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THE TREATMENT OF SEMANTIC PARADOXES FROM 1400 TO 1700

E. J. ASHWORTH

During the middle ages, semantic paradoxes, particularly in the form of "Socrates speaks falsely", where this is taken to be his sole utterance, were discussed extensively under the heading of *insolubilia*. Some attention has been paid to the solutions offered by Ockham, Buridan, and Paul of Venice, but otherwise little work seems to have been done in this area.[1] My own particular interest is with the generally neglected period of logic between the death of Paul of Venice in 1429 and the end of the seventeenth century; and the purpose of this paper is to cast some light both upon the new writings on paradoxes and upon the marked change in emphasis which took place during the sixteenth century. Although the traditional writings on *insolubilia* were available throughout the period, the detailed discussions of the fifteenth and early sixteenth centuries were soon entirely replaced by briefer comments whose inspiration seems wholly classical. Even the mediaeval word *insolubile* was replaced by the Ciceronian *inexplicabile*. In this area at least there is strong evidence for the usual claim that the insights of scholastic logic were swamped by the new interests and studies of renaissance humanism.

Before I go on to discuss in some detail the various types of theory which were put forward, I shall give a brief survey of my sources. I looked at 232 printed books in the British Museum, the Cambridge University Library and the Bodleian, all on logical or related topics, and I found some 47 which were relevant to my study. They fell into three groups: (1) Works devoted to, or with significant chapters on, *insolubilia*;[2] (2) works devoted to, or with sections on, *De Sophisticis Elenchis*;[3] (3) works containing discussion or references under miscellaneous and sometimes unexpected headings.[4] The first group is, of course, the most important. Among the mediaeval sources available in printed form were Ockham's *Summa Totius Logicae*, Buridan's *Sophismata*, the *Regulae Solvendi Sophismata* of William of Heytesbury [Gulielmus Hentisberus], the *Insolubilia* of Peter of Ailly [Alliacus], a French cardinal who died in 1420, and the *Logica*, both *magna* and *parva*, of Paul of Venice, together with a commentary by Mengus Blanchellus Faventinus. Also available was the *Logica* of Paul of Pergola,

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a follower of Paul of Venice, who died in 1451. In addition there are a number of fifteenth century works which cannot be dated with precision. A tract on insolubles was attributed to Peter of Spain, but since it contains references to Buridan and Marsilius of Inghen [d.1396] it must be considerably later. It was printed with commentaries by the Thomists of Cologne and by Johannes de Glogovia (d. 1507) who taught in Cracow. The *Logica* of Peter of Mantua presumably also belongs to the first part of the fifteenth century, as does the *Ars Sophistica* of Stephanus de Monte. Towards the end of the fifteenth century we have Thomas Bricot and Petrus Tartaretus, who both taught at Paris. A little later, four anonymous books were published in England, three of which are largely identical. Returning to writers of whom something is known we have first two humanists, the Frenchman Jacques Le Fèvre d'Etaples (1450?-1536) and Jodocus Clichtoveus (1472-1543) who was originally from Belgium but worked in France. Next, we have two Germans, Jodocus Trutvetter [Isenachensis] (d. 1519) who taught mainly at Erfurt, and Johannes Eckius (1486-1543) a noted theologian and polemicist. Scotland was represented by Johannes Major Scotus (1469-1550) who taught at Paris; and Spain by three men: Gaspar Lax (1487-1560), who also taught at Paris, Cardillus de Villalpandeus (1527-1571), and M. Doniensis Ormazius whose book on Dialectic (pub. 1569) is his only known work. Finally, we have the English mathematician and medical practitioner, Thomas Oliver (d. 1624) who is noteworthy for his apparently unique illustration of the written liar.[5]

The second group is neither so large nor so important. The members took their inspiration from Aristotle's mention of the liar paradox in *De Sophisticis Elenchis* 180 b as an example of the fallacy *secundum quid et simpliciter*. Commentaries on this work were written by various mediaeval figures including Albertus Magnus, Giles of Rome [Aegidius Romanus] and John Duns Scotus (or Pseudo-Scotus). Of the commentaries I have seen, the only two written during the period we are interested in were those by George of Brussels (d. 1450) and Augustinus Niphus (1473-1538 or 1546), an Italian Aristotelian. However, of these writers neither Giles of Rome nor Niphus gave the liar paradox in its true form, for they both assume that Socrates uttered a second proposition, either "An ass flies"[6] or "God does not exist."[7] Three other authors mention paradoxes in a discussion of fallacies which forms part of a general logic textbook. They are Jodocus Willichius (1501-1550), a German; Joachim Perion (1499-1559), a French member of the Benedictine order, and Jacobus Gorscius (or Gôrski), a Pole who lived in the sixteenth century.

The third group cannot be given any real characterization. Some members were reasonably orthodox in the placing of their references to the problem. Hieronymus Savonarola, the Florentine friar (d. 1498) discussed insolubles under the heading of "Sophistic Syllogisms"; and the French political writer François Hotman (1524-1590) and the sixteenth century Italian Ludovicus Carbo (not to be confused with the fifteenth century humanist) both spoke of insolubles under the heading of "sophisms". John Sanderson (d. 1602), an Englishman trained in Cambridge, but who died in

exile as a Catholic priest, and Cornelius Valerius (1512-1572) who taught at Utrecht and Louvain, both used the neutral title of "other forms of argument". John Seton (d. 1567), whose career was similar to that of Sanderson, Ludovicus Lemosius, a sixteenth century Portugese medical doctor and philosopher, and Pierre du Moulin (1600-1684), a Protestant theologian, all introduced the subject in a chapter on Dilemma. However, du Moulin barely deserves to be included, for he only gives a brief reference to the Aristotelian problem of the man who swears that he swears falsely. Hieronymus Cardano (1501-1576), another medical doctor and philosopher, gave a passing reference to the liar in his *Dialectica*. Bartholomaeus Keckermann (1571-1609) mentioned the liar in his history of logic, as did Pierre Gassendi (1592-1655). David Derodon (d. 1664), a French Huguenot, introduced the problem during a discussion of truth and falsity; Johannes Caramuel Lobkowitz (1606-1682), the Spanish Cistercian who was at one time the Archbishop of Prague, devoted a paragraph to the problem of self-falsifying propositions; and Henry Aldrich (1647-1710), the Oxford logician and architect, relegated the matter to an appendix. Finally, there are the French jurist Jacobus Cujacius (1522-1590), who refers to the liar in his work *Ad Africanum*; and the Spanish historian and philosopher Petrus Valentia (1555-1620), who discussed it in his commentary on Cicero's *Academica*. As can be seen, Savonarola is the only pre-sixteenth century writer to appear in this group, and even he can hardly be regarded as early.

