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ADDITIONAL NOTE ON LATTICE-THEORETICAL FORM OF HAUBER'S LAW

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In [1] it has been shown that the lattice-theoretical formula corresponding to the logical law of Hauber is provable in the field of distributive lattice with Boolean zero element. Actually, as it will be proved below, this formula is a consequence of a weaker lattice system, namely it is provable in the field of modular lattice with zero element. Moreover, two formulas which are akin to the form of Hauber's law mentioned above will also be investigated.

1 Assume that a system

$$\mathfrak{A} = \langle a\ , \cup, \cap\rangle$$

is a modular lattice. Then in its field the following formula is provable:

$\mathcal{A}[abcd] .\because a, b, c, d \epsilon A . a \cap b = c \cap d . a \cup b = c \cup d . \supset : a \leqslant c . b \leqslant d . \equiv .$
$\quad c \leqslant a . d \leqslant b$

Namely,[1]

$A[abcd] : a, b, c, d \epsilon A . a \cap b = c \cap d . a \cup b = c \cup d . a \leqslant c . b \leqslant d . \supset . c \leqslant a .$
$\quad d \leqslant b$

PR $[abcd]$: Hp (5) . $\supset$.

6. $a \cup c = c .$ [**L**; 1; 4]
7. $b \cup d = d .$ [**L**; 1; 5]
8. $a \cup (d \cap c) = (a \cup d) \cap c .$ [**ML**; 1; 4]
9. $b \cup (c \cap d) = (b \cup c) \cap d .$ [**ML**; 1; 5]
10. $a = a \cup (a \cap b) = a \cup (c \cap d) = (a \cup d) \cap c = [a \cup (b \cup d)] \cap c$
$\quad = [(a \cup b) \cup d] \cap c = [(c \cup d) \cup d] \cap c = (c \cup d) \cap c = c$ [**L**; 1; 2; 8; 7; 3]
11. $b = b \cup (a \cap b) = b \cup (c \cap d) = (b \cup c) \cap d = [b \cup (a \cup c)] \cap d$
$\quad = [(a \cup b) \cup c] \cap d = [(c \cup d) \cup c] \cap d = (c \cup d) \cap d = d$ [**L**; 1; 2; 9; 6; 3]

$c \leqslant a . d \leqslant b$ [**L**; 1; 10; 11]

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1. In the proof lines the bold letters **L** and **ML** indicate that a proof is obtained by an application of the theorems belonging to the lattice or modular lattice theories respectively.

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Since in the field of $\mathfrak{A}$ formula $\mathcal{A}$ follows from A at once, the proof is complete.

2 It is obvious that formula $\mathcal{A}$ is akin to the lattice-theoretical form of Hauber's law, viz.:

$$\mathcal{L}[abcd] \therefore a, b, c, d \epsilon A . a \cap b = O . c \cap d = O . a \cup b = c \cup d . \supset : a \leqslant c . b \leqslant d . \equiv . c \leqslant a . d \leqslant b$$

and to the dual of $\mathcal{L}$, viz.:

$$\mathcal{L}^* [abcd] \therefore a, b, c, d \epsilon A . a \cup b = I . c \cup d = I . a \cap b = c \cap d . \supset : a \leqslant c . b \leqslant d . \equiv . c \leqslant a . d \leqslant b$$

and, moreover, that the formulas $\mathcal{L}$ and $\mathcal{L}^*$ follows from $\mathcal{A}$ in the field of any lattice system in which $\mathcal{A}$ is provable and which contains the well defined constants $O \epsilon A$ in the case of $\mathcal{L}$ and $I \epsilon A$ in the case of $\mathcal{L}^*$.

REFERENCES

[1] Sobociński, B., "Lattice-theoretical and mereological forms of Hauber's law," *Notre Dame Journal of Formal Logic,* vol. XII (1971), pp. 81-85.

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