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EXAMINATION OF THE AXIOMATIC FOUNDATIONS OF A THEORY OF CHANGE. V

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Third Part*

§5

§5. *Independence* Independence is not an essential requirement of axiom systems; rather, it belongs to the realm of aesthetics. It is a question of whether we can reduce the number of axioms, i.e., whether any one of the axioms can be derived from the other ones. Assuming, for example, that $A_1, \ldots, A_n$ are axioms, then A_n is said to be independent of $A_1, \ldots, A_{n-1}$, if A_n cannot be derived from $A_1, \ldots, A_{n-1}$. In order to prove that A_n is not derivable from $A_1, \ldots, A_{n-1}$, we choose a domain ω of individuals such that $A_1, \ldots, A_{n-1}$ are satisfied in ω while A_n is not. The proof of independence is here limited to those axioms which were used in the derivation of Theorem 7.9:

A 3.1.2 $x \sim y \rightarrow y \sim x$
A 3.2.3 $x < y \wedge y < z \rightarrow x < z$
A 3.3 $x < y \rightarrow \neg x \sim y$
A 3.5 $x_1 < y \wedge x_2 < y \rightarrow \mathbf{G}x_1 x_2$
A 3.6 $x < y_1 \wedge x < y_2 \rightarrow \mathbf{G}y_1 y_2$
A 5.1 $\mathbf{A}x\alpha \rightarrow \mathbf{F}x\alpha$
A 5.2 $x_1 < x_2 \wedge \mathbf{F}x_2\alpha \rightarrow \mathbf{F}x_1\alpha$
A 6.2 $\mathbf{M}xy\alpha \rightarrow \exists y_0 (x \sim y_0 \wedge y_0 < y)$
A 6.3 $\mathbf{M}xy\alpha \rightarrow \mathbf{A}y\alpha$
A 6.4 $\mathbf{M}xy\alpha \wedge x \sim y_0 \leq y_1 < y \rightarrow \neg \mathbf{A}y_1\alpha$
A 6.7 $\underline{\mathbf{V}}y_1 y_2\alpha \rightarrow \exists x \exists y (x \sim y \wedge y_1 \leq y < y_2 \wedge \mathbf{M}xy_2\alpha)$
A 7.2 $\bar{\mathbf{B}}xy\alpha \wedge \mathbf{B}yz\alpha \rightarrow \mathbf{B}xz\alpha$
A 7.3 $\mathbf{P}x\alpha \wedge \mathbf{A}y\alpha \rightarrow \mathbf{W}xy\alpha$
A 7.5 $\mathbf{M}xy\alpha \wedge \mathbf{A}z\alpha \rightarrow \mathbf{B}zx\alpha$

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1. Independence of Axiom 7.5 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, a_2, a_3 \\ \\ b_1, b_2, b_3 \end{Bmatrix}, \quad \text{where} \neq a_1, a_2, a_3, b_1, b_2, b_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$ $\quad a_1 \sim a_1,\ a_2 \sim a_2,\ a_3 \sim a_3,\ b_1 \sim b_1,\ b_2 \sim b_2,\ b_3 \sim b_3$
$a_1 \sim b_1,\ a_2 \sim b_2,\ a_3 \sim b_3,\ b_1 \sim a_1,\ b_2 \sim a_2,\ b_3 \sim a_3$
$\neg a_1 \sim a_2, \neg a_1 \sim a_3, \neg a_2 \sim a_3, \neg b_1 \sim b_2$, etc. $\neg$ for all other cases

$<$ $\quad a_1 < a_2,\ a_1 < a_3,\ a_2 < a_3,\ b_1 < b_2,\ b_1 < b_3,\ b_2 < b_3$
$\neg a_2 < a_1, \neg a_3 < a_1$, etc. $\neg$ for all other cases

G $\quad \mathbf{G}a_1a_2,\ \mathbf{G}a_1a_3, \mathbf{G}a_2a_3, \mathbf{G}b_1b_2, \mathbf{G}b_1b_3, \mathbf{G}b_2b_3, \mathbf{G}a_1a_1, \mathbf{G}a_2a_2, \mathbf{G}a_3a_3,$
$\mathbf{G}a_2a_1,\ \mathbf{G}a_3a_1, \mathbf{G}a_3a_2, \mathbf{G}b_2b_1, \mathbf{G}b_3b_1, \mathbf{G}b_3b_2, \mathbf{G}b_1b_1, \mathbf{G}b_2b_2, \mathbf{G}b_3b_3,$
$\neg\mathbf{G}a_1b_1, \neg\mathbf{G}b_2a_1$, etc. $\neg$ for all other cases

M $\quad \mathbf{M}a_1b_3\alpha,\ \mathbf{M}a_2b_3\alpha,$
$\neg\mathbf{M}a_1b_2\alpha, \neg\mathbf{M}b_1a_3\alpha$, etc. $\neg$ for all other cases

A $\quad \neg\mathbf{A}a_1\alpha, \neg\mathbf{A}a_2\alpha, \neg\mathbf{A}a_3\alpha, \neg\mathbf{A}b_1\alpha, \neg\mathbf{A}b_2\alpha,\ \mathbf{A}b_3\alpha$

F $\quad \neg\mathbf{F}a_1\alpha, \neg\mathbf{F}a_2\alpha, \neg\mathbf{F}a_3\alpha,\ \mathbf{F}b_1\alpha,\ \mathbf{F}b_2\alpha,\ \mathbf{F}b_3\alpha$

P $\quad \neg\mathbf{P}a_1\alpha, \neg\mathbf{P}a_2\alpha, \neg\mathbf{P}a_3\alpha,\ \mathbf{P}b_1\alpha,\ \mathbf{P}b_2\alpha,\ \mathbf{P}b_3\alpha$

V $\quad \mathbf{V}b_1b_3\alpha,\ \mathbf{V}b_2b_3\alpha, \neg\mathbf{V}a_1a_2\alpha$, etc. $\neg$ for all other cases

B $\quad \neg\mathbf{B}a_2a_1\alpha, \neg\mathbf{B}a_3a_1\alpha, \neg\mathbf{B}b_1a_1\alpha, \neg\mathbf{B}b_2a_1\alpha, \neg\mathbf{B}b_3a_1\alpha,\ \mathbf{B}a_1a_1\alpha,\ \mathbf{B}a_2a_2\alpha,\ \mathbf{B}a_3a_3\alpha,$
$\mathbf{B}a_1a_2\alpha,\ \mathbf{B}a_1a_3\alpha,\ \mathbf{B}a_1b_1\alpha,\ \mathbf{B}a_1b_2\alpha,\ \mathbf{B}a_1b_3\alpha,\ \mathbf{B}b_1b_1\alpha,\ \mathbf{B}b_2b_2\alpha,\ \mathbf{B}b_3b_3\alpha,$
$\neg\mathbf{B}a_3a_2\alpha, \neg\mathbf{B}b_1a_2\alpha, \neg\mathbf{B}b_2a_2\alpha, \neg\mathbf{B}b_3a_2\alpha,\ \mathbf{B}a_3b_3\alpha,$
$\mathbf{B}a_2a_3\alpha,\ \mathbf{B}a_2b_1\alpha,\ \mathbf{B}a_2b_2\alpha,\ \mathbf{B}a_2b_3\alpha,\ \mathbf{B}b_3a_3\alpha,$
$\mathbf{B}b_1a_3\alpha,\ \mathbf{B}b_2a_3\alpha,\ \mathbf{B}b_1b_2\alpha,\ \mathbf{B}b_1b_3\alpha,\ \mathbf{B}b_2b_3\alpha,$
$\neg\mathbf{B}a_3b_1\alpha, \neg\mathbf{B}a_3b_2\alpha, \neg\mathbf{B}b_2b_1\alpha, \neg\mathbf{B}b_3b_1\alpha, \neg\mathbf{B}b_3b_2\alpha,$

W $\quad \mathbf{W}a_1a_2\alpha,\ \mathbf{W}a_1a_3\alpha,\ \mathbf{W}a_1b_1\alpha,\ \mathbf{W}a_1b_2\alpha,\ \mathbf{W}a_1b_3\alpha,\ \mathbf{W}a_2a_3\alpha,\ \mathbf{W}a_2b_1\alpha,\ \mathbf{W}a_2b_2\alpha,$
$\mathbf{W}a_2b_3\alpha,\ \mathbf{W}b_1a_3\alpha,\ \mathbf{W}b_2a_3\alpha,\ \mathbf{W}b_1b_2\alpha,\ \mathbf{W}b_1b_3\alpha,\ \mathbf{W}b_2b_3\alpha$, etc. $\neg$ for all other cases

I $\quad \mathbf{I}a_3b_3\alpha,\ \mathbf{I}a_1a_1\alpha,\ \mathbf{I}a_2a_2\alpha,\ \mathbf{I}b_1b_1\alpha,\ \mathbf{I}b_2b_2\alpha,\ \mathbf{I}b_3b_3\alpha,\ \mathbf{I}a_3a_3\alpha$, etc. $\neg$ for all other cases

This model verifies all axioms except 7.5 which fails in this model because $\mathbf{M}a_1b_3\alpha \wedge \mathbf{A}b_3\alpha \wedge \neg\mathbf{B}b_3a_1\alpha$ holds.

2. Independence of Axiom 7.3 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, a_2, a_3 \\ \\ b_1, b_2, b_3 \end{Bmatrix}, \text{ where} \neq a_1, a_2, a_3, b_1, b_2, b_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$ $a_1 \sim a_1,\ a_2 \sim a_2,\ a_3 \sim a_3,\ b_1 \sim b_1,\ b_2 \sim b_2,\ b_3 \sim b_3$
$a_1 \sim b_1,\ a_2 \sim b_2,\ a_3 \sim b_3,\ b_1 \sim a_1,\ b_2 \sim a_2,\ b_3 \sim a_3$
$\neg a_1 \sim a_2, \neg a_1 \sim a_3, \neg b_1 \sim b_2$, etc. $\neg$ for all other cases

$<$ $a_1 < a_2,\ a_1 < a_3,\ a_2 < a_3,\ b_1 < b_2,\ b_1 < b_3,\ b_2 < b_3$
$\neg a_2 < a_1, \neg a_3 < a_1$, etc. $\neg$ for all other cases

G $\mathbf{G}a_1a_2,\ \mathbf{G}a_1a_3, \mathbf{G}a_2a_3, \mathbf{G}b_1b_2, \mathbf{G}b_1b_3, \mathbf{G}b_2b_3, \mathbf{G}a_1a_1, \mathbf{G}a_2a_2, \mathbf{G}a_3a_3,$
$\mathbf{G}a_2a_1,\ \mathbf{G}a_3a_1, \mathbf{G}a_3a_2, \mathbf{G}b_2b_1, \mathbf{G}b_3b_1, \mathbf{G}b_3b_2, \mathbf{G}b_1b_1, \mathbf{G}b_2b_2, \mathbf{G}b_3b_3,$
$\neg\mathbf{G}a_1b_1, \neg\mathbf{G}b_1a_1$, etc. $\neg$ for all other cases

M $\mathbf{M}a_1b_3\alpha,\ \mathbf{M}a_2b_3\alpha,$
$\neg\mathbf{M}a_1b_2\alpha, \neg\mathbf{M}b_1a_3\alpha$, etc. $\neg$ for all other cases