I shall first discuss those whose inspiration was purely classical. The majority are to be found in the third group, leaving aside Savonarola, Ormazius, Cardano, du Moulin and Derodon. Gorscius, Willichius and Perion may be added from the second group, and Villalpandeus from the first. Of those whose inspiration was scholastic only Eckius and Major from the first group, and Ormazius from the third, showed any awareness of the classical sources, giving references to Cicero and Aulus Gellius.[8] On the other hand, Aldrich is the only classically inspired writer who discussed a scholastic author, namely Ockham.[9] A great many paradoxes, though not many of logical interest, were current in the Ancient world,[10] and information about them was obtainable in the sixteenth century from such sources as Diogenes Laertius, Cicero (especially *Academica Priora* II 95 and 96), and Aulus Gellius, whose *Noctes Atticae* was very popular and received a number of editions. Adrian Turnebus, who commented on Cicero's *Academica*,[11] referred to Diogenes Laertius by name, but usually it was only Cicero and Aulus Gellius who were cited. The standard liar was given by ten people, two of whom called it an example of *Asystaton, non consistens*, along with such propositions as "I am silent."[12] Nine people discussed Epimenides's claim that all Cretans are liars, and three called it *Pseudomenon* as if it were the standard liar paradox.[13] Only five people gave references to both versions.[14] Another popular paradox was the crocodile, which was referred to by nine people, two of whom attributed it to the humanist Politian's *Miscellanea*, first published in 1489.[15] Nine authors went on to give other paradoxes, or *inexplicabiles*, such as the electra, the horns, and the heap,[16] although in most cases they were listed

rather than described or discussed. Only Aldrich and Villalpandeus wrote at any length. The thinnest treatment is that of Perion, who cites Chrysippus and Cicero but gives "I swear falsely" as his sole example.[17] A reference to the *Lex Falcidia* was added by Hotman,[18] but this is the only unusual touch.

Most of these authors did not offer any solutions to the paradoxes they mentioned, but some of them solved the problem of Epimenides's statement by refusing to recognize its true nature. Carbo, Gorscius, Sanderson, Valerius, Villalpandeus and Willichius all agreed that it was fallacious to take a general or indefinite statement like "Cretans are liars" and to conclude that one particular man, Epimenides, was therefore a liar.[19] They did not consider what would be the case if Epimenides said "All Cretans without any exception are liars".

But it would be a mistake to think that none of those who were influenced by classical writers had anything of interest to say, for Aldrich and Valentia joined the more traditional authors, Savonarola, Ormazius and Derodon in offering a solution which does not seem to have been taken into account by the genuine scholastics I shall discuss, although it would be found acceptable today. This solution is based on the claim that insoluble sentences are not propositions, and hence cannot be assessed as true or false. Savonarola alone did not offer an argument to support his claim, but said that an insoluble was to a genuine proposition as a dead man was to a live one.[20] Derodon argued that an insoluble cannot be a proposition since it leads to an impossible situation, and hence, being itself impossible, says nothing.[21] This view bears considerable similarity to that known as *cassatio*, which is found in more than one mediaeval manuscript.[22] Such views were also mentioned by Paul of Venice.[23] Valentia agreed with Derodon about the status of insolubles, but his reasons, like those of Aldrich and Ormazius rested on a doctrine of presuppositions rather than the notion of impossibility. Who, asked Valentia, says "I lie" wishing to refer to that very proposition? If he is referring to some previous proposition he does indeed make a statement [*sermonem facit*]; otherwise, since he says nothing, it is neither true nor false.[24] Aldrich argued that Socrates's claim, "Socrates speaks falsely", signifies nothing unless there is some previous utterance, for "whoever makes a judgment necessarily presupposes something about which he judges".[25] Ormazius gave an even more elaborate justification, though as we will see later, the arguments he used were standard, even if their conclusion was novel. He first outlined some of the untoward consequences of taking an insoluble to be true or false, such as the acceptance of two contradictories both of which are false; and he then said that in fact insolubles were not propositions but *orationes imperfectae*, for one cannot accept a situation in which part of a proposition refers to (or stands for) the very proposition of which it is part.[26] Experience substantiates this, for if someone remarks "This proposition is false", his audience will listen avidly for some other proposition.

The time has now come to consider the work of those who offered a more traditional, and more sophisticated, treatment of semantical paradoxes. With the exception of George of Brussels, all the writers to be

discussed in detail come from the first group, and the latest in date are Eckius and Major, though references may be made to others. The usual starting point was the problem of self-reference, for it was generally recognized that insoluble propositions exhibited self-reference, or, as it was expressed, had reflexion on themselves. In order to do this, they had to contain a term like 'true', 'false', 'universal', 'known', or 'believed', whose range of reference was propositions.[27] Bricot felt that self-reference arose also from the presence of transcendental terms such as 'being' but this point was not emphasized.[28] The chief area of disagreement concerned the legitimacy of self-reference. Buridan and Paul of Venice had both accepted it explicitly, whilst Peter of Ailly accepted it only in one area.[29] Self-reference was, he felt, possible among vocal and written propositions; but this was no great concession, for it was both illegitimate and impossible for a mental proposition to refer to itself, and it was mental propositions that were the repository of true clarity and meaningfulness. [See below for more about this distinction.] In our period Trutvetter thought the most probable theory was that which interpreted insolubles as categorical propositions whose self-reference was possible; while George of Brussels, Eckius and Clichtoveus, felt that such propositions as "Every proposition is affirmative" or "This proposition is affirmative" were legitimate.[30] Indeed, the latter both signifies itself and is verified of itself, said Clichtoveus.

The chief spokesman of the theory that self-reference was illegitimate was Ockham,[31] although he did not originate the theory.[32] He claimed that part of a proposition cannot suppose for the whole, or, in other words, the proposition in which a term like 'true' appears cannot be included within that term's range of reference. Ockham's view seems to have been based upon a theory of language hierarchies, or levels of predication as Boehner put it,[33] but it could also be interpreted in terms of presuppositions. When I say "This proposition is false", it is presupposed that I am referring to some other proposition, and the truth or falsity of the presupposition has bearing upon the truth or falsity of my proposition. For Ockham, unlike Strawson, if the presupposition was false, the proposition too was false. Thus he translates "Socrates speaks falsely", when this is all that Socrates says, into "Socrates says something false other than this, 'Socrates speaks falsely'," and assesses it as being false. On the other hand, "Socrates does not speak truly" is translated into "Socrates does not say something true other than 'Socrates does not speak truly'," which is true even when Socrates has said only the one proposition. Tartaretus, Bricot, Trutvetter, George of Brussels, and Eckius all put this forward as one possible solution to the problem of insolubles, and, like Ormazius, they all appealed to common usage to support the view.[34] If you ask a man what he says or hears and he replies "I say nothing" or "I hear nothing", his answer is accepted as true, though it could not be if self-reference were allowed. None of them, however, had anything to say about language hierarchies, and in the absence of further evidence it is probably safer to attribute to them a vaguely expressed theory of presuppositions, whereby

the falsity of the presupposition entails the falsity of the proposition in question. Cardano also referred to such a view. Heytesbury held, he claimed, that "I say what is false" is equivalent to "I say what is false, since I say that I say what is false", and this whole statement is true, since the antecedent is false when nothing has been said previously.[35]

