THE FORMALIZING OF THE TOPICS IN MEDIAEVAL LOGIC

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The Topics have received little attention in modern logical studies. If they are noticed at all, it is usually for reasons that have little to do with formal logic. In fact, by those who use the distinction it is customary to assign the Topics to material rather than to formal logic.

Yet during the great period of mediaeval formal logic considerable attention was given to the Topics. The Summulae Logicales of Peter of Spain devotes one of its twelve treatises to them. Since this was a standard elementary text, this means that no mediaeval logician would be ignorant of their existence. Furthermore, when we look at their history, we find that a serious effort was made to isolate a formal element in the Topics and to formulate it as a logical rule. There is in effect, within the limits of the logical method employed, a formalizing of the Topics.

I propose to analyse this development as follows:

- I. The conception of a Topic according to Boethius, who is the auctoritas for the mediaeval Topical tradition.
- II. Abelard's criticism and revision of the Boethian conception of a Topic.
- III. The absorption of the Topics into the Theory of Consequences in 14th century logic.

I. Boethius on the Topics

How Boethius looked upon the Topics may be seen from the first detailed analysis of one of them in his *De Differentiis Topicis* (BDT,1187A-B).* Suppose, he writes, we want to know whether trees are animals. The following syllogism will decide the question:

- 1. An animal is an animate sensible substance.
- 2. A tree is not an animate sensible substance.
- 3. Therefore, a tree is not an animal.

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^{*}Such page citations are to the editions listed in the Table of References at the end of this article.

Here the fitting and proper (convenientes et congruas) propositions through which the question gets decided obtain their force, Boethius claims, from this prima et maxima propositio:

M1. Cui generis diffinitio non convenit, id ejus cujus ea diffinitio est species non est.

(That to which the definition of genus does not apply is not a species of that of which the definition is given).

By relating the parts of this maxim to the syllogistic example we can see how the syllogism may be said to get its force from M1.

In 1 we have:

- *1.1 generis diffinitio "animate sensible substance" (definition of genus)
- *1.2 ejus cujus ea diffinitio est: "animal"
 (of which the definition is given)

In 2 we have:

*2 id cui generis diffinitio non convenit: "tree" (that to which the definition of genus does not apply)

Then by M1 we may conclude:

*3 *2 species non est (is not a species of) *1.2

Boethius notes that although the question here concerns genus (baec de genere quaestio est), i.e. whether "tree" belongs in the genus "animal", the Topic warranting the conclusion is that of Definition, and M1 is a Maxim of Definition (locus nibilominus nuncupatur a diffinitione — BDT. 1187A). The reference to genus-species is accordingly irrelevant to the statement of the Maxim. By the time of Abelard it has been dropped, and, referring to this very passage, he formulates the Maxim simply as:

M1¹. A quocumque removetur definitio, et definitum (From whatever the definition is removed, so is the definitum - D.258⁸)

Boethius adopts Cicero's description of a Topic as a sedes argumenti (C.8) and interprets it as "that from which a fitting argument may be drawn for a proposed question" (id unde ad propositam quaestionem conveniens trabitur argumentum — BDT. 1174D). Since this makes the Topics a source of arguments, he agrees with Cicero in assigning them to the faculty of discovery rather than of proof (inveniendi, not judicandi — BDT. 1173B). This is to say that the Topics provide a way of finding arguments.

How they do so is illustrated in the above example. For the question, whether trees are animals, the Topic from Definition enables us to provide an argument, granted that we have the knowledge necessary for using it.

This involves two different things: one which Boethius calls the propositio maxima, which I shall translate as Topical Maxim, or simply Maxim, and another which he calls the maximae propositionis differentia, which I shall translate as Topical Difference, or simply Difference.