A $\neg\mathbf{A}a_1\alpha, \neg\mathbf{A}a_2\alpha, \neg\mathbf{A}a_3\alpha, \neg\mathbf{A}b_1\alpha, \neg\mathbf{A}b_2\alpha,\ \mathbf{A}b_3\alpha$

F $\neg\mathbf{F}a_1\alpha, \neg\mathbf{F}a_2\alpha, \neg\mathbf{F}a_3\alpha,\ \mathbf{F}b_1\alpha,\ \mathbf{F}b_2\alpha,\ \mathbf{F}b_3\alpha$

P $\neg\mathbf{P}a_1\alpha, \neg\mathbf{P}a_2\alpha, \neg\mathbf{P}a_3\alpha,\ \mathbf{P}b_1\alpha,\ \mathbf{P}b_2\alpha, \neg\mathbf{P}b_3\alpha$

V $\mathbf{V}b_1b_3\alpha, \mathbf{V}b_2b_3\alpha, \neg\mathbf{V}a_1a_3\alpha$, etc. $\neg$ for all other cases

B $\mathbf{B}a_2a_1\alpha,\ \mathbf{B}a_3a_1\alpha,\ \mathbf{B}b_1a_1\alpha,\ \mathbf{B}b_2a_1\alpha,\ \mathbf{B}b_3a_1\alpha,\ \mathbf{B}a_1a_1\alpha,\ \mathbf{B}a_2a_2\alpha,\ \mathbf{B}a_3a_3\alpha,$
$\neg\mathbf{B}b_2b_3\alpha,$
$\neg\mathbf{B}a_1a_2\alpha, \neg\mathbf{B}a_1a_3\alpha, \neg\mathbf{B}a_1b_1\alpha, \neg\mathbf{B}a_1b_2\alpha, \neg\mathbf{B}a_1b_3\alpha,\ \mathbf{B}b_1b_1\alpha,\ \mathbf{B}b_2b_2\alpha,\ \mathbf{B}b_3b_3\alpha,$
$\mathbf{B}b_3b_2\alpha,$
$\mathbf{B}a_3a_2\alpha,\ \mathbf{B}b_1a_2\alpha,\ \mathbf{B}b_2a_2\alpha,\ \mathbf{B}b_3a_2\alpha, \neg\mathbf{B}b_1a_3\alpha, \neg\mathbf{B}b_2a_3\alpha, \neg\mathbf{B}b_3a_3\alpha, \neg\mathbf{B}b_1b_2\alpha,$
$\neg\mathbf{B}b_1b_3\alpha,$
$\neg\mathbf{B}a_2a_3\alpha, \neg\mathbf{B}a_2b_1\alpha, \neg\mathbf{B}a_2b_2\alpha, \neg\mathbf{B}a_2b_3\alpha,\ \mathbf{B}a_3b_1\alpha,\ \mathbf{B}a_3b_2\alpha,\ \mathbf{B}a_3b_3\alpha,\ \mathbf{B}b_2b_1\alpha,$
$\mathbf{B}b_3b_1\alpha$

W $\mathbf{W}a_2a_1\alpha, \mathbf{W}a_3a_1\alpha, \mathbf{W}b_1a_1\alpha, \mathbf{W}b_2a_1\alpha, \mathbf{W}b_3a_1\alpha, \mathbf{W}a_3a_2\alpha, \mathbf{W}b_1a_2\alpha, \mathbf{W}b_2a_2\alpha,$
$\mathbf{W}b_3a_2\alpha, \mathbf{W}a_3b_1\alpha, \mathbf{W}a_3b_2\alpha, \mathbf{W}a_3b_3\alpha, \mathbf{W}b_2b_1\alpha, \mathbf{W}b_3b_1\alpha, \mathbf{W}b_3b_2\alpha$, etc. $\neg$ for all other cases

This model verifies all axioms except 7.3 which fails in this model because $\mathbf{P}b_1\alpha \wedge \mathbf{A}b_3\alpha \wedge \neg\mathbf{W}b_1b_3\alpha$ holds.

3. Independence of Axiom 7.2 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1,\ a_2,\ a_3 \\ \\ b_1,\ b_2,\ b_3 \end{Bmatrix}, \text{ where } \neq a_1,\ a_2,\ a_3,\ b_1,\ b_2,\ b_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$ $a_1 \sim a_1$, $a_2 \sim a_2$, $a_3 \sim a_3$, $b_1 \sim b_1$, $b_2 \sim b_2$, $b_3 \sim b_3$,
$a_1 \sim b_1$, $a_2 \sim b_2$, $a_3 \sim b_3$, $b_1 \sim a_1$, $b_2 \sim a_2$, $b_3 \sim a_3$,
$\neg a_1 \sim a_2$, $\neg a_1 \sim a_3$, etc. $\neg$ for all other cases

$<$ $a_1 < a_2$, $a_1 < a_3$, $a_2 < a_3$, $b_1 < b_2$, $b_1 < b_3$, $b_2 < b_3$,
$\neg a_2 < a_1$, $\neg a_3 < a_1$, etc. $\neg$ for all other cases

G $\mathbf{G}a_1a_2$, $\mathbf{G}a_1a_3$, $\mathbf{G}a_2a_3$, $\mathbf{G}b_1b_2$, $\mathbf{G}b_1b_3$, $\mathbf{G}b_2b_3$, $\mathbf{G}a_1a_1$, $\mathbf{G}a_2a_2$, $\mathbf{G}a_3a_3$,
$\mathbf{G}a_2a_1$, $\mathbf{G}a_3a_1$, $\mathbf{G}a_3a_2$, $\mathbf{G}b_2b_1$, $\mathbf{G}b_3b_1$, $\mathbf{G}b_3b_2$, $\mathbf{G}b_1b_1$, $\mathbf{G}b_2b_2$, $\mathbf{G}b_3b_3$,
$\neg\mathbf{G}a_1b_1$, $\neg\mathbf{G}b_1a_1$, etc. $\neg$ for all other cases

M $\mathbf{M}a_1b_3\alpha$, $\mathbf{M}a_2b_3\alpha$, $\neg\mathbf{M}a_1b_2\alpha$, $\neg\mathbf{M}b_1a_3\alpha$, etc. $\neg$ for all other cases

A $\neg\mathbf{A}a_1\alpha$, $\neg\mathbf{A}a_2\alpha$, $\neg\mathbf{A}a_3\alpha$, $\neg\mathbf{A}b_1\alpha$, $\neg\mathbf{A}b_2\alpha$, $\mathbf{A}b_3\alpha$

F $\neg\mathbf{F}a_1\alpha$, $\neg\mathbf{F}a_2\alpha$, $\neg\mathbf{F}a_3\alpha$, $\mathbf{F}b_1\alpha$, $\mathbf{F}b_2\alpha$, $\mathbf{F}b_3\alpha$

P $\neg\mathbf{P}a_1\alpha$, $\neg\mathbf{P}a_2\alpha$, $\neg\mathbf{P}a_3\alpha$, $\mathbf{P}b_1\alpha$, $\mathbf{P}b_2\alpha$, $\neg\mathbf{P}b_3\alpha$

V $\mathbf{V}b_1b_3\alpha$, $\mathbf{V}b_2b_3\alpha$, $\neg\mathbf{V}a_1a_3\alpha$, etc. $\neg$ for all other cases

B $\neg\mathbf{B}a_2a_1\alpha$, $\mathbf{B}a_3a_1\alpha$, $\mathbf{B}b_1a_1\alpha$, $\mathbf{B}b_2a_1\alpha$, $\mathbf{B}b_3a_1\alpha$, $\mathbf{B}a_1a_1\alpha$, $\mathbf{B}a_2a_2\alpha$, $\mathbf{B}a_3a_3\alpha$, $\mathbf{B}b_1b_3\alpha$,
$\mathbf{B}a_1a_2\alpha$, $\neg\mathbf{B}a_1a_3\alpha$, $\neg\mathbf{B}a_1b_1\alpha$, $\neg\mathbf{B}a_1b_2\alpha$, $\neg\mathbf{B}a_1b_3\alpha$, $\mathbf{B}b_1b_1\alpha$, $\mathbf{B}b_2b_2\alpha$, $\mathbf{B}b_3b_3\alpha$, $\neg\mathbf{B}b_3b_1\alpha$,
$\neg\mathbf{B}a_3a_2\alpha$, $\mathbf{B}b_1a_2\alpha$, $\mathbf{B}b_2a_2\alpha$, $\mathbf{B}b_3a_2\alpha$, $\mathbf{B}b_1a_3\alpha$, $\mathbf{B}b_2a_3\alpha$, $\mathbf{B}b_3a_3\alpha$, $\mathbf{B}b_1b_2\alpha$, $\mathbf{B}b_2b_3\alpha$,
$\mathbf{B}a_2a_3\alpha$, $\neg\mathbf{B}a_2b_1\alpha$, $\neg\mathbf{B}a_2b_2\alpha$, $\neg\mathbf{B}a_2b_3\alpha$, $\neg\mathbf{B}a_3b_1\alpha$, $\neg\mathbf{B}a_3b_2\alpha$, $\mathbf{B}a_3b_3\alpha$, $\neg\mathbf{B}b_2b_1\alpha$, $\neg\mathbf{B}b_3b_2\alpha$

W $\mathbf{W}a_1a_2\alpha$, $\mathbf{W}a_3a_1\alpha$, $\mathbf{W}b_1a_1\alpha$, $\mathbf{W}b_2a_1\alpha$, $\mathbf{W}b_3a_1\alpha$, $\mathbf{W}a_2a_3\alpha$, $\mathbf{W}b_1a_2\alpha$, $\mathbf{W}b_2a_2\alpha$, $\mathbf{W}b_2a_3\alpha$,
$\mathbf{W}b_3a_2\alpha$, $\mathbf{W}b_1a_3\alpha$, $\mathbf{W}b_1b_2\alpha$, $\mathbf{W}b_1b_3\alpha$, $\mathbf{W}b_2b_3\alpha$, etc. $\neg$ for all other cases

I $\mathbf{I}a_3b_3\alpha$, $\mathbf{I}a_1a_1\alpha$, $\mathbf{I}a_2a_2\alpha$, $\mathbf{I}a_3a_3\alpha$, $\mathbf{I}b_1b_1\alpha$, $\mathbf{I}b_2b_2\alpha$, $\mathbf{I}b_3b_3\alpha$, etc. $\neg$ for all other cases

This model verifies all axioms except 7.2 which fails in this model because $\mathbf{B}a_2a_3\alpha \wedge \mathbf{B}a_3a_1\alpha \wedge \neg\mathbf{B}a_2a_1\alpha$ holds.

