THE LOGIC OF ESSENTIALLY ORDERED CAUSES

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I   In an article in the *Philosophical Review* of 1966, Patterson Brown set out to clarify one of the medieval proofs for the existence of God.[1] The argument with which he concerned himself can be referred to as the argument from essentially ordered causes; two main proponents of it are Aquinas and Scotus.

Aquinas and Scotus, borrowing from Avicenna and Aristotle, say that causes may be ordered in two ways: essentially or accidentally. As a case of essentially ordered causes the medievals typically gave the example of a man pushing a stone with a stick. Accidentally ordered causes were continually exemplified by a father begetting a son, who in turn begets a son.

These two examples will serve for discussing the three differences which Scotus proposes between essentially and accidentally ordered causes:

(1) In essentially ordered causes, according to Scotus, the second depends on the first precisely in the act of causation. This is not so in accidentally ordered causes.

(2) In essentially ordered causes, there is causality of another nature or order, since the higher is more perfect. This is not so among accidentally ordered causes.

(3) All essentially ordered causes are simultaneously required to cause the effect; accidentally ordered causes can be successive.

In terms of the two examples the differences are as follows: First, in the very act of pushing a stone the stick depends on the man to cause it to push; but while it is true that Isaac depends on Abraham for his existence, he requires no direct help from Abraham in begetting Jacob. The second difference is clear; for if we were asked in regard to the first example, which of the causes was more important, we should unhesitatingly choose the man who pushed the stick over the stick which was pushed by the man. Abraham and Isaac, however, are both fathers; neither is a superior type or kind of cause. Finally, the third difference is present in the examples, according to the medievals, because at the very moment the stick pushes, the man pushes with the stick. But it is obvious that Isaac does not beget Jacob at the very moment Abraham begets Isaac.

The purpose of introducing the distinction between accidentally and essentially ordered causes was in order to handle the problem of infinite regress. Aristotle had proposed that the world had no beginning in time, and while only a handful of medievals believed he was right, most medievals agreed that they were not able to demonstrate he was wrong. Aristotle's position, they thought, was consistent with philosophy, if not with theology. But if the world is eternal, causes could be traced back *ad infinitum*; Jacob's father is Isaac, Isaac's is Abraham, Abraham had a father, who in turn had a father, and so on. Scripture states that Adam was the first man, but philosophy cannot demonstrate that there is a first of this sequence. It is perfectly consistent with philosophy, say the medievals, that we should never be able to reach the first in the sequence of begetters.

Were this true of all causal relations, we should not be able to prove that there is a first in any causal sequence, and therefore, we could not prove that God exists by proving that there is a first cause. But proofs of the existence of God normally took the form of proving that there is a first in some causal sequence; this first cause was then identified with God.

The medievals stated that accidentally ordered causes can regress to infinity, but essentially ordered causes cannot. Arguments to show this were based on the three differences mentioned above. The most popular argument was taken from the third difference: Since all essentially ordered causes are required simultaneously, if there were an infinite regress among them, there would, at one moment, be an actually infinite number of causes. But the medievals felt Aristotle had shown this to be absurd.[2] Therefore there can be no infinite regress among essentially ordered causes. The sequence must come to a stop at some first cause—which we call God.

This argument does not apply to accidentally ordered causes, for they need not all operate at once; they can operate successively. A regress to infinity here would involve only a potential infinity; of this kind of infinity the Philosopher approves.

Given the distinction between essentially and accidentally ordered causes, the medievals felt that they could, even granting the eternity of the world, prove the existence of God. Using metaphors, their points can be made by saying that while one can grant a regress to infinity on the horizontal level of accidentally ordered causes, concomitant with this, one can show that there can be but a finite number of vertical or ascending essentially ordered causes. God is the first in this ascending series of causes.

In essentially ordered causes the medievals made a three-fold division. First, there is the ultimate effect, then there are the intermediate causes (these are also called secondary or instrumental causes), finally the primary cause. Their argument is that there cannot be an infinite regress of intermediate causes (these are causes which are themselves caused); at some point, a first uncaused cause will be reached.

