a good consequence (*A superiori distributo ad inferius distributum est bona consequentia* – O. 3-3.2,385). E.g.:

T3. 'Every animal is running, therefore every man is running'.

R2. Negatively from a distributed superior to a distributed inferior there is a simple consequence (*A superiori distributo ad inferius distributum negative est consequentia simplex* – O. 3-3.4,390). E.g.:

T4. 'No animal is running, therefore no ass is running'.

R3. From a superior to an inferior with a preceeding negation there is a good consequence (*A superiori ad inferius praeposita negatione est bona consequentia* – O. 3-3.8,403). E.g.:

T5. 'Socrates is not an animal, therefore Socrates is not a man'.

R4. From the affirmative of one genus to the negative of another non-subalternate genus there is a good consequence (*Ab affirmativa de uno genere ad negativa de alio genere non subalterno est bona consequentia* – O. 3-3.9,411). E.g.:

T6. 'Socrates is an animate body, therefore Socrates is not an inanimate body'.

How does Ockham's treatment differ from that of Peter of Spain? First, as already noted, Ockham states what he calls Rules, not Maxims, and does so, in three cases out of four, in terms of Superior-Inferior rather than Genus-Species.

Secondly, he does not restrict his attention to the negative alone, but considers an entirely affirmative consequence in R1, and an affirmative to a negative in R4. The entirely negative rules, R2-R3 correspond to those of Peter, e.g. to M1-M2.

Thirdly, unlike Peter of Spain he shows an explicit concern for the

quantity of the propositions involved. This together with the second difference leads to a different division in the treatment of the Topics, as I shall show in detail later. But there are still other differences which have not yet appeared because they depend on technical distinctions that Ockham makes among consequences.

Ockham draws nine distinctions among consequences. In expounding them, I shall not follow his order, but shall take the simplest first, which are most immediately related to the differences already noted. With my numbering then, we have:

D1. Consequences differ according as the consequent is universal or particular (O. 3-3.1,385).

D2. Consequences differ according as the consequent is affirmative or negative (*loc. cit.*).

D3. Consequences differ according as the consequence is from an affirmative antecedent to an affirmative consequent, from a negative to a negative, from an affirmative to a negative, or from a negative to an affirmative (*loc. cit.*).

D4. Consequences differ according as the consequent is an assertoric or a modal proposition, i.e. involving modal quantifiers such as 'necessarily', 'possibly' (*loc. cit.*).

D5. Consequences differ according as the predicate of the consequent is or is not the name of one of the five predicables or an equivalent–accident, genus, property, definition, difference–, as in "'Animal' is the genus of 'man'" (3-3.1,384).

D6. Consequences differ according as the subject of the consequent is in personal and significative supposition or in simple or material supposition. This is said to agree closely with the above, D5 (*convenit multum cum praecedente – loc. cit.*). This is the difference, e.g. between the propositions, 'An animal is running' and "'Animal' is a 'genus'." The subject of the first is said to be in personal and significative supposition because it 'stands for' its usual referents, i.e. for animals, whereas in the second it 'stands for' an *intentio mentis*, in this case a class.

D7. Consequences differ according as the consequence is factual (*ut nunc*) or simple (*simplex*). A factual consequence is described as one in which "the antecedent can be true at some time without the consequent being true; e.g. 'Every animal is running, therefore Socrates runs', since at that time when Socrates is an animal, the antecedent cannot be true without the consequent; and yet there is a time in which the antecedent can be true without the consequent." A simple consequence is one in which at no time can the antecedent be true without the consequent; e.g. 'No animal is running, therefore no man is running'; for at no time could it be true, 'No animal is running' without it also being true, 'No man is running', granted that the proposition were formed" (*loc. cit.*).

