

ON IDENTITY AND INTERCHANGEABILITY IN
LEIBNIZ AND FREGE

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“Eadem sunt quorum unum potest substitui alteri salva veritate”. This famous and so called Leibnizian principle was formally assumed by Frege in *Grundlagen der Arithmetik* §65 with the following comment: “In der allgemeinen Ersetzbarkeit sind nun in der That alle Gesetze der Gleichheit enthalten”. Without explicitly mentioning Leibniz, the content of the principle had been already accepted by Frege in *Begriffsschrift* §8. Afterwards, the principle reappears in *Sinn und Bedeutung* (p. 35), where, as in *Grundlagen* it is explicitly assumed as a Leibnizian principle. In *Grundlagen der Arithmetik* §65 Frege quotes the Leibnizian formula according to Erdmann’s edition¹; the passage referred to by Frege includes a particularly clear and strong explanation of the famous formula, which is meant by Leibniz as a *biconditional*, namely if $A = B$ then A and B are interchangeable in any context *salva veritate*, and if A and B are interchangeable in any context *salva veritate* then $A = B$ ². (A certain confusion of use and mention in this or other formulations of the principle, which was pointed out for instance by A. Church³, is irrelevant to the present discussion, and may be easily repaired).

It should be observed that the principle is inconsistent with respect to another Fregean insight, according to which different names of a thing *make a difference*. This fundamental notion in Frege’s philosophy appeared for the first time in *Begriffsschrift* §8, i.e., in the same paragraph where Frege also formulates for the first time (without mentioning Leibniz) the “eadem sunt. . .” in the above sense of a biconditional. Later on, Frege will introduce a special terminology for this view: “*Sinn*” and “*Bedeutung*”. Frege says that the different names of a thing make a difference (i.e. names have *Sinn* and *Bedeutung*) *because* the aspects of a thing make a difference (i.e. things have or may be given under different aspects). *Sinn-Bedeutung* is primarily an ontological distinction. (“To each of these two ways of determining it there answers a separate name”, *Begriffsschrift* §8).

The ontological distinction underlying *Sinn-Bedeutung* is familiar in

ontology since the Greeks. It may be found in Aristotle's analysis of change as well as in any standard textbook of the Aristotelian tradition (*cf.* for instance the distinction *objectum formale-objectum materiale*). Aristotle and Frege start from a common insight into the fact that an entity may be given under different guises. But then Aristotle says that "there is no necessity" that an entity given in some particular way have all the attributes of the same entity given in another particular way,⁴ while Frege, on the contrary, disregards this warning and hastily adopts the "eadem sunt. . .". Thereby Frege seems to have been a victim of an insufficient knowledge of the Leibnizian *corpus*. The fact is that the Leibniz we know today can scarcely be assigned the view that "in der allgemeinen Ersetzbarkeit sind alle die Gesetze der Gleichheit enthalten".

The Leibniz we know today may be described as a thinker who has formulated on a metalinguistic level the Aristotelian view that "there is no necessity" that all the attributes of the Morning Star be also attributes of the Evening Star⁵. The Aristotelian warning "there is no necessity. . ." became in Leibniz a severe restriction to the "eadem sunt. . .", and this was done in a very simple way: "excipiendae sunt propositiones reduplicativae"⁶. "Reduplicativa enuntiatio, quae constat signo reduplicante, uti sunt "qua", "quatenus", "in quantum", "prout", "qua ratione", "ἤ", "καθ' ὅσον", et similes"⁷.

Scholars like L. Couturat⁸ or Professor K. Dürr⁹ do not seem to have considered this significant point of Leibniz's philosophy. It is convenient to quote some texts. Two of these texts have been published by Couturat. The third belongs to a still unpublished manuscript¹⁰. Some terms or phrases of these texts would require an explanation, and they are certainly interesting contributions to the philosophy of "reduplicative" sentences; for instance the "est in eo aliquid materiale" of text 1 and the last sentence of text 2. Nevertheless, we may dispense with a full commentary of these parts, in so far as the essential point for our present purpose is clear.