As if the matter were not difficult enough, the medieval reverence for Aristotle made it even worse. In the passage of the *Metaphysics* which the medievals considered authoritative on this question of series of causes,

Aristotle made the parenthetical remark, ''It makes no difference whether there is one intermediate or more, nor whether they are infinite or finite in number.''[3]   It had seemed that the whole point of essentially ordered causes was that there could not be an infinite number of intermediate causes.   But if Aristotle's remark is taken seriously—as the medievals took it—it now seems there can be an infinite number of intermediate causes.   To call the medieval doctrine of essentially ordered causes puzzling, would seem to be almost straining the bounds of philosphical charity.

II Our problem is to see if we can make sense out of the doctrine of essentially ordered causes. It seems to be blatantly contradictory. Can we interpret it in such a way that it will remain consistent to a greater extent than is normally granted it by modern commentators? Patterson Brown's article is the best account I have seen to date. The crucial point, he suggests, is that for the medievals the relation between essentially ordered causes was transitive, whereas that between accidentally causes was not. The relation of pushing, for example, is transitive, because from the fact that the man pushed the stick and the stick pushed the stone, we could conclude that the man pushed the stone. The relation of begetting, however, is intransitive; for from the fact that Abraham begot Isaac and Isaac begot Jacob, we can conclude that Abraham did not beget Jacob. Brown thinks that transitivity is what the medievals are trying to propose by the first difference they give between essentially and accidentally ordered causes.

The difficulty as Brown sees it, is to find causal relations which are transitive. He proposes that the medievals had in mind some quasi-legal sense of cause such as that which would be present in the following example:

> Consider the following case. Mr. Alpha is in his automobile, stopped at an intersection. Immediately behind him sits Mr. Beta in his own car. Behind Mr. Beta is Mr. Gamma, behind whom is Mr. Delta, and so on indefinitely. Suddenly Alpha's car is rammed from the rear, damaging his bumper. So Alpha, desiring to recover the expense of repairing his automobile, accuses Beta of having caused the accident, and brings suit against him. Beta, however, successfully defends himself in court on the grounds that he had himself been rammed into Alpha by Gamma. So Alpha now sues Gamma. But the latter, it turns out, had in turn been rammed by Delta. So Alpha takes legal action against Delta. And so on indefinitely. Now if this series of rammings extended *ad infinitum,* there would be no one whom Alpha could successfully sue as having caused the dent in his bumper; there would, in short, have been *no* cause for the accident at all. But if there were no cause, no mover, then there would be no effect, no moved, either—which is patently false, since Alpha's bumper *is* dented and his car *was* moved. Therefore there cannot be a regress to infinity of ramming automobiles, but rather someone was the first cause of the whole series of accidents; someone can properly be said to have moved Beta into Alpha, Gamma into Beta, Delta into Gamma, and so on. Therefore there is someone from whom Alpha can collect his expenses.

The proper account for essentially ordered causes, says Brown, ''is precisely the sense of a cause *responsible* for its effect.''[4]

To seek to explain essentially ordered causes by means of the notion of responsibility seems to me to be a case of *obscurum per obscurius*. The legal problem of fixing responsibility is itself the problem of determining when responsibility is transitive and when not.

Assume that Mr. Alpha's suit has finally reached to little old Miss Omega. She is a proper sort and defends herself by arguing that she was caused to forget to brake by the shocking sight of a bull and a cow engaged in a very natural activity right at the fence alongside the road. Since Farmer Jones was responsible for allowing such brazen activity, he ought to pay Alpha's damages.

It seems all right to me to say that the sight of this activity was responsible for Miss Omega's lapse in driving. It also seems all right to say that Farmer Jones was responsible for allowing the activity to take place in the field by the road. But the problem still remains for the judge to decide whether or not we here have responsibility which is transitive. Just talking about responsibility does not answer that question.

To take another example, parents are legally responsible for children who are minors. If their children break a window, the parents are held responsible for the damages. The law punishes the parent. But the law does not punish the parents of a child who has murdered someone. I realize there may be legal niceties to be considered in this example; I only wish to point out that neither responsibility nor legal responsibility can serve to give a clear explication of when causal relations are transitive and when not.