D8. Consequences differ according as they hold through an intrinsic medium (*per medium intrinsecum*) or through an extrinsic medium (*per medium extrinsecum*). "A consequence holds through an intrinsic medium when it holds by virtue of some proposition formed from the same terms [as those

in the consequence]. Thus this consequence, 'Socrates is not running, therefore a man is not running', holds by virtue of this medium, 'Socrates is a man'; for unless this were true the consequence would not be valid (*non valeret*). A consequence holds through an extrinsic medium when it holds through some general rule which no more respects those terms [in the consequence] than any others; e.g. this consequence, 'Only man is an ass, therefore every ass is a man', holds through this general rule: An exclusive affirmative proposition converts with a universal affirmative with transposed terms. It does not hold through another proposition formed from the terms, 'man' and 'ass', but through that general rule. It is through such extrinsic media that all syllogisms hold. If it should be urged against this distinction that the previous consequence, 'Socrates is not running, therefore a man is not running', holds through this extrinsic medium: From the singular to the indefinite and from the inferior to the superior with a following negation there is a good consequence, and that consequently it does not hold through an intrinsic medium, to this it must be said that it holds through this extrinsic medium mediately and as it were remotely and insufficiently, because in addition to this general rule more is required, namely that Socrates is a man; and therefore it holds more immediately and sufficiently through this medium, 'Socrates is a man', which is an intrinsic medium." (*ibid.* 383-384). Here it should be noted that a consequence may thus be said to hold through both an intrinsic and an extrinsic medium.

D9. Consequences differ according as they are formal or material. "A formal consequence is two-fold, since it sometimes holds through an extrinsic medium which respects the form of the propositions, such as these rules: From an exclusive to a universal with transposed terms there is a good consequence, or from a necessary major and an assertoric minor there follows a necessary conclusion, and the like. Sometimes, however, it holds immediately through an intrinsic medium and mediately through an extrinsic medium respecting the general conditions of the propositions, and not,* namely, truth, falsity, necessity, impossibility. Of such sort is the previous consequence, 'Socrates is not running, therefore a man is not running'." (*ibid.* 384). The material consequence is not immediately relevant to our Topical analysis, and since its interpretation involves peculiar textual difficulties, I shall postpone its consideration.

With these distinctions we can now return to the example of Topical analysis and continue noting how Ockham's treatment differs from that of Peter of Spain. However, it will help for formalize Ockham's examples and their rules. In stating his examples Ockham usually joins the two parts of the consequence by 'therefore' ('*ergo*', sometimes '*igitur*'). Yet, as already noted, he considers them to be conditional propositions, and a conditional is defined as being "composed of two categoricals connected by means of the conjunction 'if' ('*si*') or its equivalent" (O. 2.31,315). For this I shall

*Here I have added 'not' to the 1675 text on the manuscript authority provided by Salamucha (S. 108).

use the ordinary sign of material implication, but shall take it in a neutral sense without determining whether Ockham's '*si*' is in every respect equivalent.

Using '*A*' for 'animals', '*Rx*' for '*x*-is-running', '*M*' for 'men', '*D*' for 'asses', '*B*' for animate bodies', and '*s*' for 'Socrates', we may formalize Ockham's examples as follows:

T3* $[x] . x \in A \supset Rx . \supset . [x] . x \in M \supset Rx$

T4* $[x] . \sim (x \in A . Rx) . \supset . [x] . \sim (x \in D . Rx)$

T5* $s \sim \in A . \supset . s \sim \in M$

T6* $s \in B . \supset . x \sim \in - B$

Turning now to Ockham, we can follow his analysis of a consequence as it is represented in these examples. Of each he asks whether it is a simple or a factual consequence. According to this distinction (D7), a consequence is factual, or *ut nunc*, if at some time the antecedent can be true without the consequent, i.e. he appeals to the truth-conditions of the component parts. By this test he claims that T3 is a factual consequence, since at a time when there are no men the proposition 'Every animal is running' may be true while the proposition 'Every man is running' is false; i.e. it is possible to verify the antecedent and falsify the consequent. By the same test the other examples turn out to be simple consequences, since if the antecedent is true so is the consequent.