1. "A ∞ B significant A et B esse idem, seu ubique sibi posse substitui. (*Nisi prohibeatur, quod fit in iis, ubi terminus aliquis certo respectu considerari declaratur ver. g. licet trilaterum et triangulum sint idem, tamen si dicas triangulum, quatenus tale, habet 180 gradus; non potest substitui trilaterum. Est in eo aliquid materiale.*)¹¹
2. "... sequitur ut modum probandi tradamus; is vero continetur hoc potissimum axioma: Praedicatum in locum subjecti propositionis universalis affirmativae, vel consequens in locum antecedentis propositionis affirmativae, salva veritate substitui potest in alia propositione ubi subjectum prioris est praedicatum, vel ubi antecedens prioris est consequens. *Excipiendae autem sunt propositiones reduplicativae in quibus nos testamur de termino aliquo ita stricte loqui ut alium substitui nolimus, sunt enim reflexivae et respectu cogitationum se habent ut propositiones materiales respectu vocum.*¹²
3. Distincta seu diversa dixerim quae capacia sunt praedicatorum oppositorum unde intelligimus non esse idem sed diversa. Albus Socrates et Musicus Socrates sunt unum idem [que] nam etsi Socrates qua Musicus bene canit, et qua albus non canit, verum tamen est album Socratem canere et quicquid de Musico Socrate etiam de Albo dici potest, nisi quod praedicationes hic excludimus reduplicativas quibus formales

rationes praedicatorum albedo scilicet et musica [...] *distinguuntur*. Et sane revera "Socrates qua Musicus bene canit" est [...] enuntiatio praegnans, [...] constans ex his: "Socrates canit, quia Socrates est Musicus, et nisi Socrates esset Musicus non bene caneret".¹³

The interesting phenomenon of *reduplicatio* seems to have been forgotten in contemporary philosophy; perhaps Bolzano was the first and the last modern logician having paid attention to it, and in a very interesting way indeed.¹⁴ *Reduplicatio* considered as an operation on a name, has the effect that not only the *Bedeutung* is taken into account, but also the *Sinn* of the name, and in such a way that any other *Sinn* corresponding to the same *Bedeutung* (any other name) "would not do". If the chosen ontology is such that entities are supposed to have different "aspects" ("rationes" in classical terminology, "raisons formelles" or "différens rapports d'une même chose" in Leibniz's words¹⁵, "Bestimmungsweise"¹⁶, "Seiten"¹⁷, or "Arten des Gegebenseins"¹⁸ in Fregean vocabulary) and if *consequently* the chosen semantics is such that names are supposed to have not only a reference but also a sense (in Fregean style), then it seems clear that the list of possible subjects of predication usually considered by modern philosophers of logic: individuals, properties of individuals, properties of properties, etc., is *not* exhaustive, but that it has to be "ramified"¹⁹ (at least as regards individuals) into the various aspects (of an individual) under which one wishes to speak about the individual²⁰.

The reason is that—as Bolzano's analysis of reduplicative sentences shows—when one speaks of an individual *sub quadam ratione* one does not speak either about the individual only or about some property only. The situation cannot be either described as if one were speaking about that sort of property which Reichenbach has called "specific property"²¹ (and which corresponds to traditional individual accidents, or to Husserlian "moments"). One is speaking *both* about an individual *and* about some of its properties (leaving aside whether the latter are viewed as "abstract" or "concrete" properties²²). On such grounds, despite the fact that "triangulum" and "trilaterum" happen to be always interchangeable *salva veritate* for instance in the context of Euclid's *Elements*, (as Leibniz points out²³) it will not be true that they are interchangeable in any context (as Leibniz also points out):

"... on peut tousjours dire dans l'abstrait, que le triangle n'est pas le trilatère, ou que les raisons formelles du triangle et du trilatère ne sont pas les mêmes, comme parlent les Philosophes. Ce sont de différens rapports d'une même chose."²⁴