4. Independence of Axiom 6.7 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, a_2, a_3 \\ b_1, b_2, b_3 \\ c_1, c_2, c_3 \end{Bmatrix}, \text{ where} \neq a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$	$a_1 \sim a_1,\ a_2 \sim a_2,\ a_3 \sim a_3,\ b_1 \sim b_1,\ b_2 \sim b_2,\ b_3 \sim b_3,\ c_1 \sim c_1,\ c_2 \sim c_2,\ c_3 \sim c_3,$ $a_1 \sim b_1,\ a_1 \sim c_1,\ b_1 \sim c_1,\ a_2 \sim b_2,\ a_2 \sim c_2,\ b_2 \sim c_2,\ a_3 \sim b_3,\ a_3 \sim c_3,\ b_3 \sim c_3,$ $b_1 \sim a_1,\ c_1 \sim a_1,\ c_1 \sim b_1,\ b_2 \sim a_2,\ c_2 \sim a_2,\ c_2 \sim b_2,\ b_3 \sim a_3,\ c_3 \sim a_3,\ c_3 \sim b_3,$ $\neg a_1 \sim a_2, \neg a_1 \sim a_3$, etc. $\neg$ for all other cases
$<$	$a_1 < a_2,\ a_1 < a_3,\ a_2 < a_3,\ b_1 < b_2,\ b_1 < b_3,\ b_2 < b_3,\ c_1 < c_2,\ c_1 < c_3,\ c_2 < c_3,$ $\neg a_2 < a_1, \neg a_3 < a_1$, etc. $\neg$ for all other cases
G	$\mathsf{G}a_1a_2,\ \mathsf{G}a_1a_3,\ \mathsf{G}a_2a_3,\ \mathsf{G}b_1b_2,\ \mathsf{G}b_1b_3,\ \mathsf{G}b_2b_3,\ \mathsf{G}c_1c_2,\ \mathsf{G}c_1c_3,\ \mathsf{G}c_2c_3,\ \mathsf{G}a_1a_1,$ $\mathsf{G}a_2a_1,\ \mathsf{G}a_3a_1,\ \mathsf{G}a_3a_2,\ \mathsf{G}b_2b_1,\ \mathsf{G}b_3b_1,\ \mathsf{G}b_3b_2,\ \mathsf{G}c_2c_1,\ \mathsf{G}c_3c_1,\ \mathsf{G}c_3c_2,\ \mathsf{G}a_2a_2,$ $\mathsf{G}a_3a_3,\ \mathsf{G}b_1b_1,\ \mathsf{G}b_2b_2,\ \mathsf{G}b_3b_3,\ \mathsf{G}c_1c_1,\ \mathsf{G}c_2c_2,\ \mathsf{G}c_3c_3,$ $\neg\mathsf{G}a_1b_1, \neg\mathsf{G}b_1a_1$, etc. $\neg$ for all other cases
M	$\mathsf{M}a_1c_3\alpha,\ \mathsf{M}a_2c_3\alpha, \neg\mathsf{M}a_1b_3\alpha, \neg\mathsf{M}b_1c_3\alpha$, etc. $\neg$ for all other cases
A	$\neg\mathsf{A}a_1\alpha, \neg\mathsf{A}a_2\alpha, \neg\mathsf{A}a_3\alpha, \neg\mathsf{A}b_1\alpha, \neg\mathsf{A}b_2\alpha,\ \mathsf{A}b_3\alpha, \neg\mathsf{A}c_1\alpha, \neg\mathsf{A}c_2\alpha,\ \mathsf{A}c_3\alpha$
F	$\neg\mathsf{F}a_1\alpha, \neg\mathsf{F}a_2\alpha, \neg\mathsf{F}a_3\alpha,\ \mathsf{F}b_1\alpha,\ \mathsf{F}b_2\alpha,\ \mathsf{F}b_3\alpha,\ \mathsf{F}c_1\alpha,\ \mathsf{F}c_2\alpha,\ \mathsf{F}c_3\alpha$
P	$\neg\mathsf{P}a_1\alpha, \neg\mathsf{P}a_2\alpha, \neg\mathsf{P}a_3\alpha,\ \mathsf{P}b_1\alpha,\ \mathsf{P}b_2\alpha, \neg\mathsf{P}b_3\alpha,\ \mathsf{P}c_1\alpha,\ \mathsf{P}c_2\alpha, \neg\mathsf{P}c_3\alpha$
V	$\mathsf{V}b_1b_3\alpha,\ \mathsf{V}b_2b_3\alpha,\ \mathsf{V}c_1c_3\alpha,\ \mathsf{V}c_2c_3\alpha,$ $\neg\mathsf{V}a_1a_3\alpha, \neg\mathsf{V}b_1b_2\alpha$, etc. $\neg$ for all other cases
B	$\mathsf{B}a_2a_1\alpha,\ \mathsf{B}a_3a_1\alpha,\ \mathsf{B}b_1a_1\alpha,\ \mathsf{B}b_2a_1\alpha,\ \mathsf{B}b_3a_1\alpha,\ \mathsf{B}c_1a_1\alpha,\ \mathsf{B}c_2a_1\alpha,\ \mathsf{B}c_3a_1\alpha,$ $\neg\mathsf{B}a_1a_2\alpha, \neg\mathsf{B}a_1a_3\alpha, \neg\mathsf{B}a_1b_1\alpha, \neg\mathsf{B}a_1b_2\alpha, \neg\mathsf{B}a_1b_3\alpha, \neg\mathsf{B}a_1c_1\alpha, \neg\mathsf{B}a_1c_2\alpha, \neg\mathsf{B}a_1c_3\alpha,$ $\mathsf{B}a_3a_2\alpha,\ \mathsf{B}b_1a_2\alpha,\ \mathsf{B}b_2a_2\alpha,\ \mathsf{B}b_3a_2\alpha,\ \mathsf{B}c_1a_2\alpha,\ \mathsf{B}c_2a_2\alpha,\ \mathsf{B}c_3a_2\alpha,\ \mathsf{B}b_2c_2\alpha,\ \mathsf{B}b_2c_3\alpha,$ $\neg\mathsf{B}a_2a_3\alpha, \neg\mathsf{B}a_2b_1\alpha, \neg\mathsf{B}a_2b_2\alpha, \neg\mathsf{B}a_2b_3\alpha, \neg\mathsf{B}a_2c_1\alpha, \neg\mathsf{B}a_2c_2\alpha, \neg\mathsf{B}a_2c_3\alpha,\ \mathsf{B}c_2b_2\alpha, \neg\mathsf{B}c_3b_2\alpha,$ $\mathsf{B}b_1a_3\alpha,\ \mathsf{B}b_2a_3\alpha,\ \mathsf{B}b_3a_3\alpha,\ \mathsf{B}c_1a_3\alpha,\ \mathsf{B}c_2a_3\alpha,\ \mathsf{B}c_3a_3\alpha,\ \mathsf{B}b_1b_2\alpha,$ $\neg\mathsf{B}a_3b_1\alpha, \neg\mathsf{B}a_3b_2\alpha,\ \mathsf{B}a_3b_3\alpha, \neg\mathsf{B}a_3c_1\alpha, \neg\mathsf{B}a_3c_2\alpha,\ \mathsf{B}a_3c_3\alpha, \neg\mathsf{B}b_2b_1\alpha,$ $\mathsf{B}b_1b_3\alpha,\ \mathsf{B}b_1c_1\alpha,\ \mathsf{B}b_1c_2\alpha,\ \mathsf{B}b_1c_3\alpha,\ \mathsf{B}b_2b_3\alpha, \neg\mathsf{B}b_2c_1\alpha,\ \mathsf{B}a_1a_1\alpha,\ \mathsf{B}a_2a_2\alpha,\ \mathsf{B}a_3a_3\alpha,$ $\neg\mathsf{B}b_3b_1\alpha,\ \mathsf{B}c_1b_1\alpha, \neg\mathsf{B}c_2b_1\alpha, \neg\mathsf{B}c_3b_1\alpha, \neg\mathsf{B}b_3b_2\alpha,\ \mathsf{B}c_1b_2\alpha,\ \mathsf{B}b_1b_1\alpha,\ \mathsf{B}b_2b_2\alpha,\ \mathsf{B}b_3b_3\alpha,$ $\neg\mathsf{B}b_3c_1\alpha, \neg\mathsf{B}b_3c_2\alpha,\ \mathsf{B}b_3c_3\alpha,\ \mathsf{B}c_1c_2\alpha,\ \mathsf{B}c_1c_3\alpha,\ \mathsf{B}c_2c_3\alpha,\ \mathsf{B}c_1c_1\alpha,\ \mathsf{B}c_2c_2\alpha,\ \mathsf{B}c_3c_3\alpha,$ $\mathsf{B}c_1b_3\alpha,\ \mathsf{B}c_2b_3\alpha,\ \mathsf{B}c_3b_3\alpha, \neg\mathsf{B}c_2c_1\alpha, \neg\mathsf{B}c_3c_1\alpha, \neg\mathsf{B}c_3c_2\alpha$
W	$\mathsf{W}a_2a_1\alpha,\ \mathsf{W}a_3a_1\alpha,\ \mathsf{W}b_1a_1\alpha,\ \mathsf{W}b_2a_1\alpha,\ \mathsf{W}b_3a_1\alpha,\ \mathsf{W}c_1a_1\alpha,\ \mathsf{W}c_2a_1\alpha,\ \mathsf{W}c_3a_1\alpha,$ $\mathsf{W}a_3a_2\alpha,\ \mathsf{W}b_1a_2\alpha,\ \mathsf{W}b_2a_2\alpha,\ \mathsf{W}b_3a_2\alpha,\ \mathsf{W}c_1a_2\alpha,\ \mathsf{W}c_2a_2\alpha,\ \mathsf{W}c_3a_2\alpha,\ \mathsf{W}b_1a_3\alpha,\ \mathsf{W}b_2a_3\alpha,$ $\mathsf{W}b_1b_2\alpha,\ \mathsf{W}b_1b_3\alpha,\ \mathsf{W}b_2b_3\alpha,\ \mathsf{W}b_1c_2\alpha,\ \mathsf{W}b_2c_3\alpha,\ \mathsf{W}b_1c_3\alpha,\ \mathsf{W}c_1b_2\alpha,\ \mathsf{W}c_1b_3\alpha,\ \mathsf{W}c_2b_3\alpha,$ $\mathsf{W}c_1a_3\alpha,\ \mathsf{W}c_2a_3\alpha,\ \mathsf{W}c_1c_2\alpha,\ \mathsf{W}c_1c_3\alpha,\ \mathsf{W}c_2c_3\alpha$, etc. $\neg$ for all other cases

I $\mathsf{I}b_1c_1\alpha$, $\mathsf{I}b_2c_2\alpha$, $\mathsf{I}a_3b_3\alpha$, $\mathsf{I}a_3c_3\alpha$, $\mathsf{I}b_3c_3\alpha$, $\mathsf{I}a_1a_1\alpha$, $\mathsf{I}a_2a_2\alpha$, $\mathsf{I}a_3a_3\alpha$,
$\mathsf{I}b_1b_1\alpha$, $\mathsf{I}b_2b_2\alpha$, $\mathsf{I}b_3b_3\alpha$, $\mathsf{I}c_1c_1\alpha$, $\mathsf{I}c_2c_2\alpha$, $\mathsf{I}c_3c_3\alpha$, etc. $\neg$ for all other cases

This model varfies all axioms except 6.7 which fails in this model because $\mathsf{V}b_1b_3\alpha$, $x \sim y \wedge b_1 \leq y < b_3 \rightarrow y = b_1 \vee y = b_2$, $x \sim b_1 \rightarrow x = a_1 \vee x = b_1 \vee x = c_1$, and $x \sim b_2 \rightarrow x = a_2 \vee x = b_2 \vee x = c_2$ hold, while $\neg\mathsf{M}a_1b_3$, $\neg\mathsf{M}b_1b_3$, $\neg\mathsf{M}c_1b_3$, $\neg\mathsf{M}a_2b_3$, $\neg\mathsf{M}b_2b_3$, $\neg\mathsf{M}c_2b_3$ also hold.