Let us see if we are able to account for the transitivity of essentially ordered causes without appealing to some other notion—like responsibility—whose transitivity is equally vague.

III We borrow some logical notions that can be found in introductory texts.[5] Let us say that a property F is hereditary with respect to a relation R when, if something has the property F and is related by R to something else, that something else must have the property F,

(4) $$(x)(y)(Fx \cdot Rxy \cdot \supset Fy).$$

Some interpretations: Where the universe of discourse is the natural numbers, if x is greater than 5, and x precedes y, then y is greater than 5. Where the universe is animals, if x is human and x begot y, then y is human. Finally, where the universe of discourse is cases of bumping, if Miss Omega is responsible for x and x caused y, then Miss Omega is responsible for y. It seems to me that this is the *only* characteristic Brown wants to obtain by introducing the notion of responsibility. My suggestion is to forget about responsibility and concentrate on this characteristic.[6]

Number (4) is Carnap's version of heredity. Quine has a variant form,

(5) $$(x)(y)(Fx \cdot Ryx \cdot \supset Fy).$$

Numbers (4) and (5) are not equivalent unless the relation R is symmetric, so it makes some difference which we work with. I am going to be using Quine's notion of heredity and so we shall have the definition,

(6) $\text{Her}(F,R) \equiv (x)(y)(Fx . Ryx . \supset Fy)$

 The characteristic of being hereditary forms part of Frege's famous definition of ancestor. Some informal introduction to this notion is appropriate. The problem is to see whether we can define the relative term 'ancestor' on the basis of 'parent'. For the moment let us take 'R' as a constant, so that 'Rxy' will be read 'x is a parent of y'. Now we must try to write 'x is an ancestor of y' using only 'R' and logical symbols.

 How can we specify the ancestors of y? The notion of heredity now comes into play. Imagine that y has a property F such that this property F is hereditary, in Quine's sense, with respect to the relation R, i.e., $\text{Her}(F,R)$. Where R is the parent relation, then F is a property such that if anyone has that property then all parents of that person have that property. It follows that y's parents will have that property; their parents in turn will also have that property, and so on. In fact, any ancestor of y will have that property. As a first attempt at defining 'x is an ancestor of y' we can try.

(7) $Fy . (w)(z)(Fw . Rzw . \supset Fz) . \supset Fx,$

or in abbreviated form,

(8) $Fy . \text{Her}(F,R) . \supset Fx.$

This can be read, "If y has a property F which is hereditary with respect to the parent relation, then x has that property F." Now while it is true that if y has such a property F, all his ancestors will have it, yet many things besides his ancestors could also have it. For example, read 'Fy' as 'y existed before 2000 A. D.', assume y has F, and surely F is hereditary with respect to R. It follows that all ancestors of y have F. But it is also true that F is possessed by everyone alive today and all their ancestors. Further, F is possessed by such unlikely candidates as Cleopatra's barge and the cathedral at Chartres. Numbers (7) and (8) give a necessary condition for x being an ancestor of y, but not a sufficient condition. We must try to be more restrictive in our genealogy.

 The trouble is that there is an unlimited number of properties possessed by y which are hereditary with respect to R, and some of these properties are also possessed by disparate groups of things. We must try to express what is unique about y's ancestors. It turns out that y's ancestors form the only group of individuals that possess *every* R-hereditary property possessed by y. Let us then define 'x is an ancestor of y' — which we shall represent by '$*Rxy$' — as follows:

(9) $*Rxy \equiv (F)\left[Fy . \text{Her}(F,R) . \supset Fx \right] \quad .$

Some readings of (9) may help. To say that x is an ancestor of y is to say, if y has *any* property which is hereditary with respect to R, then x has that property. It is to say that x has *every* property which belongs to y and is hereditary with respect to R; or, it is to say, pick any property you choose, if that property belongs to y and is hereditary with respect to R, then that property belongs to x.