Boethius describes the Maxim (of which M1 is an example) variously as follows:

- It is known through itself (per se nota) so as not to need proving from anything else; whence it is called maxima et principalis (BDT.1185A)
- maxima et universalis et principalis et indemonstrabilis atque per se nota propositio (BDT.1185D)
- These universal and maximal propositions are called Topics because they contain other propositions and through them a conclusion becomes consequent and sound (consequens et rata). Just as place contains within itself the quantity of a body so these propositions which are maximal hold within themselves all the later force and consequence of the conclusion itself (intra se omnem vim posteriorem atque ipsius conclusionis consequentiam tenent). (BDT.1185D-1186A)

The Topical Difference, or more literally the Difference of the Maximal Proposition, is that by which one Topic differs from another (BDT.1186A). Thus the Topic of Definition, for instance, differs from that of Whole and Part in that the Maxim of the one warrants an inference among terms in which a Definition occurs, while the other warrants an inference among terms in which Whole and Part occur.

Topical Differences, according to Boethius, "are drawn forth from the terms constituting the question and then discoursed about" (BDT.1186A). Thus in our example, it is the question, whether trees are animals, that makes it possible to appeal to the Topic of Definition, since, knowing the definition of "animal" and that trees do not satisfy it, we are warranted by the Topical Maxim to conclude that trees are not animals.

The De Differentiis Topicis is little more than a listing of such Topical Differences with representative Maxims for each. Book II gives the compilation of Topics made by Themistius from Aristotle; Book III that of Cicero, followed by a comparison of the two. Book I is a general introduction dealing with the terms used for analysing an argument, and Book IV, the final book, considers the Topics used by rhetoricians.

This work became the source for mediaeval Topical doctrine. It seems to be the only work Abelard used for his extensive treatise on the Topics. Peter of Spain made a précis of it (primarily of the second book) and provided additional Maxims in the fifth tract of his Summulae. Since this became a standard elementary text in logic from the late 13th through the 15th centuries, Boethius thus remained indirectly the auctoritas for the Topics,

and this seems to have remained true even after the recovery of the Aristotelian *Topica* in the late 12th century.*

II. Abelard on the Topics

For Abelard to study the Topics is to analyse the hypothetical proposition "whose sense is put forward under the condition of consecution" (quorum sensus sub consecutionis conditione proponitur — D.253¹¹), i.e. conditional propositions and particularly those formed with si ("if. . .then"). For this reason in his Dialectica the treatise on the Topics immediately preceeds that on the hypothetical syllogism, to which it is said to bear the same relation as the treatise on the categorical proposition does to that on the categorical syllogism (D.253⁴). It is from the Topics, according to Abelard, that "the truth or falsity of hypothetical propositions is best known, since it is mostly from them that they get their inferential basis and evidence for their truth" (inferentiae suae sedem ac veritatis evidentiam ex locis quam maxime tenent — D.253¹²). Towards the close of his very extensive consideration of the Topics, in fact after 201 pages in the De Rijk edition, he claims that he has shown the inferential force (vis inferentiae) of all conditionals (D.454³³).

A Topic taken broadly is for Abelard a vis inferentiae (D.25316). This would appear to agree well enough with Boethius, for whom, as we have seen, a Topic is an inference-warrant. However, Abelard is far from satisfied with the analysis of the Topics in Boethius. He notes that Boethius cites as Maxims many that in fact are not and excuses him for this on the ground that in these cases Boethius is either citing the opinion of others or testing the reader (G.242²⁸). But what concerns him most is that Boethius made no attempt to distinguish necessary from probable Topics. Boethius indeed allowed for both. Accepting Aristotle's definition of the probable as "that which seems so to all or to most or to the wise..." (A.I.1, 100b21), he notes that probable arguments are either necessary or nonnecessary (BDT.1180C). The Topics as sources or warrants for probable arguments would thus contain logically necessary as well as only logically probable arguments. But although Boethius made this distinction, he did little to apply it to the Topics themselves. Abelard, on the other hand, made it in his major purpose in re-working the Topical tradition received from Boethius to sort out the necessary from the non-necessary Topics.