5. Independence of Axiom 6.4 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, a_2, a_3 \\ b_1, b_2, b_3 \\ c_1, c_2, c_3 \end{Bmatrix}, \text{ where } \neq a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$ $a_1 \sim a_1$, $a_2 \sim a_2$, $a_3 \sim a_3$, $b_1 \sim b_1$, $b_2 \sim b_2$, $b_3 \sim b_3$, $c_1 \sim c_1$, $c_2 \sim c_2$, $c_3 \sim c_3$,
$a_1 \sim b_1$, $a_1 \sim c_1$, $b_1 \sim c_1$, $a_2 \sim b_2$, $a_2 \sim c_2$, $b_2 \sim c_2$, $a_3 \sim b_3$, $a_3 \sim c_3$, $b_3 \sim c_3$,
$b_1 \sim a_1$, $c_1 \sim a_1$, $c_1 \sim b_1$, $b_2 \sim a_2$, $c_2 \sim a_2$, $c_2 \sim b_2$, $b_3 \sim a_3$, $c_3 \sim a_3$, $c_3 \sim b_3$,
$\neg a_1 \sim a_2$, $\neg a_1 \sim a_3$, etc. $\neg$ for all other cases

$<$ $a_1 < a_2$, $a_1 < a_3$, $a_2 < a_3$, $b_1 < b_2$, $b_1 < b_3$, $b_2 < b_3$, $c_1 < c_2$, $c_1 < c_3$, $c_2 < c_3$,
$\neg a_2 < a_1$, $\neg a_3 < a_1$, etc. $\neg$ for all other cases

G $\mathsf{G}a_1a_2$, $\mathsf{G}a_1a_3$, $\mathsf{G}a_2a_3$, $\mathsf{G}b_1b_2$, $\mathsf{G}b_1b_3$, $\mathsf{G}b_2b_3$, $\mathsf{G}c_1c_2$, $\mathsf{G}c_1c_3$, $\mathsf{G}c_2c_3$, $\mathsf{G}a_1a_1$,
$\mathsf{G}a_2a_1$, $\mathsf{G}a_3a_1$, $\mathsf{G}a_3a_2$, $\mathsf{G}b_2b_1$, $\mathsf{G}b_3b_1$, $\mathsf{G}b_3b_2$, $\mathsf{G}c_2c_1$, $\mathsf{G}c_3c_1$, $\mathsf{G}c_3c_2$, $\mathsf{G}a_2a_2$,
$\mathsf{G}a_3a_3$, $\mathsf{G}b_1b_1$, $\mathsf{G}b_2b_2$, $\mathsf{G}b_3b_3$, $\mathsf{G}c_1c_1$, $\mathsf{G}c_2c_2$, $\mathsf{G}c_3c_3$,
$\neg\mathsf{G}a_1b_1$, $\neg\mathsf{G}b_1a_1$, etc. $\neg$ for all other cases

M $\mathsf{M}a_1b_3\alpha$, $\mathsf{M}a_2b_3\alpha$, $\mathsf{M}a_1c_3\alpha$, $\mathsf{M}a_2c_3\alpha$,
$\neg\mathsf{M}a_1b_2\alpha$, $\neg\mathsf{M}a_1c_2\alpha$, etc. $\neg$ for all other cases

A $\neg\mathsf{A}a_1\alpha$, $\neg\mathsf{A}a_2\alpha$, $\neg\mathsf{A}a_3\alpha$, $\mathsf{A}b_1\alpha$, $\mathsf{A}b_2\alpha$, $\mathsf{A}b_3\alpha$, $\neg\mathsf{A}c_1\alpha$, $\neg\mathsf{A}c_2\alpha$, $\mathsf{A}c_3\alpha$

F $\neg\mathsf{F}a_1\alpha$, $\neg\mathsf{F}a_2\alpha$, $\neg\mathsf{F}a_3\alpha$, $\mathsf{F}b_1\alpha$, $\mathsf{F}b_2\alpha$, $\mathsf{F}b_3\alpha$, $\mathsf{F}c_1\alpha$, $\mathsf{F}c_2\alpha$, $\mathsf{F}c_3\alpha$

P $\neg\mathsf{P}a_1\alpha$, $\neg\mathsf{P}a_2\alpha$, $\neg\mathsf{P}a_3\alpha$, $\neg\mathsf{P}b_1\alpha$, $\neg\mathsf{P}b_2\alpha$, $\neg\mathsf{P}b_3\alpha$, $\mathsf{P}c_1\alpha$, $\mathsf{P}c_2\alpha$, $\neg\mathsf{P}c_3\alpha$

V $\mathsf{V}c_1c_3\alpha$, $\mathsf{V}c_2c_3\alpha$, $\neg\mathsf{V}b_1b_3\alpha$, $\neg\mathsf{V}b_2b_3\alpha$, etc. $\neg$ for all other cases

$\mathsf{B}a_2a_1\alpha$, $\mathsf{B}a_3a_1\alpha$, $\mathsf{B}b_1a_1\alpha$, $\mathsf{B}b_2a_1\alpha$, $\mathsf{B}b_3a_1\alpha$, $\mathsf{B}c_1a_1\alpha$, $\mathsf{B}c_2a_1\alpha$, $\mathsf{B}c_3a_1\alpha$,
$\neg\mathsf{B}a_1a_2\alpha$, $\neg\mathsf{B}a_1a_3\alpha$, $\neg\mathsf{B}a_1b_1\alpha$, $\neg\mathsf{B}a_1b_2\alpha$, $\neg\mathsf{B}a_1b_3\alpha$, $\neg\mathsf{B}a_1c_1\alpha$, $\neg\mathsf{B}a_1c_2\alpha$, $\neg\mathsf{B}a_1c_3\alpha$,
$\mathsf{B}a_3a_2\alpha$, $\mathsf{B}b_1a_2\alpha$, $\mathsf{B}b_2a_2\alpha$, $\mathsf{B}b_3a_2\alpha$, $\mathsf{B}c_1a_2\alpha$, $\mathsf{B}c_2a_2\alpha$, $\mathsf{B}c_3a_2\alpha$, $\neg\mathsf{B}b_2c_2\alpha$,
$\mathsf{B}b_2c_3\alpha$,

B $\neg \mathsf{B}a_2a_3\alpha$, $\neg \mathsf{B}a_2b_1\alpha$, $\neg \mathsf{B}a_2b_2\alpha$, $\neg \mathsf{B}a_2b_3\alpha$, $\neg \mathsf{B}a_2c_1\alpha$, $\neg \mathsf{B}a_2c_2\alpha$, $\neg \mathsf{B}a_2c_3\alpha$, $\mathsf{B}c_2b_2\alpha$, $\mathsf{B}c_3b_2\alpha$,
$\mathsf{B}b_1a_3\alpha$, $\mathsf{B}b_2a_3\alpha$, $\mathsf{B}b_3a_3\alpha$, $\mathsf{B}c_1a_3\alpha$, $\mathsf{B}c_2a_3\alpha$, $\mathsf{B}c_3a_3\alpha$, $\mathsf{B}b_1b_2\alpha$,
$\mathsf{B}a_3b_1\alpha$, $\mathsf{B}a_3b_2\alpha$, $\mathsf{B}a_3b_3\alpha$, $\neg \mathsf{B}a_3c_1\alpha$, $\neg \mathsf{B}a_3c_2\alpha$, $\mathsf{B}a_3c_3\alpha$, $\mathsf{B}b_2b_1\alpha$,
$\mathsf{B}b_1b_3\alpha$, $\neg \mathsf{B}b_1c_1\alpha$, $\neg \mathsf{B}b_1c_2\alpha$, $\mathsf{B}b_1c_3\alpha$, $\mathsf{B}b_2b_3\alpha$, $\neg \mathsf{B}b_2c_1\alpha$, $\mathsf{B}a_1a_1\alpha$, $\mathsf{B}a_2a_2\alpha$, $\mathsf{B}a_3a_3\alpha$,
$\mathsf{B}b_3b_1\alpha$, $\mathsf{B}c_1b_1\alpha$, $\mathsf{B}c_2b_1\alpha$, $\mathsf{B}c_3b_1\alpha$, $\mathsf{B}b_3b_2\alpha$, $\mathsf{B}c_1b_2\alpha$, $\mathsf{B}b_1b_1\alpha$, $\mathsf{B}b_2b_2\alpha$, $\mathsf{B}b_3b_3\alpha$,
$\neg \mathsf{B}b_3c_1\alpha$, $\neg \mathsf{B}b_3c_2\alpha$, $\mathsf{B}b_3c_3\alpha$, $\mathsf{B}c_1c_2\alpha$, $\mathsf{B}c_1c_3\alpha$, $\mathsf{B}c_2c_3\alpha$, $\mathsf{B}c_1c_1\alpha$, $\mathsf{B}c_2c_2\alpha$, $\mathsf{B}c_3c_3\alpha$,
$\mathsf{B}c_1b_3\alpha$, $\mathsf{B}c_2b_3\alpha$, $\mathsf{B}c_3b_3\alpha$, $\neg \mathsf{B}c_2c_1\alpha$, $\neg \mathsf{B}c_3c_1\alpha$, $\neg \mathsf{B}c_3c_2\alpha$,

W $\mathsf{W}a_2a_1\alpha$, $\mathsf{W}a_3a_1\alpha$, $\mathsf{W}b_1a_1\alpha$, $\mathsf{W}b_2a_1\alpha$, $\mathsf{W}b_3a_1\alpha$, $\mathsf{W}c_1a_1\alpha$, $\mathsf{W}c_2a_1\alpha$, $\mathsf{W}c_3a_1\alpha$, $\mathsf{W}a_3a_2\alpha$,
$\mathsf{W}b_1a_2\alpha$, $\mathsf{W}b_2a_2\alpha$, $\mathsf{W}b_3a_2\alpha$, $\mathsf{W}c_1a_2\alpha$, $\mathsf{W}c_2a_2\alpha$, $\mathsf{W}c_3a_2\alpha$, $\mathsf{W}c_1b_1\alpha$, $\mathsf{W}c_2b_1\alpha$, $\mathsf{W}c_1b_2\alpha$,
$\mathsf{W}c_2b_2\alpha$, $\mathsf{W}c_1a_3\alpha$, $\mathsf{W}c_2a_3\alpha$, $\mathsf{W}c_1b_3\alpha$, $\mathsf{W}c_2b_3\alpha$, $\mathsf{W}c_1c_2\alpha$, $\mathsf{W}c_1c_3\alpha$, $\mathsf{W}c_2c_3\alpha$,
etc. $\neg$ for all other cases

I $\mathsf{I}a_3b_1\alpha$, $\mathsf{I}a_3b_2\alpha$, $\mathsf{I}b_1b_2\alpha$, $\mathsf{I}b_1b_3\alpha$, $\mathsf{I}b_1c_3\alpha$, $\mathsf{I}b_2b_3\alpha$, $\mathsf{I}b_2c_3\alpha$,
$\mathsf{I}a_3b_3\alpha$, $\mathsf{I}a_3c_3\alpha$, $\mathsf{I}b_3c_3\alpha$, $\mathsf{I}a_1a_1\alpha$, $\mathsf{I}a_2a_2\alpha$, $\mathsf{I}a_3a_3\alpha$, $\mathsf{I}b_1b_1\alpha$,
$\mathsf{I}b_2b_2\alpha$, $\mathsf{I}b_3b_3\alpha$, $\mathsf{I}c_1c_1\alpha$, $\mathsf{I}c_2c_2\alpha$, $\mathsf{I}c_3c_3\alpha$, etc. $\neg$ for all other cases

This model verifies all axioms except 6.4 which fails in this model because $\mathsf{M}a_1b_3\alpha \wedge a_1 \sim b_1 \leq b_2 < b_3 \wedge \mathsf{A}b_2\alpha$ holds.