 Definition (9) is the more important formulation, but it requires one

refinement in order to avoid an embarrassment of riches. The difficulty with (9) is that it allows y to be counted among his own ancestors; for surely whatever R-hereditary property y has is possessed by y. Let us call '$*R$' the *improper* ancestral of R, and in terms of it define the *proper* ancestral of R, ($^{\#}R$) as

$$(10) \qquad\qquad {}^{\#}Rxy \equiv (\exists u)(Ruy . *Rxu) \ .$$

Recall that, for the moment, we are taking 'Rxy' as the constant, 'x is a parent of y'. We assume, naturally enough, that this relation is asymmetric. It follows that the relation R is also irreflexive. In the definiens of (10), it is possible that u and x be the same individual. That is, u may be x, or a parent of x, or a parent of a parent of x, etc. But by including Ruy, we rule out the possibility of u and y being identical since R is irreflexive. To say that x is a proper ancestor of y is to say that there is a u who is a parent of y, and x is an improper ancestor of u. This may sound a bit bawdy, but the logic is all right. We shall have more to say about ancestrals, but now let us return to the question of essentially ordered causes.

Brown argues that the notion of cause used in regard to essentially ordered causes differs from the "ordinary" notion of cause in two major ways. First, essentially ordered causes are transitive, whereas ordinary causes are not. Second, the responsibility for the effect passes back along the members of the essentially ordered series of causes. We have already argued that this "passing back" or responsibility is merely an instance of a property being hereditary with respect to causality. Let us introduce 'C' as a primitive, undefined term such that we read 'Cxy' as 'x causes y'. We demand only that C be irreflexive—as a medieval would say, "*Quidquid causatur, ab alio causatur.*" Then, instead of bothering with all the nuances of 'responsible', we can state that we are interested in any property F such that,

$$(11) \qquad\qquad \text{Her}\,(F, C).$$

The medievals thought—as is well known and much discussed—that *existence* was a property hereditary with respect to causality.

But even if the above handles the second characteristic Brown demands of essentially ordered causes, there still remains the problem of transitivity. Not all causal relations are transitive, and in recognition of this we do not demand transitivity for the relation C. Can we then construct a notion of cause out of our non-transitive C such that this new notion of cause will be transitive?

The answer is easy once we learn two facts, namely, that every relation has an ancestral, and that ancestrals are transitive. Where R is now variable, we can have as a theorem

$$(12) \qquad\qquad (x)(y)(Rxy \supset *Rxy).$$

Another theorem is

$$(13) \qquad\qquad (x)(y)(z)(*Rxy . *Ryz . \supset *Rxz).^{7}$$

From (12), since R is variable, we get as an instance,

(14) $(x)(y)(Cxy \supset {}^*Cxy)$

and from (13)

(15) $(x)(y)(z)({}^*Cxy . {}^*Cyz . \supset {}^*Cxz)$.

From (14) and (15) we see that we can, beginning with a notion of cause which is not transitive, arrive at a related notion which is. Let us propose ${}^*Cxy$ as a tentative first reading of 'x is an essentially ordered cause of y', with the definition,

(16) ${}^*Cxy \equiv (F) [Fy . (w)(z)(Fw . Czw . \; \vdots \; Fz) . \supset Fx]$.

We are proposing that to say x is an essentially ordered cause of y is to say that if y has any property which is hereditary with respect to causality, then x has that property.

The medievals, however, demanded that essentially ordered causes also be irreflexive. We know from our discussion of (9) that ${}^*Cxy$ will not be irreflexive, i.e., it would follow that anything was its own essentially ordered cause. As before, we therefore pass from the improper to the proper ancestral, defining

(17) ${}^\#Cxy \equiv (\exists u)(Cuy . {}^*Cxu)$.

Let ${}^\#Cxy$ be our second reading of 'x is an essentially ordered cause of y'. Proof that it is in fact irreflexive must wait, but it fits fairly well with an informal understanding of what is going on in the process of discovering essentially ordered causes. If we wish to discover essentially ordered causes, we look for a y which has some cause u. It seems to me that this must surely be part of what a medieval like Aquinas means when he says we begin with *contingent* beings. We then trace back the causal ancestry of u, that is, x is either u, or a cause of u, or a cause of a cause of u, etc. But this is just the analogue of the medieval argument that, beginning with a contingent thing, we look to its cause. That cause will be either uncaused or caused. If uncaused, QED. If caused, we look in turn to its cause, which again will be uncaused or caused. We continue this process until we reach an uncaused cause. They maintain that this process cannot go on to infinity.