Since he conceives of a Topic as a vis inferentiae, Abelard to accomplish this purpose has accordingly to analyse and distinguish various kinds of vis inferentiae. He does so at great length and with great logical acumen, but only with much difficulty at least for a modern reader. The source of most the difficulty comes from lack of a technical vocabulary in which to make the necessary distinctions. Lacking this, there is often considerable doubt whether, having made the distinctions, he succeeds in holding to them.

^{*}I am presently at work on the history of the Topics in mediaeval logic.

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It is *inferentia* and its various forms that offers the greatest difficulty. It is used in at least three significantly different senses:

- S1. Vis inferentiae, as above, where it seems to be used for "warrant" in the widest sense to apply to both formal and non-formal reasoning and including both what would now be called inference-schemes and implicative statements.
- S2. Inferentia as consisting in necessity of consecution (in necessitate consecutionis), which is said to be such that the sense of the antecedent compels the sense of the consequent (quod ex sensu antecedentis sententia exigitur consequentis D.253²⁸). This may come about purely because of the logical form regardless of the meaning of the substantive terms, and we then have a perfecta inferentia; or because of the habitudo holding between the substantive terms, when we then have an imperfecta inferentia. This distinction, we shall see, corresponds to that made later between formal and material Consequences.
- S3. Inferentia as an implication or implicative-statement distinct from an inference-scheme. Abelard makes this distinction, and it is rarely made so precisely in mediaeval logic, in terms of the difference between si ("if") and ergo ("therefore"), according as the antecedent is inconcessa or concessa (D.457¹²). An example is found in the difference between the categorical and the hypothetical syllogism the only difference being that in the former there is a "concession of the antecedent propositions" (D.254³³), since "the auditor has to consent to them, i.e. receive them as true" (D.253⁹), whereas this is lacking in the hypothetical syllogism, which is a conditional proposition.

Turning now to Abelard's analysis of the Topics, in which these distinctions find their use, we find that he is primarily interested in those in which there is a necessity of consecution (inferentia S2). Taking probare in the sense of producing conviction (fidem facere), as does Boethius following Cicero, Abelard admits as probative those arguments which derive only from what is conceeded (concessi tantum) and not from any inferring force (vis cuiuslibet inferentis — D.455³). As an example of such an argument, he says that it might be argued that "from the fact that I ran off with a girl it is known that I love her".*

Among the Topics some are warrants for arguments of this sort. But since they are probable only and "seldom used by logicians", Abelard relegates them to the second of the two books of his treatise on the Topics and considers them only "lest he seem to subtract from the Topical tradition" (D.413³²). These, he says, are called Topics as sedes argumenti only in a very loose sense (nimium laxe – D.455¹). Thus if such a Topic is a vis inferentiae, it is so only in our first very broad sense of inferentia, which can accommodate such non-formal inference-warrants as well as the formal ones, to which we can now turn.

^{*}Although it is tempting to see here a reference to Abelard's notorious passional life, the temptation must be withstood, since the same example occurs in Boethius—(BDT.1199B).

To distinguish a necessary Topic Abelard appeals to the notion of necessitas consecutionis. But this, as already noted (S2), occurs in both perfect and imperfect inference. By a perfect inference Abelard understands "one is which the truth of the consequent is manifested from the complexio of its antecedent, and the construction of the antecedent is so disposed that it also contains in itself the construction of the consequent" (D.253³²). "Antecedent" and "consequent" as used here may refer either to the two parts of a conditional proposition or to the two premisses and conclusion of a syllogism. In fact, Abelard argues at some length in his Gloss on the BDT that the categorical syllogism is not a conditional proposition (G.322-323). Thus a perfect inference may be either an implicative-statement or an inference-scheme (S3).

What above all else characterizes a perfect inference is the fact that its consecutio holds good (valet consecutio) no matter what terms occur in it or whether its propositions are true or false (D.255³²). It holds because of its complexio (D.256²¹). In other words, it is a question of logical form. Although Abelard regularly cites examples with concrete terms, it is thus no wrenching of his thought to formalize them with the use of variables. In citing examples now of the various kinds of perfect inference that he distinguishes, I shall accordingly follow his examples with their formal analogue in modern logic.