So far in ascertaining the truth of the parts of the consequence we have been assuming that there is a meaningful relation between the categorematic terms of antecedent and consequent, namely that men and asses *are* animals. To state this explicitly is to form a proposition from a term taken from the antecedent and a term taken from the consequent of each consequence. Such a proposition is what Ockham calls in (D8) an *intrinsic medium*. When this proposition stating the intrinsic medium is added to the consequence, we get an exemplification or realization of the corresponding rule, which according to Ockham "no more respects those terms [appearing in the consequence] than any other." This he calls an Extrinsic Medium, and in the case of a simple consequence it is the analogue of a logical law. Since these rules are independent of the terms in which they are realized, we may formalize them with variables as follows, if we understand the superior-inferior relation as class inclusion:

R1* $\alpha \subset \beta : \supset : [x] \, x \in \beta \supset \phi x . \supset . [x] \, x \in \alpha \supset \phi x$

R2* $\alpha \subset \beta : \supset : [x] . \sim (x \in B . \phi x) . \supset . [x] . \sim (x \in \alpha . \phi x)$

R3* $\alpha \subset \beta : \supset : x \sim \in \beta . \supset . x \sim \in \alpha$

R4* $\alpha \cup - \alpha = V : \supset : x \in \alpha . \supset . x \sim \in - \alpha$

These last two rules are stated for the indefinite proposition with which Ockham equates both the particular and the singular proposition (3-3.6, 391). For this reason I have used an unbound variable.

With the distinction between intrinsic and extrinsic media we are now in a position to see how Ockham has re-interpreted the traditional Topical analysis. D9 in distinguishing a formal consequence that "holds immediately through an intrinsic medium and mediately through an extrinsic medium" in effect provides a formal definition of the traditional Topic. This becomes evident on placing side by side Ockham's analysis of such a consequence with the corresponding Topical analysis of Peter of Spain:

OCKHAM:

Extrinsic medium: Negatively from a distributed superior to a distributed inferior there is a simple consequence R2

$$\alpha \subset \beta : \supset : [x] \,.\, \sim (x \in \beta \,.\, \phi\, x) \,.\, \supset \,.\, [x] \,.\, \sim (x \in \alpha \,.\, \phi\, x) \qquad \text{R2*}$$

Intrinsic medium: 'Every ass is an animal'

$$D \subset A$$

Consequence: 'No animal is running therefore no ass is running' T4

$$[x] \,.\, \sim (x \in A \,.\, Rx) \,.\, \supset \,.\, [x] \,.\, \sim (x \in D \,.\, Rx) \qquad \text{T4*}$$

PETER OF SPAIN:

Topical Maxim: Whatever is removed from the genus is also removed from the species M1

(same as R2*)

Topical Difference: 'Animal' is the genus of the species 'man'

$$M \subset A$$

Topical argument: 'An animal is not running, therefore a man is not running' T1

$$[x] \,.\, \sim (x \in A \,.\, Rx) \,.\, \supset \,.\, [x] \,.\, \sim (x \in M \,.\, Rx)$$

The two formalize in the same way, i.e. they are realizations of the same form. The fact that Peter of Spain talks in terms of genus-species and Ockham in terms of superior-inferior is of little significance, since, as we have seen Peter himself speaks of the genus as superior to the species as inferior. There is, however, a difference between the expression of the intrinsic medium and the Topical Difference. Ockham puts it at once in the object-language, whereas Peter phrases it metalinguistically. This corresponds to the difference in supposition that Ockham draws in distinction D6, which, as we shall see, he uses for another purpose. In the case of the example, however, the two statements are equivalent.

The intrinsic-extrinsic distinction is used by both, but for different purposes. For Peter of Spain, as we have seen, it provides a criterion for dividing the Topics into different kinds. For Ockham, so far, we have seen it used to analyse the formal structure of a consequence. Yet he also uses

it to divide kinds of consequences that fall among the traditional Boethian Topics. Thus corresponding to the Topic from Authority, which is classed among the Extrinsic Topics by Peter of Spain, Okham has the following rule:

R5. What the expert says is true (*Quid dicit sapiens est verum* – O. 3-3.7, 400). But as he immediately points out, this is neither a general nor a formal rule.

