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A MODAL TRUTH-TABULAR INTERPRETATION FOR NECESSARY AND SUFFICIENT CONDITIONS

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An event, D, is a necessary condition for an event, B, if and only if it is *never* the case that B occurs and D does not occur.[1] On the other hand, D is a sufficient condition for B if and only if it is *never* the case that D occurs and B does not occur. These familiar definitions lend themselves readily to truth-tabular schematization. In the tables below we can interpret 'P' to mean that the event is present or did occur. The 'A' is then read 'is absent'. The formulae '$(D \textcircled{N} B)$', '$(D \textcircled{S} B)$', and '$(D \textcircled{NS} B)$' are to be read "Event D is a necessary condition for event B", "Event D is a sufficient condition for event B", and "Event D is a necessary and sufficient condition for event B" respectively.

D	B	$(D \textcircled{N} B)$	$(D \textcircled{S} B)$	$(D \textcircled{NS} B)$
P	P	T	T	T
P	A	T	F	F
A	P	F	T	F
A	A	T	T	T

The striking similarity that the table for '$(D \textcircled{S} B)$' bears to the ordinary truth table for the horseshoe, and the similarity that the table for '$(D \textcircled{NS} B)$' bears to that of the triple bar lead one to suspect that certain normal truth-functional procedures would apply to more complex statements about necessary and sufficient conditions. Indeed, the suspicion is borne out. Consider the law that an event, B, is a necessary condition for an event, D, if and only if D is a sufficient condition for B.[2] This law can be symbolized

(1) $$(D \textcircled{N} B) \equiv (B \textcircled{S} D).$$

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1. It would be better to use 'event-type, B,' or 'an event of type B'.

2. Skyrms, Brian, *Choice and Chance,* pp. 47-51.

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A truth table for (1) produced by appeal to the earlier tables reveals that (1) is, as we supposed, a tautology. This is also the case for the other laws of necessary and sufficient conditions:

(2) (*D* Ⓢ *B*) ≡ (-*B* Ⓢ -*D*)
(3) (*D* Ⓢ *B*) ≡ (-*D* Ⓝ -*B*)
(4) (*D* Ⓝ *B*) ≡ (-*B* Ⓝ -*D*)
(5) (*D* Ⓝ *B*) ≡ (-*D* Ⓢ -*B*)

The use of these truth tables in the interpretation and presentation of the concepts of necessary and sufficient conditions would seem to be established, however a problem emerges. Suppose that we were interested in a condition relationship between two events and still we were unsure as to whether to call the relationship one of necessary, or sufficient conditioning. Suppose we elected a safe course and claimed merely that event *D* "conditions" event *B*. This might easily be symbolized by '(*D* © *B*)'; and interpreted to hold true whenever *D* was either a necessary or a sufficient condition for *B*. Sadly '(*D* © *B*)' becomes a tautology when examined in the light of our tables. It is sad because it is simply not true that each event conditions each other event.

The situation seems to call for a more subtle interpretation, thus the following plenary sets of tables[3] are offered as a modal interpretation of '(*D* Ⓝ *B*)', '(*D* Ⓢ *B*)', '(*D* (NS) *B*)', and '(*D* © *B*)'. The virtue of this modal interpretation is that the five laws of conditions remain tautologies and the formula '(*D* © *B*)' is no longer a tautology. Thus our intuitions are heeded, but at the expense of a measure of simplicity that would have allowed these matters to be easily taught in that type of logic course that normally deals with necessary and sufficient conditions.[4]

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3. Massey, Gerald, *Understanding Symbolic Logic.*

4. (*D* Ⓢ *B*) ≡ □(*D* ⊃ *B*), (*D* (NS) *B*) ≡ □(*D* ≡ *B*).

TABLE	*D*	*B*	(*D* Ⓝ *B*)	(*D* Ⓢ *B*)	(*D* (NS) *B*)	(*D* © *B*)
1	P	P	T	T	T	T
2	P	A	T	F	F	T
3	A	P	F	T	F	T
4	A	A	T	T	T	T
5	P P	P A	T T	F F	F F	T T
6	P A	P P	F F	T T	F F	T T
7	P A	P A	T T	T T	T T	T T
8	P A	A P	F F	F F	F F	F F
9	P A	A A	T T	F F	F F	T T
10	A A	P A	F F	T T	F F	T T
11	P P A	P A P	F F F	F F F	F F F	F F F
12	P P A	P A A	T T T	F F F	F F F	T T T
13	P A A	P P A	F F F	T T T	F F F	T T T
14	P A A	A P A	F F F	F F F	F F F	F F F
15	P P A A	P A P A	F F F F	F F F F	F F F F	F F F F

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