Abelard distinguishes three types of perfect inference, as follows:

1. Categorical syllogism (D.254³)

Every man is an animal. -AxyEvery animal is animate. -AyzTherefore every man is animate. -Axz

2. Hypothetical syllogism (D.254³⁵)

If every man is animal and every animal is animate, every man is animate.

CKAamAmbAab

- 3. Simple Consequences (D.311²⁰⁻³⁵)
- 3.1. If since it is man it is animal, then if since it is risible it is man, then since it is risible it is animal.

He cites this as an example of the following rule (regula)

3.1' Si aliquid infert aliud, quicquid infert antecedens et consequens. (If one thing implies another, whatever implies the antecedent implies the consequent), where infert is used in sense of S3.

3.2. If man is a species of animal. then if it is man it is animal.

This is cited as an example of the following rule:

3.2' Si aliquid est species alterius, ipsum positum ponit alterum. (If one thing is the species of another, positing the former posits the latter)

$$\alpha \in \beta : \supset : (x) . x \in \alpha . \supset . x \in \beta$$

Abelard cites many examples of 3 and of what he calls the "rules of simple consequences". But although these become the principal object of concern in the later theory of Consequences, in Abelard they are secondary to his interest in analysing the necessary Topics, to which we now come in considering imperfect inference.

Necessitas consecutionis is found in imperfect inference as well as in perfect inference. As an example of such an inference, Abelard cites this example $(D.256^{12})$.

4. If it is man, it is animal.

This is a good and necessary inference, Abelard claims. But its goodness does not lie in its *complexio*, as may be seen by substituting other terms for the substantive term in either antecedent or consequent:

- 5. If it is stone, it is animal = 4. man/stone $(D.256^{17})$
- 6. If it is man, it is stone = 4. animal/stone $(D.256^{19})$

Neither 5 nor 6 is a good inference; they lack necessitas consecutionis. What then is the ground for it in 4? It lies, Abelard claims, in the habitudo, or relation, between the terms "man" and "animal" and its Topic. But a Topic, as we have seen from Boethius, involves two aspects or elements, the Difference and the Maxim. In this case the habitudo of "man" to "animal" reveals the Topical Difference of Species, since "man" is a species of "animal". For this there is the corresponding Topical Maxim of Species, which Abelard cites under the form:

M2. De quocumque predicatur species, et genus.

(Of whatever the species is predicated, so is the genus - D.313²³).

With these two we have all that is necessary, Abelard claims, for a perfect inference, and 4 is a rata consecutio (D.262³³). The Topic warrants the inference in 4 (valere inferentiam – D.257⁵). "It is the term "man", Abelard writes, "that proves 'animal', for we see that 'man' is a species of 'animal' and know that the nature of species is such that it necessarily posits its genus. After assigning the Difference [i.e. recognizing what habitudo of terms we have], the Maxim is brought in from outside to show the mode of proof (modum probationis – D.257⁶)." Thus he can claim that "it is the property of a Topic, from the habitude which it has to the illated term, to confer inferential force upon a consequence" (vim inferentiae ex habitudine quam habet ad terminum illatum conferre consequentiae – D.256³⁷).

To see that we have a necessary consecution in 4 we need only write out in full what is needed to make it a perfect inference, thus:

- 41. "Man" is a species of "animal".
- Topical Difference
- 411. Of whatever the species is predicated so is the genus.

Topical Maxim M2

4. If it is man, it is animal.

Since this is a perfect inference, it will hold good whatever terms we have. We may formalize it then, and interpreting species-genus as class inclusion, we have:

4'*.
$$\alpha \subset \beta$$

4''*. $\alpha \subset \beta : \supset : (x) . x \in \alpha . \supset . x \in \beta$
4*. $(x) . x \in \alpha . \supset . x \in \beta$

