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A NOTE ON R-MINGLE AND SOBOCIŃSKI'S THREE-VALUED LOGIC

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The system **R**-Mingle (**RM**) of Dunn [2] is the result of adding the axiom schema $A \rightarrow . A \rightarrow A$ to the system **R** of relevant implication—*cf.* Belnap [1]. Consider the matrices

$\rightarrow$	0	1	2	$\sim$
0	2	2	2	2
*1	0	1	2	1
*2	0	0	2	0

&	0	1	2
0	0	0	0
*1	0	1	1
*2	0	1	2

v	0	1	2
0	0	1	2
*1	1	1	2
*2	2	2	2

The values 1 and 2 are designated. Since axioms of **RM** always take designated values and *modus ponens* and *adjunction* preserve this property, we have

Lemma 1. *Theorems of* **RM** *uniformly take designated values when evaluated by the above matrices.*

In [3], Sobociński proved that the system **S** based on the above matrices for $\rightarrow$ and $\sim$ is axiomatized by the following schemas together with *modus ponens*:

S1. $A \rightarrow B \rightarrow . B \rightarrow C \rightarrow . A \rightarrow C$
S2. $A \rightarrow . A \rightarrow B \rightarrow B$
S3. $(A \rightarrow . A \rightarrow B) \rightarrow . A \rightarrow B$
S4. $A \rightarrow . B \rightarrow . \sim B \rightarrow A$
S5. $\sim A \rightarrow \sim B \rightarrow . B \rightarrow A$

Lemma 1, together with Sobociński's result, yields

Lemma 2. *If A is a theorem of the pure theory of implication and negation of the calculus* **RM**, *then A is a theorem of* **S**.

In as much as the axioms of **S** are theorems of **RM** (proofs are either routine or given in Dunn [2]) and *modus ponens* is a rule of **RM**, we have

Lemma 3. *If A is a theorem of* **S**, *then A is a theorem of* **RM**.

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Lemmas 2 and 3 yield the following

Theorem. *A is a theorem of the pure theory of implication and negation of the calculus* **RM** *iff A is a theorem of* **S**.[1]

It might be noted that Sobociński's axiom S4 is independent of Dunn's axioms for $\rightarrow$ and $\sim$ so that these do not capture the pure theory of implication and negation of **RM**. Consider the matrix

$\rightarrow$	0	1	2	3	$\sim$
0	3	3	3	3	3
*1	0	1	0	3	1
*2	0	0	2	3	2
*3	0	0	0	3	0

Dunn's axioms for $\rightarrow$ and $\sim$ uniformly take designated values and *modus ponens* preserves this property, but $A \rightarrow . B \rightarrow . \sim B \rightarrow A$ takes the value 0 when A takes the value 1 and B takes the value 2.

REFERENCES

[1] Belnap, N. D., Jr., "Intensional models for first degree formulas," *The Journal of Symbolic Logic*, vol. 32 (1967), pp. 1-22.

[2] Dunn, J. M., "Algebraic completeness results for R-Mingle and its extensions," *The Journal of Symbolic Logic*, vol. 35 (1970), pp. 1-13.

[3] Sobociński, B., "Axiomatization of a partial system of three-valued calculus of propositions," *The Journal of Computing Systems*, vol. 1 (1952), pp. 23-55.

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1. This has been proved independently by Robert K. Meyer and communicated to A. R. Anderson. The result of the next paragraph is due to Meyer.