So far I have talked of self-reference in general, but in fact most writers concentrated upon a special kind of self-reference, namely self-falsification. Eckius said "An insoluble proposition is a proposition which signifies itself to be false directly or as a consequence [*consecutive*]";[36] and although there were verbal differences among the definitions offered, the principle was not disputed.[37] Some difficulties were, however, raised over its application to propositions like "No proposition is negative", "I am silent", or paradoxes of the crocodile type, although this particular example was not quoted until the sixteenth century. Those who considered the matter did not usually accept "No proposition is negative", or others of the same class as insoluble. Major said that a genuine insoluble must "signify that things are, as they are",[38] which is not the case here. Clichtoveus pointed out that if one assumed the falsity of such a proposition, its truth did not follow, and so there was no paradox.[39] Nor does it falsify itself explicitly in some specifiable circumstances. Peter of Ailly remarked that a proposition could in one sense signify itself to be false, but that for it to be insoluble its falsity must follow from it as a logical consequence.[40] "No proposition is negative" refutes itself by example, but this is not enough. On the other hand, the author of the tract attributed to Peter of Spain seemed to feel that such propositions could be accepted as insoluble.[41]

A few authors used these examples, together with those of the next group, such as "I do not speak", "I am silent", "I am drinking", to substantiate Buridan's claim that a proposition can be possible, because it signifies things as they can be, without being possibly true.[42] "No proposition is negative" or "I am silent" are obviously false whenever they are uttered with reference to the present, but the state of affairs described could well exist. As Eckius and Clichtoveus put it, such propositions can be true at the time of utterance but not for the time of utterance.[43] Thus, they were worthy of discussion, but not because they were insolubles. On the other hand, both the Thomist commentators on Peter of Spain and Johannes de Glogavia were convinced that "I do not speak" is a genuine insoluble. They agreed that it contained no word referring to a property of a proposition, but said that nevertheless it contained an implicit reference to truth and falsity.[44]

Paradoxes of the crocodile type appeared with some frequency. A favourite example concerned a bridge whose keeper (often called Plato or Socrates, though Eckius chose 'Eckebertus') said that he would throw anyone who spoke falsely into the water. The man who wished to cross said "You will throw me in the water." Did he speak truly or falsely?[45] Another puzzle concerned a country where all the healthy people, and none of the sick people, spoke the truth. One of their number said "I am ill,"[46] thus generating the paradox. Paul of Venice, Paul of Pergola, the anony-

mous author of *Libellus Sophistarum*, and David Derodon were all happy to accept such paradoxes as insolubles, though Paul of Venice did refer to them as "Insolubilia que insolubilia non apparent".[47] However, Peter of Ailly, Eckius, Major and Clichtoveus were more acute.[48] Peter of Ailly pointed out that "You will throw me from the bridge" contains no word referring to a proposition, and it does not signify its own falsity. He concluded that the bridgekeeper's conditions and the other man's statement were possible in themselves, but they were incompatible with one another. In other words, this is a paradox which can be resolved by the judgment that the situation described is impossible. Similar reasons and a similar verdict were given by the other three logicians I mentioned.

Finally, a brief reference ought to be made to those paradoxes which can be called pragmatic, on the grounds that they arise "from a relation of a language to its users".[49] I have in mind examples like "Socrates believes that he is deceived", "Socrates knows that he errs", when these are his sole beliefs, and "Socrates doubts the proposition written on the wall" when this is the only proposition written on the wall. Buridan discussed such problems at length, but they also appear in the lists of insolubles given by such logicians as Albert of Saxony, Paul of Venice, Paul of Pergola, Stephanus de Monte, and the author of *Libellus Sophistarum*.[50] Once again, not all those who mentioned them were willing to give them the status of a genuine insoluble or semantic paradox; and Peter of Ailly and Johannes Major both rejected them explicitly.[51] On the other hand, one should perhaps note in defence of Paul of Pergola that when he considered all these doubtful cases he redefined the notion of an insoluble so as to include all types of paradox under the one heading. An insoluble, he said, is not restricted to the true and the false. It is enough to be able to establish that the case is as adequately signified by the proposition in question if and only if it is not as adequately signified.[52]

Once what was to count as an insoluble had been settled, some authors went further and divided insolubles into various classes. One type of classification, to be found in Paul of Venice and in the *Libellus Sophistarum*, involves a distinction between those insolubles, like "This is false", which arise from a property of language, and those, like "Socrates believes he is deceived", which arise from a property of an act, whether interior or exterior.[53] A third kind, exemplified by "Socrates says what is false", involves a combination of language and act. Such a classification would be useful for the discussion of paradoxes in general, including pragmatic paradoxes, but it does not seem to have any bearing upon the problem of semantic paradoxes. The standard division, found in a great many authors, was between those insolubles which falsified themselves immediately and those which did so mediately, through some further proposition or propositions.[54] The author of the *Libellus Sophistarum*, again echoing Paul of Venice, added that in the case of immediate falsification some propositions falsify themselves alone, and some, like "Every proposition is false" have a more general reference.[55] The most elaborate classification was offered by Peter of Ailly, whose scheme was later largely repeated by Tartaretus.[56]

First, we have what seems to be a type-token distinction, between those propositions which are self-falsifying *per se*, in any circumstances, like "This (very) proposition is false"; and those which are self-falsifying *per accidens*, in a particular situation, like "Socrates says what is false". The second distinction was between propositions which falsify themselves directly and those, like "This is not true", which do so indirectly or as a consequence. This latter group was further divided into those, like the last example, which falsify themselves immediately, even though the word 'false' does not appear, and those which do so mediately, through some other proposition or propositions. Finally, a proposition can be mediately falsified through another of which it is not a part, or through another of which it is a part, as with the second conjunct in "God exists and this conjunction is false".

Now that all these preliminaries have been dealt with, it is time to consider the types of solution offered by those logicians who believed that insolubles were genuine propositions and who were unwilling to dispose of them by ruling that all self-reference was illegitimate. I shall begin by examining a solution which rested on the claim that a proposition implies its own truth, whether logically, or virtually as Buridan preferred to assert.[57] The solutions of both Buridan and Paul of Venice have already been discussed by Moody, Prior and Bocheński,[58] so I shall concentrate upon that outlined by Le Fèvre d'Etaples, and explained by Clichtoveus in the accompanying commentary.[59] First a number of rules were laid down: (1) Insolubles are to be assessed by means of propositions which are equivalent to them; (2) every proposition implies itself; (3) every proposition implies its own truth; (4) if a proposition implies several others, it also implies their conjunction; (5) every proposition is equivalent to a conjunction of that proposition and an assertion of its truth; (6) equivalent propositions have the same truth value and imply one another. Clichtoveus pointed out that (5) was derived from (2), (3) and (4) with the aid of a further rule that a conjunction implies one of its conjuncts. That is, we start with '$p \supset p$' and '$p \supset \mathbf{T}p$', get '$p \supset (p \,.\, \mathbf{T}p)$' by (4), and with the aid of '$(p \,.\, \mathbf{T}p) \supset p$', finish with (5), '$p \equiv (p \,.\, \mathbf{T}p)$'. If we take the insoluble, "I say something false", and call this 'a', we can see at once how an insoluble is to be solved through an equivalent proposition, for this insoluble is, by (5), equivalent to "I say something false and a is true". The second conjunct is obviously false, for if a is true and a is what I say, I cannot be saying something false. Since this conjunct is false, the whole conjunction is, and hence so is the proposition equivalent to it, namely the original insoluble. One can also argue the other way, by showing the first conjunct to be false, for whatever interpretation one adopts, the two parts are incompatible. Equally, the negation of the insoluble is true, for the disjunction equivalent to the negated conjunction, "I do not say something false or a is not true", is true. Clichtoveus apparently begs the question at this point, for he says that the disjunction is true because it is the contradictory of a false conjunction, and he does not examine the crucial question of whether the assumption that the insoluble proposition is false will lead to an assertion that it is true. Since he agrees