Hopefully no one will misconstrue me as arguing that the medievals anticipated Frege's discovery. That is not the point at all. I am proposing the notion of the ancestral of C as an explication of the medieval notion of essentially ordered cause. The precision of the modern notion will help in trying to locate some of the trouble spots in the medieval argument. I am attempting to give the most benevolent interpretation possible to the medieval argument. Lest benevolence override justice, it is necessary to give further support to the appropriateness of the ancestral of C as an interpretation of essentially ordered causality. This is the task of the next two sections. The medievals maintain that there cannot be an infinite process among essentially ordered causes. Let us see if we can make any sense of this remark in terms of our ancestrals.

IV   The argument I am going to present is purely dialectical; it is an *argumentum ad homines contemporaneos*. I do not mean to suggest that it vindicates the medieval argument, but hopefully, it will bring us to be more sympathetic towards their problems. Finally, there is a perversity about it that delights me.

The medievals began with the consideration of some "ultimate effect" whose cause they could recognize. They checked that cause to see whether it was uncaused or caused. If caused they checked the prior cause and so on. In terms of ancestrals we can say that they sought to trace back the improper ancestry of the cause they first recognized. Let us represent this recognizable cause by the constant 'a'. Essentially ordered causes then become the x's such that,

(18) $(F)[Fa . (w)(z)(Fw . Czw . \supset Fz) . \supset Fx]$.

The medievals argued that there cannot be an infinite regress among such x's.

In contemporary philosophy we run across the same schema in an interesting situation. Taking 'O' as a constant, 'Sxy' as '$x = y + 1$' (this can also be read, 'x is the immediate successor of y'), we meet the schema

(19) $(F)[FO . (w)(z)(Fw . Szw . \supset Fz) . \supset Fx]$,

whose closure gives us

(20) $(F)[FO . (w)(z)(Fw . Szw . \supset Fz) . \supset (x)Fx]$,

which is one form of mathematical induction. We can abbreviate (19) as

(21) $*SxO$,

and this is, in fact, a standard way of defining that x is a natural number.[8]

Because in their discussion of finitude, philosophers often restrict themselves to positive integers, (19) is often presented as a definition of *finite* numbers.[9]

I certainly do not wish to confuse numbers and causes. But what I am interested in is the kind of informal account often given of (19). Russell states "Thus we may define finite numbers as those that can be reached by mathematical induction, starting from 0 and increasing by 1 at each step, ..." and again he says that we may define the finite numbers "as those which, starting from 0 or 1, can be reached by mathematical induction. This principle, therefore, is not to be taken as an axiom or a postulate, but as the definition of finitude."[10]

Frequently it will be said, for any finite number x, no matter how large it may be, it can be reached in a finite number of successor steps... But then it should also follow that for any essentially ordered cause x, given that (18) defines it, we will reach x in a finite number of causal steps, i.e., the causal steps do not regress to infinity. In fact, logicians give a general law of ancestral induction, of which mathematical induction is just an instance.[11] If this is so, we could say that essentially ordered causes are the *inductive* causes.

This would perhaps explain Aristotle's remark that there could be a finite or infinite number of intermediates, yet without an infinite regress. This remark avoids contradiction given that 'infinite' is ambiguous. Aristotle, who understands a series that has no greatest as a case of potential infinity, would speak of infinite where we have been speaking of finite. The series could be potentially infinite, but it would never reach actual infinity—the regress back to any essentially ordered cause will never demand more than a finite number of steps.

It should be made clear that I am not suggesting the proposed definition of essentially ordered cause will show there has to be a *first*. Just as (19) will never generate a greatest finite number, so (18) will not generate a first cause. For the moment I am only suggesting that we can, perhaps, make sense of the medieval dictum that the series will not regress to infinity. Or rather, we can give a dialectical argument which defends this dictum against many objections that have been put forward.