This is why the "eadem sunt..." should be weakened into "aequipollentia sunt quorum unum potest alteri substitui salvis legibus calculi"²⁵. "Aequivalentia" or "aequipollentia" are indeed the right words for the right meaning in the present question²⁶. Interchangeability in any context is not a law deriving from the "essence" of identity²⁷ (except perhaps in the sense that two equiform signs-event may be interchanged everywhere *salva*

veritate, i.e. except perhaps in what Leibniz and the philosophical tradition have called “identitas formalis”²⁸). And interchangeability in some context (say Euclid’s *Elements*) can hardly be said to imply identity. In the fact that entities have aspects, lies the deep motivation for a logic of sense and denotation, independently of how this logic has been concretely approached up to now.

Although Frege’s initial move was to disregard his own insight into entities as given under different aspects (and names reflecting these aspects), dogmatically adhering to a Leibnizian formula, he was compelled afterwards to pay tribute to that insight, a tribute which may be described as a painstaking reconquest of opaque contexts. It might be replied that once this tribute is completely paid (if ever), the situation turns out to be the same as if the insight into entities as given under different aspects had been respected since the very beginning. But there is still a difference between making a mistake plus correcting it and simply avoiding a mistake. Frege’s situation, because of relying upon an insufficient knowledge of Leibniz, was unfortunately the former. As for Leibniz, it may be said that at least for some time during his life²⁹ he was in the second category, i.e. he was aware that his ontology determined his semantics in such a way that the antinomy of the name-relation could not appear, because the third principle of the name-relation would lack plausibility³⁰ or because “substitutivity” could not be accepted as “one of the fundamental laws governing identity”³¹.

NOTES

1. p. 94: *Non inelegans specimen demonstrandi in abstractis*; also in Gerhardt’s edition, vol. VII, p. 228.
2. This has been translated into English by C. I. Lewis, *A Survey of Symbolic Logic*, Appendix.
3. *Introduction to Mathematical Logic*, note 502.
4. *Sophist. Elen.* 179a 36; *Phys.* 202b 10 f. In my view, Prof. Bocheński’s interpretation of these texts is not adequate, cf. *Ancient Formal Logic*, North-Holland Pu.Co., thesis 11.46.
5. Aristotle would give “Coriscus” and “the man who is approaching” instead of the famous Fregean example (cf. the texts referred to in preceding note).
6. Couturat: *Opuscules et Fragments inédits de Leibniz* (reprint Olms 1961) p. 403 (see also below, text 2).
7. J. Jungius: *Logica Hamburgensis*, ed. R. Meyer, Hamburg, 1957, p. 91.
8. Couturat: *La Logique de Leibniz d’après des documents inédits* (Paris, Alcan, 1901), for instance chapter VI, 16.
9. K. Dürr: *Die mathematische Logik von Leibniz*, in *Studia Philosophica* (Basel), vol. VII (1947), 87-102.

Y. Belaval offers some references concerning the occurrence of “reduplicatio” in Leibniz’s *Confessio Philosophi* (Vrin, Paris, 1961, p. 127). It is curious to observe, however, that Y. Belaval does not mention the philosophical significance of reduplication (I owe this remark to E. de Olaso).

10. According to an indication of Dr. H. Lackmann (Niedersaechsische Landesbibliothek, Handschriften-Abteilung).
11. Couturat, *Opuscules et Fragments inédits de Leibniz* (reprint Olms 1961), p. 261.
12. *Ibid.* p. 403.
13. This is the initial passage of a manuscript recorded in Bodemann, *Die Leibniz-Handschriften der koen. oeff. Bibliothek zu Hannover*, Hannover, 1895, p. 113. Bodemann himself gives an extensive *incipit*. Doubtful texts have been inserted in square brackets or replaced by "...". I am grateful to Dr. H. Lackmann (see note 10) for having sent a microfilm of this manuscript. The title of the latter is: "Notationes quaedam quae P. Aloysii Temmik 'Philosophia veram theologiae et medicinae. . .' in itinere aliquo percurrenti inciderunt". Couturat, curiously, leaves unpublished this important *Handschrift* (*Opuscules...*, p. 529).