6. Independence of Axiom 6.3 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, a_2, a_3 \\ b_1, b_2, b_3 \\ c_1, c_2, c_3 \end{Bmatrix}, \text{ where } \neq a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$ as in 5.

$<$ as in 5.

G as in 5.

M $\mathsf{M}a_1b_3\alpha$, $\mathsf{M}a_2b_3\alpha$, $\mathsf{M}a_1c_3\alpha$, $\mathsf{M}a_2c_3\alpha$,
$\neg \mathsf{M}a_1b_2\alpha$, $\neg \mathsf{M}a_1c_2\alpha$, etc. $\neg$ for all other cases

A $\neg \mathsf{A}a_1\alpha$, $\neg \mathsf{A}a_2\alpha$, $\neg \mathsf{A}a_3\alpha$, $\neg \mathsf{A}b_1\alpha$, $\neg \mathsf{A}b_2\alpha$, $\neg \mathsf{A}b_3\alpha$, $\neg \mathsf{A}c_1\alpha$, $\neg \mathsf{A}c_2\alpha$, $\mathsf{A}c_3\alpha$

F $\neg \mathsf{F}a_1\alpha$, $\neg \mathsf{F}a_2\alpha$, $\neg \mathsf{F}a_3\alpha$, $\mathsf{F}b_1\alpha$, $\mathsf{F}b_2\alpha$, $\mathsf{F}b_3\alpha$, $\mathsf{F}c_1\alpha$, $\mathsf{F}c_2\alpha$, $\mathsf{F}c_3\alpha$

P $\neg \mathsf{P}a_1\alpha$, $\neg \mathsf{P}a_2\alpha$, $\neg \mathsf{P}a_3\alpha$, $\mathsf{P}b_1\alpha$, $\mathsf{P}b_2\alpha$, $\mathsf{P}b_3\alpha$, $\mathsf{P}c_1\alpha$, $\mathsf{P}c_2\alpha$, $\neg \mathsf{P}c_3\alpha$

V $\mathsf{V}c_1c_3\alpha$, $\mathsf{V}c_2c_3\alpha$, $\neg \mathsf{V}b_1b_3\alpha$, $\neg \mathsf{V}b_2b_3\alpha$, etc. $\neg$ for all other cases

$\mathbf{B}a_2a_1\alpha,\ \mathbf{B}a_3a_1\alpha,\ \mathbf{B}b_1a_1\alpha,\ \mathbf{B}b_2a_1\alpha,\ \mathbf{B}b_3a_1\alpha,\ \mathbf{B}c_1a_1\alpha,\ \mathbf{B}c_2a_1\alpha,\ \mathbf{B}c_3a_1\alpha,$
$\neg\mathbf{B}a_1a_2\alpha, \neg\mathbf{B}a_1a_3\alpha, \neg\mathbf{B}a_1b_1\alpha, \neg\mathbf{B}a_1b_2\alpha, \neg\mathbf{B}a_1b_3\alpha, \neg\mathbf{B}a_1c_1\alpha, \neg\mathbf{B}a_1c_2\alpha, \neg\mathbf{B}a_1c_3\alpha,$
$\mathbf{B}a_3a_2\alpha,\ \mathbf{B}b_1a_2\alpha,\ \mathbf{B}b_2a_2\alpha,\ \mathbf{B}b_3a_2\alpha,\ \mathbf{B}c_1a_2\alpha,\ \mathbf{B}c_2a_2\alpha,\ \mathbf{B}c_3a_2\alpha,\ \mathbf{B}b_2c_2\alpha,$
$\mathbf{B}b_2c_3\alpha,$
$\neg\mathbf{B}a_2a_3\alpha, \neg\mathbf{B}a_2b_1\alpha, \neg\mathbf{B}a_2b_2\alpha, \neg\mathbf{B}a_2b_3\alpha, \neg\mathbf{B}a_2c_1\alpha, \neg\mathbf{B}a_2c_2\alpha, \neg\mathbf{B}a_2c_3\alpha,\ \mathbf{B}c_2b_2\alpha,$
$\neg\mathbf{B}c_3b_2\alpha,$
$\mathbf{B}b_1a_3\alpha,\ \mathbf{B}b_2a_3\alpha,\ \mathbf{B}b_3a_3\alpha,\ \mathbf{B}c_1a_3\alpha,\ \mathbf{B}c_2a_3\alpha,\ \mathbf{B}c_3a_3\alpha,\ \mathbf{B}b_1b_2\alpha,$
B $\neg\mathbf{B}a_3b_1\alpha, \neg\mathbf{B}a_3b_2\alpha, \neg\mathbf{B}a_3b_3\alpha, \neg\mathbf{B}a_3c_1\alpha, \neg\mathbf{B}a_3c_2\alpha,\ \mathbf{B}a_3c_3\alpha,\ \mathbf{B}b_2b_1\alpha,$
$\mathbf{B}b_1b_3\alpha,\ \mathbf{B}b_1c_1\alpha,\ \mathbf{B}b_1c_2\alpha,\ \mathbf{B}b_1c_3\alpha,\ \mathbf{B}b_2b_3\alpha,\ \mathbf{B}b_2c_1\alpha,\ \mathbf{B}a_1a_1\alpha,\ \mathbf{B}a_2a_2\alpha,$
$\mathbf{B}a_3a_3\alpha,$
$\mathbf{B}b_3b_1\alpha,\ \mathbf{B}c_1b_1\alpha,\ \mathbf{B}c_2b_1\alpha, \neg\mathbf{B}c_3b_1\alpha,\ \mathbf{B}b_3b_2\alpha,\ \mathbf{B}c_1b_2\alpha,\ \mathbf{B}b_1b_1\alpha,\ \mathbf{B}b_2b_2\alpha,$
$\mathbf{B}b_3b_3\alpha,$
$\mathbf{B}b_3c_1\alpha,\ \mathbf{B}b_3c_2\alpha,\ \mathbf{B}b_3c_3\alpha,\ \mathbf{B}c_1c_2\alpha,\ \mathbf{B}c_1c_3\alpha,\ \mathbf{B}c_2c_3\alpha,\ \mathbf{B}c_1c_1\alpha,\ \mathbf{B}c_2c_2\alpha,$
$\mathbf{B}c_3c_3\alpha,$
$\mathbf{B}c_1b_3\alpha,\ \mathbf{B}c_2b_3\alpha, \neg\mathbf{B}c_3b_3\alpha,\ \mathbf{B}c_2c_1\alpha, \neg\mathbf{B}c_3c_1\alpha, \neg\mathbf{B}c_3c_2\alpha$

$\mathbf{W}a_2a_1\alpha,\ \mathbf{W}a_3a_1\alpha,\ \mathbf{W}b_1a_1\alpha,\ \mathbf{W}b_2a_1\alpha,\ \mathbf{W}b_3a_1\alpha,\ \mathbf{W}c_1a_1\alpha,\ \mathbf{W}c_2a_1\alpha,\ \mathbf{W}c_3a_1\alpha,$
$\mathbf{W}a_3a_2\alpha,$
W $\mathbf{W}b_1a_2\alpha,\ \mathbf{W}b_2a_2\alpha,\ \mathbf{W}b_3a_2\alpha,\ \mathbf{W}c_1a_2\alpha,\ \mathbf{W}c_2a_2\alpha,\ \mathbf{W}c_3a_2\alpha,\ \mathbf{W}b_1a_3\alpha,\ \mathbf{W}b_2a_3\alpha,$
$\mathbf{W}b_3a_3\alpha,$
$\mathbf{W}b_1c_3\alpha,\ \mathbf{W}b_2c_3\alpha,\ \mathbf{W}b_3c_3\alpha,\ \mathbf{W}c_1c_3\alpha,\ \mathbf{W}c_2c_3\alpha,\ \mathbf{W}c_1a_3\alpha,\ \mathbf{W}c_2a_3\alpha$
etc. $\neg$ for all other cases

$\mathbf{I}b_1b_2\alpha,\ \mathbf{I}b_1b_3\alpha,\ \mathbf{I}b_2b_3\alpha,\ \mathbf{I}b_1c_1\alpha,\ \mathbf{I}b_1c_2\alpha,\ \mathbf{I}b_2c_1\alpha,\ \mathbf{I}b_2c_2\alpha,\ \mathbf{I}b_3c_1\alpha,$
I $\mathbf{I}b_3c_2\alpha,\ \mathbf{I}a_3c_3\alpha,\ \mathbf{I}c_1c_2\alpha,\ \mathbf{I}a_1a_1\alpha,\ \mathbf{I}a_2a_2\alpha,\ \mathbf{I}a_3a_3\alpha,\ \mathbf{I}b_1b_1\alpha,\ \mathbf{I}b_2b_2\alpha,$
$\mathbf{I}b_3b_3\alpha,\ \mathbf{I}c_1c_1\alpha,\ \mathbf{I}c_2c_2\alpha,\ \mathbf{I}c_3c_3\alpha,$ etc. $\neg$ for all other cases

This model verifies all axioms except 6.2 which fails in this model because $\mathbf{M}a_2b_3\alpha \wedge \neg\mathbf{A}b_3\alpha$ holds.

7. Independence of Axiom 6.2 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, a_2, a_3 \\ b_3 \\ c_1, c_2, c_3 \end{Bmatrix}, \text{ where } \neq a_1, a_2, a_3, b_3, c_1, c_2, c_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$a_1 \sim a_1,\ a_2 \sim a_2, a_3 \sim a_3, b_3 \sim b_3, c_1 \sim c_1, c_2 \sim c_2, c_3 \sim c_3, a_1 \sim c_1,$
$a_2 \sim c_2,$
$\sim$ $c_1 \sim a_1,\ c_2 \sim a_2, a_3 \sim b_3, a_3 \sim c_3, b_3 \sim c_3, b_3 \sim a_3, c_3 \sim a_3, c_3 \sim b_3,$
$\neg a_1 \sim a_2, \neg a_2 \sim a_1,$ etc. $\neg$ for all other cases

$<$ $a_1 < a_2,\ a_1 < a_3, a_2 < a_3, c_1 < c_2, c_1 < c_3, c_2 < c_3,$
$\neg a_2 < a_1, \neg a_1 < b_3,$ etc. $\neg$ for all other cases

G
Ga_1a_1, Ga_2a_2, Ga_3a_3, Gc_1c_1, Gc_2c_2, Gc_3c_3, Ga_1a_2, Ga_1a_3, Ga_2a_3, Gb_3b_3,
Ga_2a_1, Ga_3a_1, Ga_3a_2, Gc_1a_2, Gc_1c_3, Gc_2c_3, Gc_2c_1, Gc_3c_1, Gc_3c_2,
$\neg Ga_1b_3$, $\neg Gb_3a_1$, etc. $\neg$ for all other cases

M
$Ma_1b_3\alpha$, $Ma_2b_3\alpha$, $Ma_1c_3\alpha$, $Ma_2c_3\alpha$,
$\neg Ma_3b_3\alpha$, $\neg Ma_1c_2\alpha$, etc. $\neg$ for all other cases