that the assertion of its truth implies its falsity, this is the test of the validity of his solution, for it is only a genuine solution if it is impossible to obtain the biconditional "*a* is true if and only if *a* is false". That he succeeded by following a line of argument very similar to that of Paul of Venice and which involves no distinction between use and mention, or different language levels, could be shown informally as follows. Suppose we assume that '*a*', where '*a*' is '*a* is false', is false. If '*a*' is false, then its equivalent '*a*' and '*a* is true' will also be false and we obtain "It is false that (*a* and *a* is true)". By De Morgan we get "Either it is false that *a* or it is false that *a* is true". But we can replace '*a*' in the first disjunct by '*a* is false' and argue that if it is false that *a* is false, *a* must be true. Hence we get '*a*', when false, is equivalent to "Either *a* is true or it is false that *a* is true". Since "*a* is either true or not true" is a tautology, the original assumption has not led to any contradictory consequences and a paradoxical conclusion has been avoided. It may be added that the claim that the disjunction is a taugology accords well with Clichtoveus's explicit statement that the original conjunction is made from repugnant parts, and hence is a logical contradiction.

The solution offered by Clichtoveus was not novel in its substance,[60] but the formulation, especially the use of propositional rules, was particularly neat. This did not, however, lead to its acceptance. Eckius outlined the argument with care, but rejected the claim that every proposition implies its own truth on various grounds. For instance, since a proposition, viewed as an occurrent, may not exist, it can be as the antecedent of the consequence signifies without being as the consequent signifies. A proposition must exist in order to be true. Again, if the consequence is acceptable, one must also accept that the possible can imply the impossible, as in "No proposition is negative, therefore it is true that no proposition is negative".[61] Eckius is here relying upon Buridan's view of propositions.[62] Peter of Ailly had earlier rejected the same claim, but on the grounds that if it were true, then every proposition would reflect on itself, and that it is possible to represent the proposition "Man is an ass" without conceiving or understanding it to be true.[63] He did not ask whether in this case "Man is an ass" would be accepted as a genuine proposition.

A second solution to the problem of insolubles involved a distinction between two kinds of meaning or *significatio*, and, as will become apparent, it was closely linked both with the first solution and with the third, which involved a theory of two kinds of truth conditions. According to this second solution, a proposition was said to have both primary or direct signification and secondary or indirect signification.[64] The primary meaning belonged to the proposition by virtue of the meaning of the individual terms (*ratione impositionis terminorum*, as Tartaretus put it) and was that whereby the proposition indicated some state of affairs or other. The secondary meaning concerned the truth or falsity of the proposition itself and was thus the source of all the difficulties caused by insolubles. Although Paul of Venice and his follower Paul of Pergula did not base their own solutions entirely upon this distinction, they made use of it.[65] They postulated that an

insoluble could be taken precisely, or with respect to its primary signification alone, or it could be taken to signify in accordance with both types of meaning. In the first case a contradiction is generated, and the insoluble is judged to be impossible; but in the second case the insoluble is admissible, for it can be shown to be false along the lines already discussed in relation to the first solution. They argued that it is in accordance with primary signification that a proposition is judged to be possible or impossible; but that judgments about truth and falsity concern the secondary signification.[66]

The chief distinction between those who adopted the first solution and those who adopted the second was that the former concentrated upon the claim that every proposition signifies its own truth, whereas the latter emphasized the claim that every insoluble indirectly or secondarily signifies itself to be false. The adherents of this second view were not, however, in agreement as to what kind of proposition was involved. Some held that an insoluble proposition was categorical, some that it was hypothetical, and some that it corresponded to more than one proposition. The author of the tract attributed to Peter of Spain was one of those who believed an insoluble to be a categorical proposition, on the grounds that "the secondary signification is as it were adventitious and accidental" and "there is not a multiple proposition but rather one, because both significations are involved in judging about truth or falsity."[67] Those who followed Ockham and those who adopted the third solution also held that insolubles were categorical propositions;[68] and they all agreed that they were false. In this case, they are false because of the incompatibility of the primary and the secondary signification. One cannot assert both '*p*' and 'it is false that *p*' without destroying one's assertion.[69]

Those who believed that an insoluble proposition should be interpreted in terms of a hypothetical proposition said that it was equivalent to a conjunction whose first conjunct signified that things were as described and whose second conjunct signified the first to be false, in accordance with the insoluble's secondary signification.[70] Thus, Trutvetter in his account of the theory said, "Henry says what is false" is equivalent to "Henry is a thing saying what is false and that proposition which signifies precisely that Henry says what is false is false". The first part of the conjunction is true, but the second false, so the whole conjunction is false. The contradictory of the original insoluble is equivalent to "Either the thing which is Henry is not the thing which is saying what is false or this proposition is not false", where 'this' refers to the first disjunct; and this disjunction is true. No paradox arises on this interpretation, but it was not always found acceptable. As both Peter of Ailly and Tartaretus remarked, there seemed to be no good reason to suppose that an insoluble is equivalent to a hypothetical proposition, and if it is so equivalent, why choose a conjunction rather than the other types?[71]

The view of which Peter of Ailly was the chief exponent, but which was taken into account by most of his successors,[72] involved the interpretation of an insoluble proposition as a *propositio plures*, or as equivalent to

several unconjoined parts. The theory can only be explained with the help of Peter of Ailly's apparatus of vocal, written and mental propositions. Vocal and written propositions signify conventionally (*ad placitum*) as do mental propositions improperly so-called, which are presumably those propositions that run through our mind but are not actually spoken or written. Confused self-reference is possible in all these kinds of proposition. Mental propositions properly so-called are those which signify naturally, and they thus enshrine the meaning which is common to utterances in various languages. No mental proposition can refer to itself, let alone falsify itself, on the grounds that no created thing, including concepts, can have a proper, formal, and distinct cognition of itself; and as a result no spoken or written insoluble can correspond directly to a single mental proposition. If I say "This is false", referring to that very proposition, my utterance corresponds to two mental propositions, one of which refers to the same state of affairs as my proposition, and one of which states that the first mental proposition is false. If I say "This is true", and point to its contradictory, "This is not true", then the first mental proposition is "This is not true" and the second, referring to the first, is "This is false".

Referent		Insoluble
This is not true	←	This is true [spoken]
This is not true	←	This is false
[1st mental proposition]		[2nd mental proposition]

The first mental proposition is true and the second false, so it turns out that an insoluble is in a sense both true and false.