The ambiguity of the notion of infinite was recognized in the Middle Ages, as is evidenced by the following quotation from Ockham:

> God can never make so many individuals of a given kind that He could not make more, and yet at any given time He cannot make an infinite number of individuals . . . It is impossible to give a definite number of individuals such that God cannot make more, . . . and yet as many individuals as are made there will always be but a finite number of them . . . .
>
> God can make more individuals, even more *in infinitum*; and yet He will always make a finite, never an infinite, number of them.
>
> If there is said to be a process *in infinitum*, it will still always be the case that what is actually produced will be but finite . . . if God were to continually *(semper)* make more individuals, He would never make an infinite but only a finite number of them.[12]

Ockham's remarks serve to show that the notion of a regress *in infinitum* may be understood in two ways: either as a process—corresponding to the informal accounts of mathematical induction we mentioned—which will go on and on but never produce more than a finite number of members, or as a process which will go on and on to such an extent that it will ultimately reach infinity. Now the reason that the medievals always give for denying an infinite regress is that if there were one, there would be an actually infinite number of individuals. They appeal to Aristotle in order to show that an actual infinity is absurd. Then by Modus Tollens they conclude there can be no infinite regress.

The conditional beginning this argument could not be true if one understood an infinite regress in the first of the two senses mentioned; for by definition it will never give an infinite number of individuals. Therefore the only notion of infinite regress appropriate to this argument is the second sense. The only infinite regress we *must* deny is that where the regress would actually *reach* infinity.

On my interpretation, then, the medieval denial of an infinite regress need not be, as some think, a denial that there are series that can go on and on. It must be a denial that the step-by-step series they envisaged would

ever actually reach infinity; and everybody wants to deny that. It would be perfectly conceivable for a medieval that causality would have to be traced through the entire hierarchy of angels, whose number is legion and whom no man can number. All the medievals need is to be able to exclude in principle the chaotic concept of actual infinity, a concept Aristotle had said was unknowable.[13]

V If '$^\#Cxy$' is to define 'x is an essentially ordered cause of y', we must show that it meets other requirements the medievals demanded of it in their arguments. As Brown has pointed out, it must be transitive. Given (12) and (13) we can prove as much. Further it must, unlike '$*Cxy$', be irreflexive. Intuitively this seems fairly easy given the assumption that 'C' is irreflexive, but the formal proof does not come at all easily. As far as I can see, it demands an added assumption. We seek to prove

(22) $(x) -^\#Cxx$

Proof:

*(1)	Cux	
*(2)	$*Cux$	(1), T12
*(3)	$(x) -Cxx$	Axiom, Irreflexive C
*(4)	$-Cxx$	(3) UI
*(5)	$Cux . u = x . \supset Cxx$	Subst. of Identity
*(6)	$u \neq x$	(1) (4) (5)
**(7)	$*Cxu$	
?**(8)	$(x)(y)(*Cxy . *Cyx . \supset x = y)$	Antis (*C) ?
**(9)	$*Cux . *Cxu . \supset u = x$	(8) UI, UI
**(10)	$*Cux . *Cxu$	(2) (7)
**(11)	$u = x$	(9) (10)
**(12)	$u \neq x . u = x$	(6) (11)
*(13)	$*Cxu \supset . u \neq x . u = x$	(12) Conditionalization
*(14)	$-*Cxu$	(13)
(15)	$Cux \supset -*Cxu$	(14) Conditionalization
(16)	$-(Cux . *Cxu)$	(15)
(17)	$(u) - (Cux . *Cxu)$	(16) u UG
(18)	$-(\exists u)(Cux . *Cxu)$	(17)
(19)	$-^\#Cxx$	(18) Def. 17
(20)	$(x) -^\#Cxx$	(19) x UG

I have chosen to place a question mark to indicate the crucial step. The proof obviously presents no problems if line (8) is granted. The question becomes: is *C antisymmetric? Our second proof deals with this problem. In fact I shall only give a sketch of a proof and, as some of the steps are a bit more interesting than in the attempt to prove (22), I shall intersperse some informal remarks to aid the reader.