As an example of a consequence holding through an extrinsic medium, Ockham cites one corresponding to a Topic which Peter of Spain classifies among the Mixed kind, as the Topic from Conjugates (PH. 5.44), e.g.:

T7. 'Justice is a virtue, therefore one who is just is virtuous, and one who does something justly does something virtuously.'

For this there is the rule:

R6. If the abstract is predicated of the abstract, then the concrete is also predicated of the concrete and also the adverb of the corresponding adverb

(*Si principale de principali, et conjugatum de conjugato, et casus de casu* – O. 3-3.7,398)

Although Ockham states the rule in terms of *principale*, *conjugatum*, and *casus*, he equates these with 'abstract', 'concrete', and 'adverb' respectively. In this rule Ockham conflates two of the traditional Topics, namely the Topics of Case and of Conjugates, i.e. Nos. 19-20 in Peter of Spain's listing.

From these two rules it is clear why Ockham should claim that they yield consequences that hold through extrinsic medium alone. To validate them there is no need for an intrinsic medium composed of a proposition formed from a term from the antecedent and another from the consequent. It is sufficient to see that the consequence is a realization of a general rule governing all expressions of a certain form.

From these examples it is clear that Ockham is re-thinking and re-interpreting the traditional doctrine of the Topics. How extensively is most readily shown by analysing in some detail his re-organization of the Boethian Topics. I shall do this by showing schematically in the following table how he applies his distinctions, D1-D9, to reorganize the Boethian Topics. After this I shall show in another table how the Topics enumerated in Peter of Spain's list fall within Ockham's new division.

TABLE II

Ockham's Division of the Topics in Sum. tot. log. 3-3

1.00 Consequences with terms in Personal-Significative Supposition (The Boethian Tradition)

1.10 Assertoric Consequences

TABLE II – continued

1.11	With a universal consequent inferred through an intrinsic medium from a universal antecedent	
1.111	Affirmative antecedent & affirmative consequent	
	1. in respect of all predicates	..... cap. 2
	2. not in respect of all predicates	..... cap. 3
1.112	Negative antecedent & negative consequent	
	1. in respect of all predicates	..... cap. 4
	2. not in respect of all predicates	..... cap. 5
1.113	Affirmative antecedent & negative consequent	"patere ex praedicto" (p. 391)
1.114	Negative antecedent & affirmative consequent	
1.12	With indefinite, particular, or singular antecedent and consequent	
1.121	Affirmative antecedent & affirmative consequent	
	1. through an intrinsic medium	..... cap. 6
	2. through an extrinsic medium	..... cap. 7
1.122	Negative antecedent and negative consequent	..... cap. 8
1.123	Affirmative antecedent & negative consequent	..... cap. 9
1.124	Negative antecedent & affirmative consequent	
1.20	Modal Consequences	..... cap. 10-16
2.00	Consequences with terms in Simple or Material Supposition, Accident, Genus, Property, Definition, Species, Difference, Same & Other (The Aristotelian Tradition)	..... cap. 17-30

The contents of the remaining fifteen chapters has been noted earlier (*sup.* p. 68).

It is evident at once that this much of the treatise, and this covers two-thirds of it, is organized in a highly systematic fashion, despite Boehner's claim to the contrary (PB. 55).

TABLE III

Where Ockham treats the Topics enumerated by Peter of Spain

Table II	*Table I*
1.1111	4.1, 1, 2, 3
1.1112	12.1, 4.2, 19
1.1121	4.1, 1, 2, 3
1.1122	12.1, 5.2, 19
1.1211	1, 2, 3, 5.1, 4.4, 4.3, 12.2, 8, 9
1.1212	19, 20, 18, 13
1.122	1, 2, 3, 5.1, 4.1, 5.4
1.123-124	12.2, 12.3, 21

TABLE IV

Ockham's Eliminations, i.e. Topics occurring in PH & not in O's Consequences

Table I

4.5	Whole in Place
4.6	Whole in Time
5.3	Quantitative Part
5.5	Part in Place
5.6	Part in Time
6.	Cause
7.	Effect
10.	Use
11.	Common accidents
12.4	Contradictories
14.	The Less
15.	The Similar
16.	Proportion
17.	Transumption