The third solution, and the one which both Trutvetter and Bricot thought the most probable, rested on the claim that two conditions must be satisfied before an affirmative proposition can be said to be true.[73] That is, it must signify things to be as they are, and it must not falsify itself. A negative proposition, on the other hand, is true just in case either it signifies that things are not as they are not or it has a self-falsifying contradictory. This distinction between affirmative and negative propositions had two important results. In the first place, rather paradoxically, it means that "This is false" and "This is not true" have different truth-values when they are both self-referential; and indeed it turns out that all propositions which formally signify themselves to be false are false and all propositions which formally signify themselves not to be true are true. This is because the first group are affirmative and have to satisfy two conditions for truth, whereas the second are negative and hence have to satisfy only one condition. In the second place, it means that of any two contradictory propositions one will be true and one false. "This is false" is false because it falsifies itself, and "This is not false" is true because its contradictory falsifies itself. Thus Bricot, along with Tartaretus and Trutvetter, managed to avoid a trap into which fell some of his contemporaries like Major and the anonymous author of the *Insolubilia*.[74] The latter authors had adopted the approach whereby a true proposition satisfies the two conditions mentioned, and a false proposition fails to satisfy at least one of these

conditions, irrespective of whether it is negative or affirmative.[75] As a result, "This is false" is false because it falsifies itself, but "This is not false" is also false, because it denies that things are as they are. However, even this less sophisticated version of the third solution does avoid the original paradox whereby "This is false" is true if and only if it is false. Nor is the solution to be scorned, despite an *ad hoc* air, for it has the merit of applying only to self-falsifying propositions, without impinging upon less harmful forms of self-reference. On the other hand, if the solution is accepted, the formula "**T**'p' $\equiv$ p" can no longer be appealed to, for p may be the case while 'p' is false, as when p is "This is false".

The Aristotelian solution, whereby insolubles were classed as examples of the fallacy *secundum quid ad simpliciter*, was sometimes mentioned in relation to this solution. The author of the *Libellus Sophistarum* wrote that those who objected to his solution on the grounds that Aristotle had said something different, should be aware that to argue "This proposition signifies precisely as things are, therefore it is true" commits just that fallacy.[76] The proposition in question is indeed true in a certain respect, but not simply, because there is a second condition it has to satisfy. This fallacy was sometimes mentioned in other contexts. Trutvetter, for instance, appealed to it in his account of Ockham's solution;[77] but Aristotle was obviously not regarded as having done more than provide a support for other people's solutions.

A variant of the third solution was provided by Eckius, who said that every insoluble falsifies itself 'per modum sequelae et consecutive', and that "Socrates says what is false" is false not because it signifies things to be other than they are, but because its falsity follows from the assumption of its truth; which ought to be added to the definition.[78] He claimed that Trutvetter, to whose "beautiful way of solving insolubles" he had earlier referred, would not have abhorred this solution. What Eckius seems to have been getting at was the definition "A proposition is true if and only if it signifies that things are as they are and it has no false consequences". This is supported by Paul of Pergola, who emphasized that given the rules of valid inference, no true proposition could have a false consequent, although a false one could have true consequents.[79] Buridan had offered a similar argument.[80]

Whether any of these solutions is likely to bear fruit today is for the reader to decide. It is, however, clear that the writers of the fifteenth and early sixteenth century were inspired by a genuine interest in problems of logic and language, and that they handled them with the finest tools available. That their discussions should have been so completely ignored by subsequent logicians, some of whom were doubtless their pupils, is surprising, given both the availability of their books and the persistence of other traditional doctrines like supposition.[81]

NOTES

1. For Ockham, see P. Boehner, "Ockham's Theory of Supposition and the Notion of Truth," *Franciscan Studies*, vol. 6 (1946), pp. 282-285. For Buridan, see E. A. Moody, *Truth and Consequence in Mediaeval Logic* (Amsterdam, 1953), pp. 101-110; A. N. Prior, "Some Problems of Self-Reference in John Buridan," *Proceedings of the British Academy*, vol. 48 (1962), pp. 281-296; and A. N. Prior, "On a Family of Paradoxes," *Notre Dame Journal of Formal Logic*, vol. 2 (1961), pp. 16-32. For Paul of Venice, see I. M. Bocheński, *A History of Formal Logic*, translated and edited by Ivo Thomas (Notre Dame, Indiana, 1961), pp. 238-251; and I. M. Bocheński, "Formalization of a Scholastic Solution of the Paradox of the 'Liar'" in *Logico-Philosophical Studies*, edited by A. Menne (Dordrecht, 1962), pp. 64-66. Other references are to be found in Bocheński, *A History of Formal Logic*; and in the following works: D. Morduhai-Boltovskoi, "Insolubiles in scholastica et paradoxos de infinito de nostro tempore," *Wiadomości Matematyczne*, vol. 47 (1939), pp. 111-117; V. Muñoz-Delgado, *La Logica Nominalista en la Universidad de Salamanca (1510-1530)*, (Madrid, 1964); C. Prantl, *Geschichte der Logik im Abendlande*, Vol. IV (Leipzig, 1870); L. M. de Rijk, "Some Notes on the Mediaeval Tract de Insolubilibus, with the edition of a Tract dating from the End of the Twelfth Century," *Vivarium*, vol. 4 (1966), pp. 83-115; A. Rüstow, *Der Lügner Theorie: Geschichte und Auflösung* (Leipzig, 1910); J. Salamucha, "Pojawienie się zagadnień antynomialnych na gruncie logiki średniowiecznej," *Przegląd Filozoficzny*, vol. 40 (1937), I, pp. 68-89 and II, pp. 320-343; Curtis Wilson, *William Heytesbury: Mediaeval Logic and the Rise of Mathematical Physics* (Madison, University of Wisconsin Press, 1956). Of the above sources, de Rijk's article is particularly helpful. Rüstow contains a very complete survey of references to the liar paradox from classical times onwards; and Salamucha has lengthy and helpful footnotes. There is extensive modern literature on the problem of paradoxes, and I will mention just the following sample of articles: J. Agassi, "Variations on the Liar Paradox," *Studia Logica*, vol. 15 (1964-1965), pp. 237-238; A. Koyré, "The Liar," *Philosophical and Phenomenological Research*, vol. 6 (1946), pp. 344-362; R. L. Martin, "Toward a Solution to the Liar Paradox," *Philosophical Review*, vol. 76 (1967), pp. 279-311; N. Rescher, "A Note on Self-Referential Statements," *Notre Dame Journal of Formal Logic*, vol. 5 (1964), pp. 218-220; Bas C. van Fraassen, "Presupposition, Implication and Self-Reference," *Journal of Philosophy*, vol. 65 (1968), pp. 136-152.

2. In order of their appearance in the text, the works are: William Ockham, *Summa Totius Logicae* (Oxoniae, 1675); John Buridan, *Sophismata* (Paris, 1493), but see also John Buridan, *Sophisms on Meaning and Truth*, translated and with an introduction by T. K. Scott (New York, 1966); William Heytesbury: Gulielmus Hentisberus, *Regulae Solvendi Sophismata* ([Pavia], 1481); Peter of Ailly: Petrus Alliacus, *Conceptus et Insolubilia* (Parisius, 1498); Paul of Venice, *Pauli Veneti Logica: cum Menghi Faventini super ea commentaria . . .* (Venetiis, 1498); Paul of Pergola, *Logica and Tractatus de Sensu Composito et Diviso*, edited by Sister Mary Anthony Brown (St. Bonaventure, New York, Louvain and Paderborn, 1961); Peter of Spain, *Textus omnium tractatuum Petri hispani etiam syncategorematum et parvorum logicalium cum copulatis secundum doctrinam divi Thome Aquinatis juxta processum magistrorum Colonie in bursa Montis* ([Cologne], 1493), Johannes de Glogavia, or *Glogoviensis, Exercitium super omnes tractatus parvorum logicalium Petri Hispani* (Argentine, 1517), and