A
$\neg Aa_1\alpha$, $\neg Aa_2\alpha$, $\neg Aa_3\alpha$, $Ab_3\alpha$, $\neg Ac_1\alpha$, $\neg Ac_2\alpha$, $Ac_3\alpha$

F
$\neg Fa_1\alpha$, $\neg Fa_2\alpha$, $\neg Fa_3\alpha$, $Fb_3\alpha$, $Fc_1\alpha$, $Fc_2\alpha$, $Fc_3\alpha$

P
$\neg Pa_1\alpha$, $\neg Pa_2\alpha$, $\neg Pa_3\alpha$, $\neg Pb_3\alpha$, $Pc_1\alpha$, $Pc_2\alpha$, $\neg Pc_3\alpha$

V
$Vc_1c_3\alpha$, $Vc_2c_3\alpha$, $\neg Vb_3b_3\alpha$, $\neg Va_1a_3\alpha$, etc. $\neg$ for all other cases

B
$Ba_2a_1\alpha$, $Ba_3a_1\alpha$, $Bb_3a_1\alpha$, $Bc_1a_1\alpha$, $Bc_2a_1\alpha$, $Bc_3a_1\alpha$, $Ba_1a_1\alpha$, $Ba_2a_2\alpha$, $Ba_3a_3\alpha$,
$\neg Ba_1a_2\alpha$, $\neg Ba_1a_3\alpha$, $\neg Ba_1b_3\alpha$, $\neg Ba_1c_1\alpha$, $\neg Ba_1c_2\alpha$, $\neg Ba_1c_3\alpha$, $Bc_1c_1\alpha$, $Bc_2c_2\alpha$, $Bc_3c_3\alpha$,
$Ba_3a_2\alpha$, $Bb_3a_2\alpha$, $Bc_1a_2\alpha$, $Bc_2a_2\alpha$, $Bc_3a_2\alpha$, $Bb_3a_3\alpha$, $Bc_1a_3\alpha$, $Bc_2a_3\alpha$, $Bc_3a_3\alpha$,
$\neg Ba_2a_3\alpha$, $\neg Ba_2b_3\alpha$, $\neg Ba_2c_1\alpha$, $\neg Ba_2c_2\alpha$, $\neg Ba_2c_3\alpha$, $Ba_3b_3\alpha$, $\neg Ba_3c_1\alpha$, $\neg Ba_3c_2\alpha$, $Ba_3c_3\alpha$,
$Bc_1b_3\alpha$, $Bc_2b_3\alpha$, $Bc_3b_3\alpha$, $Bc_1c_2\alpha$, $Bc_1c_3\alpha$, $Bc_2c_3\alpha$, $Bb_3b_3\alpha$,
$\neg Bb_3c_1\alpha$, $\neg Bb_3c_2\alpha$, $Bb_3c_3\alpha$, $\neg Bc_2c_1\alpha$, $\neg Bc_3c_1\alpha$, $\neg Bc_3c_2\alpha$

W
$Wa_2a_1\alpha$, $Wa_3a_1\alpha$, $Wb_3a_1\alpha$, $Wc_1a_1\alpha$, $Wc_2a_1\alpha$, $Wc_3a_1\alpha$,
$Wa_3a_2\alpha$, $Wb_3a_2\alpha$, $Wc_1a_2\alpha$, $Wc_2a_2\alpha$, $Wc_3a_2\alpha$, $Wc_1a_3\alpha$,
$Wc_2a_3\alpha$, $Wc_1b_3\alpha$, $Wc_2b_3\alpha$, $Wc_1c_2\alpha$, $Wc_1c_3\alpha$, $Wc_2c_3\alpha$,
etc. $\neg$ for all other cases

I
$Ia_3b_3\alpha$, $Ia_3c_3\alpha$, $Ib_3c_3\alpha$, $Ia_1a_1\alpha$, $Ia_2a_2\alpha$,
$Ia_3a_3\alpha$, $Ib_3b_3\alpha$, $Ic_1c_1\alpha$, $Ic_2c_2\alpha$, $Ic_3c_3\alpha$, etc. $\neg$ for all other cases

This model verifies all axioms except 6.2 which fails in this model because $\neg\exists y_0\,(y_0 < b_3) \wedge Ma_1b_3\alpha$ holds.

8. Independence of Axiom 5.2 from the others. The model consists of:

I. a) a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, & a_2, & a_3 \\ & b_2, & b_3 \\ c_1, & c_2, & c_3 \end{Bmatrix}, \text{ where } \neq a_1, a_2, a_3, b_2, b_3, c_1, c_2, c_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$
$a_1 \sim a_1$, $a_2 \sim a_2$, $a_3 \sim a_3$, $b_2 \sim b_2$, $b_3 \sim b_3$, $c_1 \sim c_1$, $c_2 \sim c_2$, $c_3 \sim c_3$,
$a_1 \sim c_1$, $a_2 \sim b_2$, $a_2 \sim c_2$, $b_2 \sim c_2$, $a_3 \sim b_3$, $a_3 \sim c_3$, $b_3 \sim c_3$,
$c_1 \sim a_1$, $b_2 \sim a_2$, $c_2 \sim a_2$, $c_2 \sim b_2$, $b_3 \sim a_3$, $c_3 \sim a_3$, $c_3 \sim b_3$,
$\neg a_1 \sim a_2$, $\neg a_2 \sim a_1$, etc. $\neg$ for all other cases

$<$ $a_1 < a_2$, $a_1 < a_3$, $a_2 < a_3$, $b_2 < b_3$, $c_1 < c_2$, $c_1 < c_3$, $c_2 < c_3$,
$\neg a_2 < a_1$, $\neg a_3 < a_1$, etc. $\neg$ for all other cases

G $\mathsf{G}a_1a_2$, $\mathsf{G}a_1a_3$, $\mathsf{G}a_2a_3$, $\mathsf{G}b_2b_3$, $\mathsf{G}c_1c_2$, $\mathsf{G}c_1c_3$, $\mathsf{G}c_2c_3$,
$\mathsf{G}a_2a_1$, $\mathsf{G}a_3a_1$, $\mathsf{G}a_3a_2$, $\mathsf{G}b_3b_2$, $\mathsf{G}c_2c_1$, $\mathsf{G}c_3c_1$, $\mathsf{G}c_3c_2$,
$\mathsf{G}a_1a_1$, $\mathsf{G}a_2a_2$, $\mathsf{G}a_3a_3$, $\mathsf{G}b_2b_2$, $\mathsf{G}b_3b_3$, $\mathsf{G}c_1c_1$, $\mathsf{G}c_2c_2$, $\mathsf{G}c_3c_3$,
$\neg\mathsf{G}a_1b_1$, $\neg\mathsf{G}b_1a_1$, etc. $\neg$ for all other cases

M $\mathsf{M}a_1c_3\alpha$, $\mathsf{M}a_2c_3\alpha$, $\neg\mathsf{M}a_1b_3\alpha$, etc. $\neg$ for all other cases

A $\neg\mathsf{A}a_1\alpha$, $\neg\mathsf{A}a_2\alpha$, $\neg\mathsf{A}a_3\alpha$, $\neg\mathsf{A}b_2\alpha$, $\mathsf{A}b_3\alpha$, $\neg\mathsf{A}c_1\alpha$, $\neg\mathsf{A}c_2\alpha$, $\mathsf{A}c_3\alpha$

F $\neg\mathsf{F}a_1\alpha$, $\neg\mathsf{F}a_2\alpha$, $\neg\mathsf{F}a_3\alpha$, $\neg\mathsf{F}b_2\alpha$, $\mathsf{F}b_3\alpha$, $\mathsf{F}c_1\alpha$, $\mathsf{F}c_2\alpha$, $\mathsf{F}c_3\alpha$

P $\neg\mathsf{P}a_1\alpha$, $\neg\mathsf{P}a_2\alpha$, $\neg\mathsf{P}a_3\alpha$, $\neg\mathsf{P}b_2\alpha$, $\neg\mathsf{P}b_3\alpha$, $\mathsf{P}c_1\alpha$, $\mathsf{P}c_2\alpha$, $\neg\mathsf{P}c_3\alpha$

V $\mathsf{V}c_1c_3\alpha$, $\mathsf{V}c_2c_3\alpha$, $\neg\mathsf{V}b_2b_3\alpha$, etc. $\neg$ for all other cases

B $\mathsf{B}a_2a_1\alpha$, $\mathsf{B}a_3a_1\alpha$, $\mathsf{B}b_2a_1\alpha$, $\mathsf{B}b_3a_1\alpha$, $\mathsf{B}c_1a_1\alpha$, $\mathsf{B}c_2a_1\alpha$, $\mathsf{B}c_3a_1\alpha$,
$\neg\mathsf{B}a_1a_2\alpha$, $\neg\mathsf{B}a_1a_3\alpha$, $\neg\mathsf{B}a_1b_2\alpha$, $\neg\mathsf{B}a_1b_3\alpha$, $\neg\mathsf{B}a_1c_1\alpha$, $\neg\mathsf{B}a_1c_2\alpha$, $\neg\mathsf{B}a_1c_3\alpha$,
$\mathsf{B}a_3a_2\alpha$, $\mathsf{B}b_2a_2\alpha$, $\mathsf{B}b_3a_2\alpha$, $\mathsf{B}c_1a_2\alpha$, $\mathsf{B}c_2a_2\alpha$, $\mathsf{B}c_3a_2\alpha$, $\mathsf{B}c_2b_2\alpha$, $\neg\mathsf{B}c_3b_2\alpha$,
$\neg\mathsf{B}a_2a_3\alpha$, $\neg\mathsf{B}a_2b_2\alpha$, $\neg\mathsf{B}a_2b_3\alpha$, $\neg\mathsf{B}a_2c_1\alpha$, $\neg\mathsf{B}a_2c_2\alpha$, $\neg\mathsf{B}a_2c_3\alpha$, $\mathsf{B}b_2c_2\alpha$, $\mathsf{B}b_2c_3\alpha$,
$\mathsf{B}b_2a_3\alpha$, $\mathsf{B}b_3a_3\alpha$, $\mathsf{B}c_1a_3\alpha$, $\mathsf{B}c_2a_3\alpha$, $\mathsf{B}c_3a_3\alpha$, $\mathsf{B}b_2b_3\alpha$, $\mathsf{B}c_1b_2\alpha$, $\neg\mathsf{B}b_2c_1\alpha$,
$\neg\mathsf{B}a_3b_2\alpha$, $\mathsf{B}a_3b_3\alpha$, $\neg\mathsf{B}a_3c_1\alpha$, $\neg\mathsf{B}a_3c_2\alpha$, $\mathsf{B}a_3c_3\alpha$, $\neg\mathsf{B}b_3b_2\alpha$, $\mathsf{B}a_1a_1\alpha$, $\mathsf{B}a_2a_2\alpha$,
$\mathsf{B}a_3a_3\alpha$,
$\mathsf{B}c_1b_3\alpha$, $\mathsf{B}c_2b_3\alpha$, $\mathsf{B}c_3b_3\alpha$, $\mathsf{B}c_1c_2\alpha$, $\mathsf{B}c_1c_3\alpha$, $\mathsf{B}c_2c_3\alpha$, $\mathsf{B}b_2b_2\alpha$, $\mathsf{B}b_3b_3\alpha$,
$\neg\mathsf{B}b_3c_1\alpha$, $\neg\mathsf{B}b_3c_2\alpha$, $\mathsf{B}b_3c_3\alpha$, $\neg\mathsf{B}c_2c_1\alpha$, $\neg\mathsf{B}c_3c_1\alpha$, $\neg\mathsf{B}c_3c_2\alpha$, $\mathsf{B}c_1c_1\alpha$, $\mathsf{B}c_2c_2\alpha$,
$\mathsf{B}c_3c_3\alpha$