(23) $(x)(y)(*Cxy . *Cyx . \supset x = y)$

Proof:

*(1) $*Cxy . *Cyx$
*(2) $(F)[Fy . (w)(z)(Fw . Czw . \supset Fz) . \supset Fx]$ (1) Simp, Def. 16
*(3) $(F)[Fx . (w)(z)(Fw . Czw . \supset Fz) . \supset Fy]$ (1) Simp, Def. 16
**(4) $x \neq y$

The intent, of course, is to derive a contradiction from (2), (3), and (4). If the universe of discourse was numbers and C the successor relation, we would expect the proof to go through.[14]   But with the universe persons and C the parent relation, we can give an interpretation where (2), (3), and (4) are all true, namely, the case where x and y are not identical but are siblings; for then, clearly, x will have every hereditary property y has— giving (2)—y will have every hereditary property x has—giving (3)—and yet they will not be the same individuals—giving (4).[15]   As things stand, therefore, we cannot give a general proof for the antisymmetry of improper ancestrals. Perhaps the realm of causation has some further assumptions which will correct this problem. So let us continue to sketch an attempt at a proof, while keeping our eyes open for suggestions that may help us.

**(5) $Fy . (w)(z)(Fw . Czw . \supset Fz) . \supset Fx$ (2) UI
**(6) $Fx . (w)(z)(Fw . Czw . \supset Fz) . \supset Fy$ (3) UI
**(7) $(w)(z)(Fw . Czw . \supset Fz) \supset . Fx \equiv Fy$ (5)(6)

Now the fun begins. We take as a reasonable assumption the Identity of Indiscernibles, viz.

**(8) $(F)(Fx \equiv Fy) \supset x = y$ Axiom

If we could then get the antecedent of (7), Modus Ponens would give $Fx \equiv Fy$, and since F does not occur free in any premises we can universally generalize on F, which with (8) would give $x = y$, QED. If desired we could even rewrite the proof without the reductio assumption.

Can we get the antecedent of (7)? Aquinas quotes a widely held principle of causality, "*quidquid perfectionis est in effectu, oportet inveniri in causa effectiva:...*"[16] If we were to translate this as, "Nothing is in an effect which is not in its cause," we would have as an axiom of causality,

**(9) $(F)[(w)(z)(Fw . Czw . \supset Fz)]$ Axiom

This, as we have said, would give us $x = y$.

It would be tempting to say this gives the medieval view, and thus further confirms our ancestral of C as an interpretation of essentially ordered causes. All it requires is to attribute a liberal degree of stupidity to the medievals. No one could seriously expect the axiom on line (9) to be true. Note that this axiom has the form of the substitutivity of identity save that it has C in place of the identity sign. But no one has ever proposed the substitutivity of causality.

Assuming the medievals aren't fools, we retranslate Aquinas' principle as, "Whatever perfection is found in an effect, will be found in its cause." The only difference is that we now have to deal with that troublesome notion

of perfection. Some properties are perfections, while others are not. The problem is to decide which is which. I think the notions we have been dealing with will help in at least giving some indication as to how one might usefully go about trying to distinguish perfections from less august properties.

Since being a perfection is true of some properties and not of others, it seems reasonable to represent being a perfection as a second-order predicate 'P' which ranges over properties. Reading 'P(F)' as 'F is a perfection', I propose the definition,

(24) $(F)[P(F) \equiv (w)(z)(Fw . Czw . \supset Fz)]$,

as a first attempt. Looking at (24), it does help to illustrate the major role perfections are meant to play in the Middle Ages. Medievals discuss perfections when they are discussing God. As often presented, the argument goes that effects—you and I, for example—have many properties which it would be improper to attribute to God. These must be ruled out. Perfections, then, become those properties such that, if we have them, God must have them. Perfections are just those properties which can be traced back through the entire causal chain. They are the properties hereditary with respect to causality.