Peter of Spain, *Tractatus Syncategorematum and Selected Anonymous Treatises*, translated by J. P. Mullally (Milwaukee, 1964); Petrus Mantuanus, *Petri Mantuani Logica* (Padua, 1477); Stephanus de Monte, *Ars Sophistica* ([Paris], 1490?); Thomas Bricot, *Tractatus Insolubilium* (Paris, 1492); Petrus Tartaretus, *Expositio in Summulas Petri Hispani. Additus est tractatus insolubilium eiusdem . . .* (Venice, 1621); *Insolubilia* (Southwark, [1527?]); *Libellus Sophistarum* (London, 1501-1502); *Libellus Sophistarum ad usum Cantabrigien* (Londinis, 1510); *Libellus Sophistarum ad usum Oxoniensium* (London, [1525?]); Jacques Le Fèvre d'Etaples, *Jacobi Fabri Stapulensis artificiales nonnulle introductiones per Iodocum Clichtoveum in unum diligenter collecte* (Parisius, 1520); Jodocus Trutvetter, *Summule totius logice* (Erphurdie, 1501); Johannes Eckius, *In Summulas Petri Hispani extemporiaria* (Augustae Vindelicorum, 1516); Johannes Major, *Insolubilia* (Paris, 1516); Gaspar Lax, *Insolubilia* (Parisius, 1512); Gasparus Cardillus Villalpandeus, *Summa Summae Summularum* (Madrid, 1615); Matthaeus Doniensis Ormazius, *De Instrumento Instrumentorum sive de dialectica libri sex* (Venetiis, 1569); Thomas Oliver, *De Sophismatum Praestigiis Cavendis Tractatus Paraeneticus* (Cambridge, 1604).

3. Albertus Magnus, *Opera Omnia*, edited by Augustus Borgnet, Vol. II (Paris, 1890); Giles of Rome: Aegidius Romanus, *Expositio supra libros elenchorum Aristotelis* (Venetiis, 1500); Augustinus Niphus, *Expositiones in libros de sophisticis elenchis Aristotelis* (Parisiis, 1540); John Duns Scotus, *Opera*, edited by L. Wadding, Vol. I (Lyons, 1639); Jodocus Willichius, *Erotematum Dialectices libri tres* (Argentorati, 1540); Joachimus Perionius, *De Dialectica libri III* (Basileae, 1549); Jacobus Gorscius, *Commentatiorum Artis Dialecticae libri decem* (Lipsiae, 1563).

4. Hieronymus Savonarola, *Compendium Logice* (Florentiae, 1497); Franciscus Hotomanus, *Dialecticae Institutionis libri IIII* (____?, 1573); Ludovicus Carbo, *Introductiones in Logicam* (Venetiis, 1597); Joannes Sandersonus, *Institutionum dialecticarum libri quatuor* (Antverpiae, 1589); Cornelius Valerius, *Tabulae totius dialectice* (Venetiis, 1564); Joannes Setonus, *Dialectica . . . annotationibus Petri Carteri* (Londini, 1574); Ludovicus Lemosius, *Paradoxorum Dialecticorum* (Salamanticae, 1558); Pierre du Moulin, *Elements de Logique* (Sedan, 1621); Hieronymus Cardano, *Opera Omnia*, Vol. I (Lyons, 1663), pp. 293-308; Bartholomaeus Keckermann, *Praecognitorum Logicorum Tractatus III* (Hanoviae, 1606); Pierre Gassendi, *De Logicae origine et varietate* in *Opera*, Vol. I (Lugduni, 1658), pp. 35-66; David Derodon, *Logica restituta* (Genevae, 1659); Johannes Caramuel Lobkowitz, *Rationalis et realis philosophia* (Lovanii, 1642); Henry Aldrich, *Artis Logicae Compendium* (Oxford, 1692); Jacobus Cujacius, *Ad Africanum Tract V* in *Opera Omnia*, Vol. I (Neapoli, 1758), cols. 1354-1357; Petrus Valentia, *Academiques de Ciceron avec le texte Latin . . . et le commentaire philosophique de Pierre Valentia, juris. Espagnol* (Londres, 1740).

5. Oliver, *op. cit.*, p. 9, gives the following illustration:

omne enuntiatum intra hoc quadratum scriptum est falsum

For a full discussion, see the article by Ivo Thomas, "The Written Liar and Thomas Oliver," *Notre Dame Journal of Formal Logic*, vol. 6 (1965), pp. 201-208.

6. Aegidius Romanus, *op. cit.*, p. 58v°.

7. Niphus, *op. cit.*, p. 23v°.

8. Eckius, *op. cit.*, fol. cviii, vol. cx v°; Major, *op. cit.* [no pagination]; Ormazius, *op. cit.*, p. 54v°.

9. Aldrich, *op. cit.*, p. 74.

10. For details, see I. M. Bocheński, *Ancient Formal Logic* (Amsterdam, 1968), pp. 100-102.

11. *M. Tulli Ciceronis Academica. Recensuit . . . et Hadr. Turnebi Petrique Fabri Commentarius adjunxit Joannes Davisius* (Cantabrigiae, 1725), p. 161, note 7.

12. Aldrich, *loc. cit.*; Cujacius, *op. cit.*, col. 1356; Gassendi, *op. cit.*, p. 40; Hotman, *op. cit.*, p. 340; Keckermann, *op. cit.*, p. 82; Lemosius, *op. cit.*, pp. 54v°-55; Sanderson, *op. cit.*, p. 154; Seton, *op. cit.* [no pagination]; Valentia, *op. cit.*, p. 62; Valerius, *op. cit.*, p. 174. The two specially mentioned are Sanderson and Valerius.

13. Aldrich, *loc. cit.*; Carbo, *op. cit.*, pp. 233-233v°; Gassendi, *loc. cit.*; Hotman, *op. cit.*, p. 341; Gorscius, *op. cit.*, p. 791; Sanderson, *loc. cit.*; Valerius, *loc. cit.*; Villalpandeus, *op. cit.*, pp. 180-180v°; Willichius, *op. cit.*, pp. 258-259. The three specially mentioned are Sanderson, Valerius, and Willichius.

14. Namely, Aldrich, Gassendi, Hotman, Sanderson and Valerius.

15. Aldrich, *op. cit.*, p. 72; Carbo, *loc. cit.*; Cujacius, *loc. cit.*; Keckerman, *loc. cit.*; Sanderson, *loc. cit.*; Seton, *loc. cit.*; Willichius, *op. cit.*, pp. 257-258; Valerius, *op. cit.*, pp. 173-174; Villalpandeus, *op. cit.*, pp. 179v°-180. The two in question are Valerius and Villalpandeus.

16. Aldrich, *loc. cit.*; Carbo, *loc. cit.*; Cujacius, *loc. cit.*; Hotman, *loc. cit.*; Keckermann, *loc. cit.*; Sanderson, *op. cit.*, p. 152; Valerius, *op. cit.*, p. 173; Willichius, *op. cit.*; p. 257; Villalpandeus, *op. cit.*, pp. 179-181.

17. Perionius, *op. cit.* [no pagination].

18. Hotman, *op. cit.*, pp. 340-341. See Rüstow, *op. cit.*, p. 41, for the original references. The *Lex Falcidia* was passed in 40 B.C.; and was discussed by Africanus, a jurist of the second century A.D.