Note that we have in 4''* the same expression as in 3.2', i.e. 4'' and 3.2' formalize as exactly the same. Yet Abelard claims that 4'' is a Topical Maxim while 3.2 is not (D.311³⁷). How then do they differ? One (4'') is part of an inference-scheme, while the other (3.2) is not, but is an implicative-statement. Abelard admits that both are rules (regulae), but a Maxim is something more in that it also involves a Difference (D.261³⁵). It is tempting to take this to mean that a Topical Maxim, or its use, involves the assertion of its antecedent, for this is what the statement of the Difference amounts to (i.e. 4'*). However, according to Abelard it is the habitudo cohaerentiae between "man" and "animal" that is the essential thing (D.313¹⁸). Thus in effect he is comparing 4 with 3.2. Noting that 3.2 is perfect by its complexio, whereas 4 is not, he says that 4 is capable of being made perfect, or formally completed, by noting the relation between its substantive terms and applying the appropriate Topical Maxim. Later, as we shall see (p. 147) the imperfect inference is analysed as an enthymeme.

From this analysis it is clear that Abelard has grasped a purely formal element in the Topics, whereas Boethius either did not see it or did not bother about it. Furthermore, in his search for necessary Topics Abelard is primarily interested in discovering Maxims like M2, which are logically necessary rules. Within the limitations of his memod and language he is engaged in formalizing the Topics.

III. From Topic to Consequence

Abelard claimed that he studied the Topics to learn about conditional propositions. However, his study of them is accurately entitled the Topics, since it is mainly concerned with the Topical tradition handed down from Boethius. However critical he may be of Boethius, the structure of his treatise reflects that of the De Differentiis Topicis. By the 14th century all of this has changed. What remains of the Topics has become absorbed into the theory of Consequences, which, at least according to Ockham, also includes the analysis of the conditional proposition — quia conditionalis [propositio] aequivalet uni consequentiae (OK.II.31,315).

Within the scope of this article all that remains to be done is to show

that this did in fact happen. It is not yet possible, because of the lack of texts in 12th-13th century logic, to show how this came about. Peter of Spain provides no evidence of it in his *Summulae*. His treatment of the Topics shows no evidence of Abelard's revisions and advances. Yet the 14th century treatises on Consequences have in effect incorporated his results, although I have yet to find him actually quoted or referred to by name.

There is no need here to develop the mediaeval theory of Consequences. Considerable study has already been devoted to it, notably that of Moody (M). For the purpose of seeing how the Topics enter into the Consequences, we need consider only the major distinction among their kinds. For this a text of the Pseudo-Scot is the crucial one, since it explicitly identifies the Topics with one kind of Consequence.

After defining a Consequence as a conditional proposition (cf.M.68), the Pseudo-Scot divides Consequences into Formal and Material. A Formal Consequence is "one that holds for all terms when there is a similar disposition and form of the terms" (PS.287B cf. P.XVII.615). As Moody points out, this amount to saying that "all conditional sentences which are 'logically true' on syntactical grounds, or which are true for any transformations of the categorematic terms occurring in the sentence, are valid by their form" (M.70-71). It corresponds to Abelard's perfecta inferentia which holds its truth from its complexio.

A Material Consequence is "one which does not hold for all terms retaining a similar disposition and form but with a variation of the terms (PS.287B cf. P.XVII.617). As Moody writes, this comprises "all conditionals such as are valid, but not in virtue of their syntactical form" (M.73).

Material Consequences divide, according to the Pseudo-Scot, into that which is *vera* or *bona simpliciter* and that which is *vera* or *bona ut nunc*. Only the first of these pertains to our concern. It is described as follows:

"It is one which can be reduced to a Formal Consequence through the assumption of a necessary proposition. Thus the following Material Consequence is bona simpliciter:

'A man runs, therefore an animal runs'.

This reduces to a Formal Consequence through this necessary proposition:

'Every man is an animal'.