TABLE V

Ockham's Additions, i.e. Consequences not included in the PH Topics but included in O's treatment of the Boethian tradition

Table II

1.1111	From one convertible to another From the difference of the superior to its inferior From a convertible with superior to its inferior
1.1122	From negation of the prior to negation of the posterior From the negation of the denominated to negation of the denominating
1.1211	From an adverb determining operation to its omission From a numeral to its part From a collective name to its part
1.1212	Adding to both parts of a good consequence From singular to plural and conversely
1.123-1.124	From an indefinite to a definite predicate and conversely

This suffices for Ockham's treatment of the Boethian tradition of the Topics. In all, in the eight chapters he devotes to it, i.e. cap. 2-9, I have counted no less than 56 explicitly formulated rules. In considering the

Aristotelian tradition, he deals with rules for finding and testing genus, species, definition, etc. With these Ockham has covered one of the two kinds of Formal Consequences that he distinguishes, that namely which holds immediately through an intrinsic medium and mediately through an extrinsic medium. (D9).

The other kind of Formal Consequence is that which holds through an extrinsic medium respecting the form of propositions, i.e. it is a realization of a general rule corresponding to a logical law. In this treatise this kind is treated in the section on Modal Consequences (cap. 10-16) and in the chapter devoted to "general rules of consequences" (cap. 37). Here are stated many rules corresponding to laws of the propositional calculus. They have been studied, and the results are well summarized by Prof. Moody (M. 82-100).

This leaves only the Material Consequence to be considered. In his definition of this Ockham differs from most of the other 14th century logicians (cf. M. 70-79). Thus for the Pseudo-Scot a Material Consequence corresponds precisely to a Topical argument (PS 288A; cf. BI. 146), i.e. to what is for Ockham one of the two kinds of Formal Consequence. Yet what Ockham intends by it seems clear enough from his examples and their corresponding rules:

T8. 'You are an ass, therefore you are God'.

R7. From the impossible anything follows.
(*Ex impossibili sequitur quodlibet* – O. 3-3.37, 479)

T9. 'You are white, therefore God is triune'

R8. The necessary follows from anything
(*Necessarium sequitur ad quodlibet* – O. *loc. cit.*)

Since a proposition about God was taken as a standard form of necessary proposition, it seems clear that by a Material Consequence Ockham intends, as Moody says (M. 74*), "those conditionals which are true merely because the antecedent is false (or impossible), or the consequent true (or necessary)"; i.e. for exemplifications of the paradoxes of material or strict implication.

The difficulty comes in trying to fit Ockham's definition of it to his examples. Following is a literal translation from the 1675 text:

> A Material Consequence is one that "holds precisely by reason of the terms and not by reason of some extrinsic medium respecting precisely the general conditions of propositions" (O. 3-3.1, 384).

The understanding of this turns on what Ockham means by the "general conditions of propositions". Earlier in distinction D9, in which this definition occurs, he had used this same phrase followed immediately by the words: *non veritatem vel falsitatem, necessitatem vel impossibilitatem.* The crux lies in knowing whether the *non* should be there. The 1675 text has *scilicet* for which there is manuscript authority; but, as already noted, I have followed

Salamucha in supplying the *non*, for which there is also manuscript authority. My reason is that he is describing in these words what he means by an extrinsic medium, and he had already said in distinction D8 that it is by an extrinsic medium that all syllogisms hold. Since a valid syllogism holds regardless of the truth or falsity of its premises, I think the *non* should be there and that by the general conditions he means purely formal considerations, including as a formal element the explicit occurrence of modal quantifiers. Turning now to the example of Material Consequences, T8-T9, we see that it is only by analysing the meaning of the terms that we know that we have an impossible proposition in the antecedent of T8 and a necessary proposition in the consequent of T9. In this sense we have to do here with a material rather than a formal element in the consequence, which distinguishes it from either kind of Ockham's Formal Consequences.

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University of Notre Dame
Notre Dame, Indiana