19. See footnote 13 for references.

20. Savonarola, *op. cit.* [no pagination], X.xvii: "Habent tamen figuram propositionum. Sicut homo mortuus habet figuram & similitudinem hominis non tamen est homo."

21. Derodon, *op. cit.*, p. 554. "Verùm respondetur: Propositionem . . . esse impossibilem, atque adeò esse purum nihil, & per consequens exactè loquendo non esse propositionem."

22. See Prantl, *op. cit.*, p. 41, note 159; de Rijk, *op. cit.*, p. 88, p. 92.

23. Bocheński, *A History of Formal Logic*, pp. 241-243.

24. Valentia, *op. cit.*, p. 63.

25. Aldrich, *op. cit.*, pp. 74-75.

26. Ormazius, *op. cit.*, pp. 54v°-56.

27. Peter of Ailly, *op. cit.*, [no pagination]. *Cf.* Tartaretus, *op. cit.*, p. 202v°; and Paul of Venice in Bocheński, *A History of Formal Logic*, p. 248.

28. Bricot, *op. cit.*, [no pagination].

29. Buridan, *Sophisms on Meaning and Truth*, pp. 192-193; Paul of Venice in Bocheński, *A History of Formal Logic*, p. 247; Peter of Ailly, *op. cit.*

30. Trutvetter, *op. cit.* [no pagination]; George of Brussels, *op. cit.*, fol. cclxx v°; Eckius, *op. cit.*, fol. cviii v°; Clichtoveus in Le Fèvre d'Etaples, *op. cit.*, p. 135 v°.

31. Ockham, *op. cit.*, pp. 487-489.

32. See Prantl, *loc. cit.*, and de Rijk, *op. cit.*, p. 88, p. 96, for references to earlier manuscripts, where the view is called *restrictio.*

33. Boehner, *op. cit.*, pp. 282-283.

34. Tartaretus, *op. cit.*, p. 205 v°; Bricot, *op. cit.*; Trutvetter, *op. cit.*; George of Brussels, *loc. cit.*; Eckius, *loc. cit.* See also Peter of Mantua, *op. cit.*, [no pagination].

35. Cardano, *op. cit.*, p. 301. *Cf.* de Rijk, *op. cit.*, p. 89.

36. Eckius, *op. cit.*, fol. cviii.

37. *Cf.* Bricot, *op. cit.*: "Sed ad hoc quod aliqua propositio sit insolubilis requiritur quod significat seipsam sub ratione falsi secundum se vel per suam contradictoriam." George of Brussels, *op. cit.*, fol. cclxx v°, quotes the opinion that "Insolubile est sophisma cuius altera pars significat ipsum aliqualiter sub ratione falsi." Major, *op. cit.* [no pagination] writes "Propositio se falsificans sive insolubile est propositio sic se habens quod ita est in re sicut ipsa significat significatione totali et ad ita esse partialiter sive per se vel cum additio infert ipsam esse falsam."

38. Major, *op. cit.*

39. Clichtoveus in Le Fèvre d'Etaples, *op. cit.*, p. 135 v°.

40. Peter of Ailly, *op. cit.*, "*Omnis propositio vocalis est particularis significat* seipsam esse falsam et tamen ad ipsam non sequitur ipsam esse falsam. . . ." *Cf.* Tartaretus, *op. cit.*, pp. 205-205v°. He refers to the view that "quod ad hoc quod aliqua propositio dicatur falsificans seipsam vel insolubilis, non sufficit quod significat seipsam sub rationi falsi: sed requiritur, quòd ad ita esse sicut per ipsa significatur, sequatur ipsam esse falsam."

41. Peter of Spain, *Tractatus Syncategorematum*, p. 137.

42. Buridan, *Sophisms on Meaning and Truth*, p. 182. See also Tartaretus, *op. cit.*, pp. 201-201v°; Bricot, *op. cit.*; Major, *op. cit.*; Clichtoveus in Le Fèvre d'Etaples, *op. cit.*, p. 134 v°. Eckius, *op. cit.*, fol. cviii, gives "No proposition is negative, therefore it is true that no proposition is negative" as an example of an inference from the possible to the impossible.

43. Eckius, *loc. cit.*; Clichtoveus, *loc. cit.*

44. Peter of Spain, *Textus omnium Tractatuum*, fol. xcvii; Glogaviensis, *op. cit.*, fol. ciiii v°.

45. E.g. Buridan, *Sophisms on Meaning and Truth*, p. 219; Paul of Venice, *op. cit.* [no pagination]; Major, *op. cit.*; Eckius, *op. cit.*, fol. cx v°; Peter of Ailly, *op. cit.*; Clichtoveus in Le Fèvre d'Etaples, *op. cit.*, pp. 139-139v°.

46. E.g. Stephanus de Monte, *op. cit.* [no pagination]; *Libellus Sophistarum* (1501-1502) [no pagination]; Clichtoveus, *loc. cit.*

47. Paul of Venice, *op. cit.*; Paul of Pergola, *op. cit.*, pp. 145-146; Derodon, *op. cit.*, p. 554.

48. Peter of Ailly, *op. cit.*; Eckius, *op. cit.*, fol. cx v°-fol. cxi; Major, *op. cit.*; Clichtoveus, *loc. cit.* Clichtoveus writes: "Respondendum est itaque non esse admittendum casum quia licet quelibet pars per se sit possibilis, ut promisso Platonis sine propositione Sortis et propositio Sortis sine promisso Platonis tamen totus casus est una copulativa impossibilis."

49. T. K. Scott in his introduction to Buridan's, *Sophisms on Meaning and Truth*, p. 59.

50. Buridan, *Sophisms on Meaning and Truth*, p. 207ff.; Albert of Saxony, quoted by Prantl, *op. cit.*, p. 80, n. 310; Paul of Venice, quoted by Bocheński, *A History of Formal Logic*, p. 240; Stephanus de Monte, *op. cit.*; Paul of Pergola, *op. cit.*, p. 145.

51. Peter of Ailly, *op. cit.*: "Sexto sequitur quod communiter multe propositiones enumerantur inter insolubilia que tamen non sunt proprie insolubiles quia non significant seipsas esse falsas patet de istis Sortes scit errare . . . . ." See also Major, *op. cit.*

52. Paul of Pergola, *op. cit.*, p. 145. "Ad ultimum principale dicitur quod multa sunt insolubilia non concernentia verum vel falsum, et omnia sunt in definitione insolubilis accepta, quia non restringuntur ad verum vel falsum, sed dicitur in generali quod ex sic esse sicut per ipsum adaequate significatur, sequitur non esse sicut per ipsum adaequate significatur, et e converso . . . ."

53. See Paul of Venice, quoted by Bocheński, *A History of Formal Logic*, p. 247. The *Libellus Sophistarum* offers the most detailed account.