W $\mathsf{W}a_2a_1\alpha$, $\mathsf{W}a_3a_1\alpha$, $\mathsf{W}b_2a_1\alpha$, $\mathsf{W}b_3a_1\alpha$, $\mathsf{W}c_1a_1\alpha$, $\mathsf{W}c_2a_1\alpha$, $\mathsf{W}c_3a_1\alpha$, $\mathsf{W}a_3a_2\alpha$,
$\mathsf{W}b_2a_2\alpha$, $\mathsf{W}b_3a_2\alpha$, $\mathsf{W}c_1a_2\alpha$, $\mathsf{W}c_2a_2\alpha$, $\mathsf{W}c_3a_2\alpha$, $\mathsf{W}b_2a_3\alpha$, $\mathsf{W}c_1a_3\alpha$, $\mathsf{W}c_2a_3\alpha$,
$\mathsf{W}b_2b_3\alpha$, $\mathsf{W}c_1b_2\alpha$, $\mathsf{W}b_2c_3\alpha$, $\mathsf{W}c_1b_3\alpha$, $\mathsf{W}c_2b_3\alpha$, $\mathsf{W}c_1c_2\alpha$, $\mathsf{W}c_1c_3\alpha$, $\mathsf{W}c_2c_3\alpha$,
etc. $\neg$ for all other cases

I $\mathsf{I}b_2c_2\alpha$, $\mathsf{I}a_3b_3\alpha$, $\mathsf{I}a_3c_3\alpha$, $\mathsf{I}b_3c_3\alpha$, $\mathsf{I}a_1a_1\alpha$, $\mathsf{I}a_2a_2\alpha$, $\mathsf{I}a_3a_3\alpha$, $\mathsf{I}b_2b_2\alpha$,
$\mathsf{I}b_3b_3\alpha$, $\mathsf{I}c_1c_1\alpha$, $\mathsf{I}c_2c_2\alpha$, $\mathsf{I}c_3c_3\alpha$, etc. $\neg$ for all other cases

This model verifies all axioms except 5.2 which fails in this model because $b_2 < b_3 \wedge \mathsf{F}b_3\alpha \wedge \neg\mathsf{F}b_2\alpha$ holds.

9. Independence of Axiom 5.1 from the others. The model consists of:

I. a) a domain S of individuals for momentantous subjects

$$S = \begin{Bmatrix} a_1, & a_2, & a_3 \\ & & b_3 \\ c_1, & c_2, & c_3 \end{Bmatrix}, \text{ where } \neq a_1, a_2, a_3, b_3, c_1, c_2, c_3$$

b) a domain Z of individuals for properties

$$Z = \{\alpha\}$$

II. an interpretation for our notions as follows:

$\sim$ as in 7.

$<$	as in 7.
G	as in 7.
M	$\mathsf{M}a_1c_3\alpha$, $\mathsf{M}a_2c_3\alpha$, $\neg\mathsf{M}a_1b_3\alpha$, etc. $\neg$ for all other cases
A	$\neg\mathsf{A}a_1\alpha$, $\neg\mathsf{A}a_2\alpha$, $\neg\mathsf{A}a_3\alpha$, $\mathsf{A}b_3\alpha$, $\neg\mathsf{A}c_1\alpha$, $\neg\mathsf{A}c_2\alpha$, $\mathsf{A}c_3\alpha$
F	$\neg\mathsf{F}a_1\alpha$, $\neg\mathsf{F}a_2\alpha$, $\neg\mathsf{F}a_3\alpha$, $\neg\mathsf{F}b_3\alpha$, $\mathsf{F}c_1\alpha$, $\mathsf{F}c_2\alpha$, $\mathsf{F}c_3\alpha$
P	$\neg\mathsf{P}a_1\alpha$, $\neg\mathsf{P}a_2\alpha$, $\neg\mathsf{P}a_3\alpha$, $\neg\mathsf{P}b_3\alpha$, $\mathsf{P}c_1\alpha$, $\mathsf{P}c_2\alpha$, $\neg\mathsf{P}c_3\alpha$
V	$\mathsf{V}c_1c_3\alpha$, $\mathsf{V}c_2c_3\alpha$, $\neg\mathsf{V}a_1a_3\alpha$, etc. $\neg$ for all other cases
B	as in 8, except in the cases where b_2 occurs.
W	as in 8, except in the cases where b_2 occurs.
I	$\mathsf{I}a_3b_3\alpha$, $\mathsf{I}a_3c_3\alpha$, $\mathsf{I}b_3c_3\alpha$, $\mathsf{I}a_1a_1\alpha$, $\mathsf{I}a_2a_2\alpha$, $\mathsf{I}a_3a_3\alpha$, $\mathsf{I}b_3b_3\alpha$, $\mathsf{I}c_1c_1\alpha$, $\mathsf{I}c_2c_2\alpha$, $\mathsf{I}c_3c_3\alpha$, etc. $\neg$ for all other cases

This model verifies all axioms except 5.1 which fails in this model because $\mathsf{A}b_3\alpha \wedge \neg\mathsf{F}b_3\alpha$ holds.

Remark. It is evident that each of the axioms 3.1.2, 3.2.3, 3.3, 3.5, and 3.6 is independent from the axioms 5.1, 5.2, 6.2, 6.3, 6.4, 6.7, 7.2, 7.3, and 7.5. Hence the next proofs of independence will be confined to the first five axioms.

10. Independence of Axiom 3.6 from the others. The model consists of:

I. a domain S of individuals for momentaneous subjects

$$S = \begin{Bmatrix} a_1, a_2, a_3 \\ b_1, b_2, b_3 \end{Bmatrix}, \text{ where } \neq a_1, a_2, a_3, b_1, b_2, b_3$$

II. an interpretation for the notions as follows:

$\sim$	$a_1 \sim a_1$, $a_2 \sim a_2$, $a_3 \sim a_3$, $b_1 \sim b_1$, $b_2 \sim b_2$, $b_3 \sim b_3$ $a_1 \sim b_1$, $a_2 \sim b_2$, $a_3 \sim b_3$, $b_1 \sim a_1$, $b_2 \sim a_2$, $b_3 \sim a_3$, $\neg a_1 \sim a_2$, $\neg a_2 \sim a_1$, etc. $\neg$ for all other cases
$<$	$a_1 < a_2$, $a_1 < a_3$, $b_1 < b_2$, $b_1 < b_3$, $b_2 < b_3$, $\neg a_2 < a_3$, $\neg a_3 < a_2$, etc. for all other cases
G	$\mathsf{G}a_1a_1$, $\mathsf{G}a_2a_2$, $\mathsf{G}a_3a_3$, $\mathsf{G}b_1b_1$, $\mathsf{G}b_2b_2$, $\mathsf{G}b_3b_3$, $\mathsf{G}a_1a_2$, $\mathsf{G}a_1a_3$, $\mathsf{G}a_2a_1$, $\mathsf{G}a_3a_1$, $\mathsf{G}b_1b_3$, $\mathsf{G}b_2b_3$, $\mathsf{G}b_2b_1$, $\mathsf{G}b_3b_1$, $\mathsf{G}b_3b_2$, $\mathsf{G}b_1b_2$, $\neg\mathsf{G}a_2a_3$, $\neg\mathsf{G}a_3a_2$, etc. $\neg$ for all other cases

This model verifies all axioms except 3.6 which fails in this model because $a_1 < a_2 \wedge a_1 < a_3 \wedge \neg\mathsf{G}a_2a_3$ holds.

11. Independence of Axiom 3.5 from the others. The model consists of:

I. a domain S of individuals for momentaneous subjects

$$S = \left\{ \begin{matrix} a_1, a_2, a_3 \\ \\ b_1, b_2, b_3 \end{matrix} \right\}, \text{ where } \neq a_1, a_2, a_3, b_1, b_2, b_3$$

II. an interpretation for the notions as follows:

$\sim$ as above in 10.

$<$ $a_1 < a_3$, $a_2 < a_3$, $b_1 < b_2$, $b_1 < b_3$, $b_2 < b_3$,
$\neg a_2 < a_1$, $\neg a_1 < a_2$, etc. $\neg$ for all other cases

G $\mathbf{G}a_1a_1$, $\mathbf{G}a_2a_2$, $\mathbf{G}a_3a_3$, $\mathbf{G}b_1b_1$, $\mathbf{G}b_2b_2$, $\mathbf{G}b_3b_3$, $\mathbf{G}a_1a_3$, $\mathbf{G}a_3a_1$, $\mathbf{G}a_2a_3$,
$\mathbf{G}a_3a_2$, $\mathbf{G}b_2b_1$, $\mathbf{G}b_1b_3$, $\mathbf{G}b_3b_1$, $\mathbf{G}b_2b_3$, $\mathbf{G}b_3b_2$, $\mathbf{G}b_1b_2$,
$\neg\mathbf{G}a_1a_2$, $\neg\mathbf{G}a_2a_1$, etc. $\neg$ for all other cases

This model verifies all axioms except 3.5 which fails in this model because $a_1 < a_3 \wedge a_2 < a_3 \wedge \neg\mathbf{G}a_1a_2$ holds.

12. Independence of Axiom 3.3 from the others. The model consists of:

I. a domain S of individuals for momentaneous subjects

$$S = \left\{ \begin{matrix} a_1, a_2, a_3 \\ \\ b_1, b_2, b_3 \end{matrix} \right\}, \text{ where } \neq a_1, a_2, a_3, b_1, b_2, b_3$$

II. an interpretation for the notions as follows:

$\sim$ $a_1 \sim a_1$, $a_2 \sim a_2$, $a_3 \sim a_3$, $b_1 \sim b_1$, $b_2 \sim b_2$, $b_3 \sim b_3$,
$a_1 \sim a_2$, $a_2 \sim a_1$, $a_1 \sim b_1$, $b_1 \sim a_1$, $a_2 \sim b_1$, $b_1 \sim a_2$,
$\neg a_1 \sim a_3$, $\neg a_2 \sim a_3$, etc. $\neg$ for all other cases

$<$ $a_1 < a_2$, $b_1 < b_2$, $b_1 < b_3$, $b_2 < b_3$,
$\neg a_2 < a_1$, $\neg a_3 < a_1$, etc. $\neg$ for all other cases

G $\mathbf{G}a_1a_1$, $\mathbf{G}a_2a_3$, $\mathbf{G}a_3a_3$, $\mathbf{G}b_1b_1$, $\mathbf{G}b_2b_2$, $\mathbf{G}b_3b_3$, $\mathbf{G}a_1a_2$, $\mathbf{G}a_2a_1$,
$\mathbf{G}b_1b_2$, $\mathbf{G}b_1b_3$, $\mathbf{G}b_2b_3$, $\mathbf{G}b_2b_1$, $\mathbf{G}b_3b_1$, $\mathbf{G}b_3b_2$,
$\neg\mathbf{G}a_1a_3$, $\neg\mathbf{G}a_1b_1$, etc. $\neg$ for all other cases

This model verifies all axioms except 3.3 which fails in this model because $a_1 < a_2 \wedge a_1 \sim a_2$ holds.