As to Temmik and his book, there are several interesting details to be mentioned. To begin with, the complete title of the book is the following "Philosophia vera theologiae et medicinae ministra, in theatrum mundi dijudicanda"; it was published in *Colonia*, 1706, a copy is to be found in the British Museum.

In Leibniz's correspondence with Des Bosses one may observe Leibniz's concern about Temmik as well as various curious circumstances about this author. For instance, "Temmik" (or other possible variants) was a pseudonym, concealing a Jesuit whose real name was "Kuemmet" (the pseudonym was, obviously, an anagram). Cf. Gerhardt, *Die Philosophischen Schriften von Leibniz*, II, pp. 326, 332, 335, 350 and especially p. 349 below (I owe these references to E. de Olaso, Consejo de Invest. Cient. y Técn., Buenos Aires). The information contained in these texts has been kindly confirmed by Prof. F. Selvaggi (Gregorian University, Rome), who has carefully examined Temmik's book.

Sommervogel, *Bibliothèque de la Compagnie de Jesus*, is at a loss about "Temmik"; he does not know that Temmik was Kuemmet. He actually mentions a certain "Gaspar Kuemmet", whose bio-bibliography fits with our Temmik; it is most probable that the denotation of the two names is the same, as Prof. C. Giaccon has kindly informed me, but this identity cannot be definitely established until we know why Gerhardt (*loc. cit.* p. 349) reports "Aloysius Kuemmet" rather than "Gaspar Kuemmet".

As to the *content* of Temmik's work, it should be observed, first, that there seems to be no study or criticism of it; it seems to be an unexplored work by an unknown author. According to Prof. F. Selvaggi, Temmik's book is an "unhappy effort at conciliating (what Temmik knew of) ancient metaphysics and (what he knew of) modern science". In the view of the present writer, however, Temmik or Kuemmet was a refined ontologist who was aware, before Hegel or Herbart, of some latent tensions in Aristotelian ontology. This appears in the initial pages of the book, which are precisely the pages which motivated Leibniz's manuscript mentioned above. Temmik begins his 700 page work with a speculation on *identity*. The fundamental situation is that *Socrates* has *two* forms, for instance *white* and *music*. Classical ontology has generally described this by saying that there is an identity *subjecto* and a distinction *ratione*. Temmik claims that this "ratione" does not mean *per conceptum et cognitionem* but that *distingui ratione significat idem ac distingui relatione* (p. 7). In such a context, Temmik employs the challenging formulation *Socrates distinguitur a se ipso*; this is not "dialectical" in a banal sense because careful qualifications are introduced as well. These qualifications are formulated by means of *reduplicatio*. In particular, we do not have "Socrates ≠ Socrates": "...et sic Socrates ut sic a se ipso non distinguitur" (p. 3). But under certain conditions we do have a difference between Socrates and himself; the following text shows this and indicates how the Leibnizian reduplicative restriction of the principle of interchangeability is deeply rooted in classical ontology:

"...ergo Socrates idem cum formis duabus facit composita duo; ergo, ut cum

- una forma est unum compositum, distinguitur a se ipso, ut cum altera forma facit alterum compositum; unde et videtur tantum ut albus, et non auditur; ut Musicus vero audi tur tantum, et non videtur" (p. 3).
14. Bolzano, *Wissenschaftslehre*, §178. As for antecedents, not of *reduplicatio* (a familiar subject in traditional logic) but of the Leibnizian association of *reduplicatio* and identity, there may be something in the Scotist tradition (cf. Prantl, *Geschichte der Logik*, III, p. 290).
 15. Quoted by R. Kauppi: *Ueber die Leibnizsche Logik*, Helsinki 1960, p. 264. See below.
 16. Frege, *Begriffsschrift* §8.
 17. Frege, *Function und Begriff*, Jena 1891, p. 5.
 18. Frege, *Sinn und Bedeutung*, in *Zeits. f. Philos. und philos. Kritik*, 100, 1892, p. 26.
 19. Of course this term is not meant in the technical sense of the theory of types. Nevertheless, it is curious to observe that types were "ramified" in *Principia* by adopting the point of view of the properties of an object *a*.
 20. L. Crahay, *Le formalisme logico-mathématique et le problème du non-sens* (Les Belles Lettres, Paris, 1957), seems to wrongly consider as predicates of predicates what are actually properties of an individual *sub quadam ratione* (in particular p. 48).
 21. Reichenbach, *Elements of Symbolic Logic*, §53.
 22. In G. Küng's terminology, cf. *Concrete and abstract properties*, Notre Dame Journal of Formal Logic, V. 31-36.
 23. Gerhardt's edition, vol. VII, p. 236 (italics ours):
 "Eadem seu coincidentia sunt quorum alterutrum ubilibet potest substitui alteri salva veritate. Exempli gratia, Triangulum et Trilaterum, in omnibus enim propositionibus ab Euclide demonstratis de Triangulo substitui potest Trilaterum, et contra, salva veritate".
 24. *Nouveaux Essais*, in Gerhardt's edition, vol. V, p. 344. The text is referred to in Kauppi, *op. cit.* p. 264.
 25. In agreement with a Leibnizian text quoted by Kauppi, *op. cit.* p. 265 (Gerhardt's edition of the philosophical works, vol. VII, p. 206).
 26. Cf. Couturat, *Opuscules et Fragments inédits de Leibniz*, p. 408 "substituendo aequivalens", p. 496 "per substitutionem terminorum aequivalentium", p. 497 "per substitutiones aequipollentium".
 27. Frege, *Grundlagen der Arithmetik*, §65:
 "... Welches sind diese? [Frege is asking about the laws of identity] Sie werden als analytische Wahrheiten aus dem Begriffe selbst entwickelt werden können"
 This is at least what Frege *pretends* to do, namely to investigate the notion of identity in order to derive from its essence the laws governing it. Unfortunately, Frege does not try to do this himself, but he dogmatically adopts the Leibnizian formula "eadem sunt...".
 Frege really believed that he had grasped the "essence" of identity. In his review of Husserl (*Zeit. f. Philos. und phil. Kritik*, 103, 1894, p. 320) he asserts that the Leibnizian "eadem sunt..." expresses *das Wesen* of the relation of identity.
 28. For Leibniz cf. Kauppi, *op. cit.* p. 71. "Formal identity" is an expression having a curious history; formerly it meant (in Fregean words) identity of senses, not only of reference; afterwards that "complete" identity was viewed in a pejorative way, as implying a sort of "emptiness" ("A = A" says nothing, is merely "formal").

29. It lies beyond the scope of the present paper to state whether Leibniz was always aware of the restriction to the “*eadem sunt...*”. A simple inspection of Leibniz’s dissertation (*Disputatio metaphysica de principio individui*, in Gerhardt’s edition, vol. IV) shows that he had all the chances to be aware of that since the beginning of his scientific career. In any case, it is clear that at least in the mature periods of his life Leibniz has restricted the “*eadem sunt...*” in a *systematical*, not in a “*rhapsodical*” way, i.e. he was not first launching a wrong principle and afterwards trying to clear up the unexpected consequences, without ever knowing whether new exceptions could arise. In stressing this point, my interpretation differs from that of R. Kauppi (*op. cit.*) who seems to view Leibniz rather in the second way, i.e., as someone more or less puzzled by unforeseen failures of the “*eadem sunt...*” (cf. p. 264: “*Es scheint, dass Leibniz diese Schwierigkeit in irgendeiner Form bemerkt hat*”). Also Kauppi’s stress on the confusion between sign and designatum, as being in some way associated with the difficulties of the “*eadem sunt...*”, is not relevant to the *quid* of the problem (pp. 72, 264, 265; cf. also note 3 here).
30. Carnap, *Meaning and Necessity* §31: “The third principle of the name-relation permits replacing a name with another name of the same entity. Although this principle seems quite plausible...”
31. Quine, *Reference and modality* (in *From a logical point of view*): “One of the fundamental principles governing identity is that of substitutivity”.

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