54. See Paul of Venice, quoted by Bocheński, *A History of Formal Logic*, p. 248; the Thomist commentary in Peter of Spain, *Textus omnium tractatuum*, fol. ccvii; Glogaviensis, *op. cit.*, fol. ciiii v°; *Insolubilia* [no pagination]; Major, *op. cit.*

55. *Libellus Sophistarum*, in the section on the second division of propositions. *Cf.* Paul of Venice, quoted by Bocheński, *loc. cit.*

56. Peter of Ailly, *op. cit.*; Tartaretus, *op. cit.*, p. 55.

57. Buridan, *Sophisms on Meaning and Truth*, p. 195.

58. See references in note 1.

59. Le Fèvre d'Etaples, *op. cit.*, pp. 135v°-136v°. P. 135v°: "Insolubilia et insolubilium opposite per suas equivalentes et suarum equivalentium oppositas: sunt cognoscende.

Omnis propositio sequitur ad seipsam.

Ad omnem propositionem sequitur ipsam esse veram.

Si ad antecedens sequantur plura consequentia: ad ipsum sequitur copulativa ex illis consequentibus constituta.

Omnis propositio equivalet copulative in qua verum de termino precise per ea supponente affirmatur constitute.

Equivalentia sunt que euisdem veritatis sunt aut falsitatis et que se convertibiliter consequuntur." P. 136 v°: "Ut posito casu quod dicam solam hanc: ego dico falsum quod vocetur a: tunc insolubile datum ego dico falsum equivalet huic copulative ego dico falsum et a est verum constitute . . . . Data enim copulativa equivalens est falsa. nam eius secunda pars est manifeste falsa. si a est verum quod si ponatur vera cum solum dicam a dico igitur verum: ergo non dico falsum quid primae partis est oppositum. Similiter ex prima parte sequitur secundae partis oppositum. . . . ille igitur copulativa ex partibus repugnantibus constitutur . . . . Et quia copulative contradicit disiunctiva de partibus contradicentibus . . . disiunctiva opposita insolubilis dati non ego dico falsum vel a non est verum . . . . Et quia huiusmodi disiunctiva est vera [nam contradictoria copulative falsa] contradictoria insolubilis non ego dico falsum illi disiunctive equivalens etiam est vera."

60. *Cf.* Paul of Pergola, *op. cit.*, p. 143: ". . . nam A significat copulative quod Sortes dicit falsum et quod A est verum et sua contradictoria disiunctive, quod Sortes non dicit falsum vel quod A non est verum, et huiusmodi significati disiunctim secunda pars est vera et consequenter tota propositio cuius est illud significatum est vera."

61. Eckius, *op. cit.*, fol. cviii v°.

62. See T. K. Scott, introduction to Buridan, *Sophisms on Meaning and Truth*, p. 51ff.

63. Peter of Ailly, *op. cit.*

64. This distinction was discussed by the following: Tartaretus, *op. cit.*, p. 204 v°; Trutvetter, *op. cit.*; George of Brussels, *op. cit.*, fol. cclxxi; Stephanus de Monte, *op. cit.*; Eckius, *op. cit.*, fol. cix; Peter of Spain, *Tractatus Syncategorematum*, pp. 135-136. It was also mentioned by Aegidius Romanus, *loc. cit.*

65. Paul of Venice, *op. cit.*; Paul of Pergola, *op. cit.*, p. 135 ff.

66. Paul of Pergola, *op. cit.*, p. 147: "Vel dicas quod insolubile habet duplex significatum . . . et penes primum cognoscitur impossibilitas vel possibilitas insolubilis. Et penes secundum attenditur veritas vel falsitas." See also Mengus Blanchellus Faventinus in Paul of Venice, *op. cit.*

67. Peter of Spain, *Tractatus Syncategorematum*, p. 137.

68. See Trutvetter, *op. cit.*, on the view that part of a proposition cannot suppose for the whole; and Major, *op. cit.* Major says: "Sit prima conclusio. Aliqua est insolubilis cathegorica significans praecise iuxta significationes terminorum. patet de multis iam datis, hec conclusio in hac tempestate non eget probatione. hoc dico propter multos etiam de artibus bene meritos. quorum aliqui tenuerunt eas omnes ypotheticas. Aliqui propositiones plures. Aliqui neque veras neque falsas simpliciter."

69. Peter of Spain, *loc. cit.*

70. This view is discussed by Tartaretus, *op. cit.*, pp. 203v°-205; Trutvetter, *op. cit.*; Thomist commentators on Peter of Spain, *Textus omnium tractatuum*, fol. xcvi; Glogoviensis, *op. cit.*, fol. ciiii v°; George of Brussels, *loc. cit.*; Stephanus de Monte, *op. cit.*

71. Peter of Ailly, *op. cit.*: ". . . quia si talis vocalis significaret eas coniuncte non appareret ratio quare plus significaret eas copulative quam disiunctive seu aliomodo ypothetice." Tartaretus, *op. cit.*, p. 205: ". . . sed non est hypothetica: quia non aequivalet sibi in significando, licet bene in inferendo: quia non videtur quod in ipsis ponatur aliquod syncategorema, quod includat coniunctionem."

72. Peter of Ailly, *op. cit.*; Paul of Venice, quoted in Bocheński, *A History of Formal Logic*, pp. 244-246; Tartaretus, *op. cit.*, pp. 203 v°-205; Trutvetter, *op. cit.*; Eckius, *op. cit.*, fol. cviii v°.

73. Trutvetter, *op. cit.*; Bricot, *op. cit.* See also Tartaretus, *op. cit.*, p. 205 v°; Major, *op. cit.*; *Insolubilia*; *Libellus Sophistarum.* Bricot wrote: "Ad propositionem affirmativam esse veram non sufficit illam omnino taliter significare qualiter est . . . sed ad ipsam esse veram requiritur quod omnino taliter sit qualiter ipsa significat: et quod ipsa non significet seipsam esse falsam . . . . Ad propositionem negativam esse veram satis est eam significare taliter non esse qualiter non est: vel quod sua contradictoria significaret se ipsam esse falsam . . . . Non est idem propositionem significare se non esse veram formaliter: et significare se ipsam esse falsam formaliter. Patet quia omnis propositio significans se non esse veram formaliter est vera: sed propositio significans se esse falsam formaliter est falsa." Trutvetter's discussion is as detailed as that of Bricot.

74. *Cf.* Ormazius, *op. cit.*, p. 55; *Libellus Sophistarum.*

75. Major, *op. cit.*, said: "Propositio vera . . . imprimis requiritur quod ita sit sicut ipsa significat: et cum hoc quod non se falsificet. alterum istorum de necessitate ad propositionis falsitatem sufficit hoc est quod non est ita in re sicut ipsa significat vel si ita est tamen se falsificat et in sua veritate recalcitrat."

76. Trutvetter, *op. cit.*, makes the same point.

77. *Cf.* Eckius, *op. cit.*, fol. cix, in relation to Paul of Venice; George of Brussels, *op. cit.*, fol. cclxxi, in relation to his own claim that insolubles are false.

78. Eckius, *op. cit.*, fol. cix v°: "Socrates dicit falsum . . . est enim falsa non quia significat rem aliter quam est: sed quia ad ipsam esse veram: sequitur ipsam esse falsa: quod ad deffinitiones addi debet . . . ."

79. Paul of Pergola, *op. cit.*, p. 140.

80. Buridan, *Sophisms on Meaning and Truth*, p. 202.

81. See my article, "The Doctrine of Supposition in the Sixteenth and Seventeenth Centuries," *Archiv für Geschichte der Philosophie*, vol. 51 (1969), pp. 260-285.

University of Waterloo
Waterloo, Ontario, Canada