13. Independence of Axiom 3.2.3 from the others. The model consists of:

I. a domain S of individuals for momentaneous subjects

$$S = \left\{ \begin{matrix} a_1, a_2, a_3 \\ \\ b_1, b_2, b_3 \end{matrix} \right\}, \text{ where } \neq a_1, a_2, a_3, b_1, b_2, b_3$$

II. an interpretation for the notions as follows:

$\sim$ as in 10.

$<$ $a_1 < a_2$, $a_2 < a_3$, $a_3 < a_1$, $b_1 < b_2$, $b_1 < b_3$, $b_2 < b_3$,
$\neg a_3 < a_2$, $\neg a_2 < a_1$, $\neg a_1 < a_3$, $\neg b_2 < b_1$, etc. $\neg$ for all other cases

G Ga_1a_2, Ga_2a_2, Ga_3a_3, Gb_1b_1, Gb_2b_2, Gb_3b_3, Ga_1a_2, Ga_2a_1, Ga_2a_2, Ga_3a_2, Ga_1a_3, Ga_3a_1, Gb_1b_2, Gb_2b_1, Gb_2b_3, Gb_3b_2, Gb_1b_3, Gb_3b_1, $\neg Ga_1b_1$, $\neg Gb_1a_1$, etc. $\neg$ for all other cases

This model verifies all axioms except 3.2.3 which fails in this model because $a_1 < a_2 \wedge a_2 < a_3 \wedge \neg(a_1 < a_3)$ holds.

14. Independence of Axiom 3.1.2 from the others. The model consists of:

I. a domain S of individuals for momentaneous subjects

$$S = \{a_1, a_2, a_3\}, \text{ where } \neq a_1, a_2, a_3$$

II. an interpretation for the notions as follows:

$\sim$ $a_2 \sim a_1$, $a_3 \sim a_2$, $a_3 \sim a_1$, $a_1 \sim a_1$, $a_2 \sim a_2$, $a_3 \sim a_3$, $\neg a_1 \sim a_2$, $\neg a_2 \sim a_3$, $\neg a_1 \sim a_3$

$<$ $a_1 < a_2$, $a_1 < a_3$, $a_2 < a_3$, $\neg a_2 < a_1$, $\neg a_3 < a_1$, $\neg a_3 < a_2$

G Ga_1a_1, Ga_2a_2, Ga_3a_3, Ga_1a_2, Ga_2a_1, Ga_1a_3, Ga_3a_1, Ga_2a_3, Ga_3a_2

This model verifies all axioms except 3.1.2 which fails in this model because $a_2 \sim a_1 \wedge \neg(a_2 \sim a_1)$ holds.

APPENDIX

On the Definition of Change in the Sense of Actualization

In §5 we expressly stated that the formalization applied to every kind of change, whether this occurred in a single moment of time or over several such moments. We focused our attention on what we considered to be the essence of a change, that is the acquisition by one and the same subject of a new determination. The more dynamic view of a process of change, hence of a special case of change, was deliberately excluded in order to avoid unnecessary complication in our formalization. We now refocus our attention on this, in order to assure ourselves that the general results which we have so far obtained are in no way invalidated.

Thus we are concerned here with the formalization of the concept of "change" in the sense of actualization. "To become changed" or "to be in the act of changing" is a process, a transition from an initial state to a final state. A thing is in the act of changing if it is on its way to its goal (the final state); thus it has "abandoned" its initial state and is in a progressive actualization towards the final state.

In the formalization thus far we have lacked a primitive notion, introducing and thus allowing for a certain topology on the states, which would express the concept of this progressive actualization of the bearer of change towards its final state. The primitive notion[1] which expresses this idea says:

1. The * indicates the introduction of new primitive notions, axioms, etc.

Pn 4.1* $\alpha \prec \beta$: α and β are typidentical, in that α precedes β; i.e.: that which is being changed by actualization to a determined final state reaches, in this actualization, the state α before the state β; conversely, if β is identified as the final state, this is reached after α.

What are the properties of the relation "$\prec$"? Let us take the example, say, of a litre of water whose initial state is 10° of heat and whose final state is 15°. Both states are typidentical, and the 10° of heat precedes the 15°. Between the two lie an infinite number of states (or a finite number, depending on the solution given in reality to the problem of the continuum). The states stand in the relation "$\prec$" to one another. A consideration of these facts suggests to us that the relation "$\prec$" is an irreflexive, asymmetrical, transitive and dense relation. These formal properties necessitate the following axioms:

A4.1.1* $\neg \alpha \prec \alpha$

The relation is irreflexive; i.e. a given state is certainly typidentical with itself, but it cannot give rise to any change in the sense of an actualization.

A4.1.2* $\alpha \prec \beta \rightarrow \neg \beta \prec \alpha$

The relation is asymmetrical; i.e. for no pair of states does the relation apply in both directions. In any given change only one direction comes into question.

A4.1.3* $\alpha \prec \beta \wedge \beta \prec \gamma \rightarrow \alpha \prec \gamma$

The relation is transitive.

A4.1.4* $\alpha \prec \gamma \rightarrow \exists \beta\ (\alpha \prec \beta \prec \gamma)$

The relation is dense; i.e. it follows from $\alpha \prec \gamma$ that there exists a state β such that $\alpha \prec \beta$ and $\beta \prec \gamma$. The primitive notion "in actu" remains unchanged, as does Axiom 4.1.

Pn 4.1 $\mathbf{A}x\alpha$: x is actual in α.
A4.1 $\exists \alpha\ \mathbf{A}x\alpha$

Given that two states are in the relation "$\prec$", the question comes to mind as to whether a momentaneous subject can be simultaneously actual in both. As we understand the concept "actual," this cannot be the case. From this arises the axiom:

A4.2* $\alpha \prec \beta \rightarrow \neg(\mathbf{A}x\alpha \wedge \mathbf{A}x\beta)$

Parallel to D3.1, D3.2, A3.5 and A3.6 we posit:

D4.1* $\alpha \preceq \beta =_{Df} \alpha \prec \beta \vee \alpha = \beta$

$\alpha \preceq \beta$ means: α and β are typidentical, so that α precedes β, respectively, is equal to β.

D4.2* $\mathbf{T}\alpha\beta =_{Df} \alpha \preceq \beta \vee \beta \prec \alpha$

$\mathbf{T}\alpha b$ means that α and β are typidentical. This defined relation is called the relation of "Typidentity."

A4.3* $\quad \alpha \prec \gamma \wedge \beta \prec \gamma \rightarrow \mathbf{T}\alpha\beta$

A4.4* $\quad \alpha \prec \beta \wedge \alpha \prec \gamma \rightarrow \mathbf{T}\beta\alpha$

With the help of Definition 4.2* it may be demonstrated that the relation of typidentity represents an equivalence relation, i.e. it is reflexive, symmetrical and transitive.

S4.1.1* $\quad \mathbf{T}\alpha\alpha$ (The relation is reflexive)

S4.1.2* $\quad \mathbf{T}\alpha\beta \rightarrow \mathbf{T}\beta\alpha$ (The relation is symmetrical)

S4.1.3* $\quad \mathbf{T}\alpha\beta \wedge \mathbf{T}\beta\gamma \rightarrow \mathbf{T}\alpha\gamma$ (The relation is transitive)

Parallel to S3.6 and S3.7 we can derive two further theorems:

S4.2* $\quad \alpha \prec \gamma \wedge \beta \prec \gamma \wedge \mathbf{A}x\alpha \wedge \mathbf{A}x\beta \rightarrow \alpha = \beta$

S4.3* $\quad \alpha \prec \beta \wedge \alpha \prec \gamma \wedge \mathbf{A}x\beta \wedge \mathbf{A}x\gamma \rightarrow \beta = \gamma$

In §5 it was shown that there are cases in which change takes place momentaneously. Typical examples of this are changes concerning states which are "singular." The concept "singular" may be defined as follows:

D4.3* $\quad \mathbf{S}\alpha =_{Df} \forall\beta\ (\mathbf{T}\alpha\beta \rightarrow \alpha = \beta)$

$\mathbf{S}\alpha$ means: α is singular.

Primitive notion 5.1, Axioms 5.1 and 5.2 as well as the defined notion 5.1 are here applied as in §5:

Pn 5.1 $\quad \mathbf{F}x\alpha$: x is capable of α

A5.1 $\quad \mathbf{A}x\alpha \rightarrow \mathbf{F}x\alpha$

A5.2 $\quad x_1 < x_2 \wedge \mathbf{F}x_2\alpha \rightarrow \mathbf{F}x_1\alpha$

D5.1 $\quad \mathbf{P}x\alpha =_{Df} \mathbf{F}x\alpha \wedge \neg\mathbf{A}x\alpha$ (x is potential in α)

Thus all presuppositions have been given, in order that the defined notion of change in the sense of actualization may now be formalized.

D5.2* $\quad \mathbf{KV}y_1y_2\alpha =_{Df} y_1 < y_2 \wedge \mathbf{A}y_2\alpha \wedge \exists\beta\,(\beta < \alpha \wedge \mathbf{A}y_1\beta \wedge$
$\forall y\,(y_1 < y < y_2 \rightarrow \exists\gamma\,(\beta < \gamma < \alpha \wedge \mathbf{A}y\gamma)) \wedge$
$\forall\gamma_1\,(\beta < \gamma_1 < \alpha \rightarrow \exists y'\,(y_1 < y' < y_2 \wedge \mathbf{A}y'\gamma_1)) \wedge$
$\neg\exists\gamma_2\exists y''\exists y'''\,(y_1 < y'' < y''' < y_2 \wedge \beta < \gamma_2 < \alpha \wedge$
$\mathbf{A}y''\gamma_2 \wedge \mathbf{A}y'''\gamma_2))$

$\mathbf{KV}y_1y_2\alpha$ means: The subject represented by y_1 and y_2 has been the bearer of a continuous change towards α in the time interval of existence of y_1 and y_2; in the change, β is the initial state and α the final state, so that α and β are typidentical; in any given Now of the time interval the subject (without any "discontinuity") is drawing nearer to its final state by the acquisition of a new state.

D5.2* gives the conditions of a (monotonic) continuous change in the strong sense. We could ignore the condition

$$\forall\gamma_1\,(\beta < \gamma_1 < \alpha \rightarrow \exists y'\,(y_1 < y' < y_2 \wedge \mathbf{A}y'\gamma_1))$$

and nevertheless still speak of change in the sense of actualization. In this way "discontinuities" in the actualization would be allowable. The typical example of a momentaneous change may be defined as follows:

D5.3* $\mathbf{MV}y_1y_2\alpha =_{Df} y_1 < y_2 \wedge \mathbf{S}\alpha \wedge \mathbf{A}y_2\alpha \wedge \forall y\,(y_1 \leq y < y_2 \rightarrow \neg\mathbf{A}y\alpha)$.

REFERENCES

References [1]-[8], [9]-[12], and [13] are given at the ends of the first, second, and third parts of this paper respectively. See *Notre Dame Journal of Formal Logic,* vol. IX (1968), pp. 371-384, vol. X (1969), pp. 277-284, and vol. X (1969), pp. 385-409. These are now supplemented by:

[IV] Larouche, L., "Examination of the axiomatic foundations of a theory of change. IV," in *Notre Dame Journal of Formal Logic,* vol. XII (1971), pp. 378-380.

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