And this (i.e. Material Consequence bona simpliciter) subdivides into many members according to the diversity of the dialectical Topics." – (PS.288A; cf. P.XVII.617)

Given this last sentence, it appears that we have here a general analysis of a Topic. In fact, the example cited is identical with that given by Peter of Spain in his consideration of the Topic from Species, for which he quotes the Maxim:

M3. Quicquid predicatur de specie et de genere (Whatever is predicated of the species is also predicated of the genus - PH.5.17)

Neither the Pseudo-Scot nor Peter of Spain give the whole lay-out of the argument. Yet after Abelard's analysis of a Topic, it is clear enough how it is to be done. The Pseudo-Scot in effect assigns the Topical Difference in the proposition, "Every man is animal", if we understand this as asserting that "man" is a species of "animal", and Peter of Spain provides the appropriate Topical Maxim in M3. Putting these together according to the pattern we have from Abelard, we have:

1. Every man is an animal.

Topical Difference

Whatever is predicated of the species is also predicated of the genus.

Topical Maxim M3

3. A man runs, therefore an animal runs.

Material Consequence bona simpliciter

Abelard also considers this Topic from Species and shows greater logical acumen than Peter of Spain by reformulating the Maxim to read:

M3* Quicquid predicatur de specie, et de genere particulariter (Whatever is predicated of the species is also predicated particularly of the genus - D.347)

For this to be necessary the consequent of the conclusion must be particular, since in the example the animal that runs must be man, and this is only part of the genus "animal".

It is evident that in the Pseudo-Scot's description of a Material Consequence bona simpliciter we have what corresponds to Abelard's imperfecta inferentia. When Abelard speaks of perfecting or completing it by appeal to a further proposition, the Pseudo-Scot says that it can be reduced to a Formal Consequence by the assumption of another proposition. However, Abelard claims that two propositions are needed, the Difference and the Maxim, whereas the Pseudo-Scot cites only one. However, leaving this aside for a moment, it is clear that for both it holds, as Moody says, that such a Consequence is "valid by reason of some connection of supposition or meaning between a term occurring in the antecedent and another term occurring in the consequent" (M.73).

Moody makes no mention of the Topics in his discussion of such Material Consequences, but claims that they are represented primarily by enthymemes (M.73). In fact, the Pseudo-Scot speaks of consequentia enthymematica (PS 302 cf. P.XVII.612). If we interpret the Material Consequence as an enthymeme, we would get a different reduction from that of the Topical analysis, since the added proposition would be taken as supplying what is needed to make a 'perfect syllogism'. For the above example we would then have the following lay-out:

- 1. Every man is an animal.
- 21. If every man is an animal and a man runs, then an animal runs.
- 31. If a man runs then an animal runs.

However, the result is much the same, since in expanding the enthymeme (3') by the addition of 1, we have to use 2' to get a syllogism, and this in fact is only an application of the Topical Maxim (M3). The Topical analysis provides a metalogical explanation of the argument obtained by expanding the enthymeme. The only other difference is that an "if"—statement occurs in 3' and a "therefore"—statement in 3. Although Abelard, as we have seen, distinguished sharply between them, by the 14th century it was common to treat them as the same (Cf. S.IV.i, 24ra; PS. 287B cf. M.68).

The difference between the two "reductions" raises the question of the relation between Topic and syllogism. That there was a difference of opinion on this is evident from Abelard's discussion of whether or not the syllogism gets its validity (firmitatem) from a Topic (D.256-263). However, this discussion does not affect our present conclusion, since by the 14th century the theory of Consequences had absorbed both Syllogistic and Topics. Thus in the Perutilis Logica of Albert of Saxony the Syllogistic occurs in chapters 7-18 and the Topics in chapters 19-24 of the treatise on Consequences.

It is more relevant to ask the question raised by Walter Burleigh, whether every Consequence is based on some dialectical Topic. He is willing to allow as much, provided that dialectic is understood to extend to the whole of logic $(W.75^{36})$. He takes a Maxim to be "a rule through which a Consequence holds" $(W.76^6)$. If there are rules that do not appear in Boethius or that have not been named, he declares that this is so because "not all Topical Differences have been given names" $(W.76^{36})$, and he even proposes new names for some of these rules $(W.77^4)$. From this it is evident that interest in the Topics, or at least in some of them, has been caught up in the study of the rules of Consequence.

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