Yet (24) is not quite right; for it would seem to classify the property of being present before a certain date as a perfection. Such a property was not traditionally considered a perfection. I am not quite sure what to do about this sort of counterexample. Pretty obviously, the medievals meant to deal with cases of causality where questions of time were irrelevant. On intuitive grounds it seems right to me that medievals thought those properties to be perfections which were hereditary with respect to essentially ordered causality, and since they thought God to be at the start of such essentially ordered series, they commonly gave the equivalent characterization that perfections are those properties possessed by God. There are numerous problems, but I suggest that a characterization along the lines of (24) might make the notion of a perfection more congenial to the modern mind, and therefore more open to modern discussion. The topic needs straightening out.

VI If we look back to the three characteristics Scotus proposed for essentially ordered causes, we can see that so far we have considered only the first. Brown thinks this is sufficient.[17] He argues that the third is not necessary and he ignores the second. There is some historical justification for ignoring the second characteristic; for Scotus states that it is a consequence of the first.[18] I think Brown and Scotus are wrong. I think it is crucial to maintain that if one thing is an essentially ordered cause of another, then the first is of a higher order than the other, or as Aquinas puts it, the first is the cause of the entire species of the second.[19]

The reason I think this is as follows: Take the example of accidental ordering which the medievals give and which Brown apparently accepts, viz., the relation of begetting. If Brown is correct, this cannot be a case of essential ordering because the relation is intransitive and responsibility

would not pass back among the *relata*. But we have shown that every rela-
tion has an ancestral, and the ancestral of begetting would be transitive—
since all ancestrals are—and would take into account the notion of 'passing
back' by means of the notion of heredity. Therefore, if Brown's are the
only requirements for essentially ordered series, it would seem that there
could not be an infinite regress among the ancestors of Jacob. But I take it
this is just the point the medievals were willing to concede to Aristotle.

Essentially ordered causes must therefore have some further distin-
guishing mark. I propose that Scotus' second characteristic contains what
we desire and that we can explicate this notion of being of a higher order in
more contemporary terms. Define a new relation '*E' as follows:

(25) $*Exy \equiv (F)[(w)(z)(Fw . Czw . \supset Fz) \supset . Fx \equiv Fy]$,

which is the same as

(26) $*Exy \equiv *Cxy . *Cyx$.

We have already met this in our attempt to prove the antisymmetry of *C.
This says that x and y have exactly the same properties that are hereditary
with respect to C. I suppose an informal reading of this relation would be
that it expresses that x and y are causal siblings. Another way of looking at
it is that it says x and y have the same causal ancestry. Or again we can
read it as saying that x and y belong to the same causal level.

It is obvious that this relation will be transitive and symmetric. Now a
relation which is transitive and symmetric is what modern logicians call an
equivalence relation.[20] From these two properties it follows that such a
relation is reflexive. That is we get the theorem,

(27) $(x)(y)\ (*Exy \supset *Exx)$.

A useful equivalent form of (27) is

(28) $(x)[(\exists y)*Exy \supset *Exx]$.

Such a relation is very similar to the identity relation. The difference is
that while it is reflexive, it is not totally reflexive. We don't have $(x)*Exx$.
If however, we were to restrict ourselves to discussing those things of
which *E held—if we restricted ourselves to the field of *E —we would have
total reflexivity. We would have, for that restricted universe of discourse,
a precise analogue of the identity relation.

This is precisely the use to which logicians put equivalence relations.
They use equivalence relations to redefine identity for classes of objects
rather than for just objects.

Assume we have a non-empty set of objects X. Let R be an equivalence
relation in X. Then for $x \epsilon X$ we can define the set $[x]$ as:

(29) $y \epsilon [x] \equiv . y \epsilon X . Rxy$.

We can read '$[x]$' as 'the R-equivalence class of x'. From this definition
and the properties of equivalence relations we can go on to prove,

(30) $[x] = [y] \equiv Rxy$,

and

(31) $$[x] \neq [y] \supset . (z)(z \in [x] \supset z \notin [y]).$$

Number (31) says that different equivalence classes have no members in common. The importance of (30) is that it serves as a principle of abstraction: objects which are identical in some respect generate identical classes. Its application reduces the number of entities being considered by passing from objects to equivalence classes of objects.[21] On applying (30) we no longer deal with objects, but with equivalence classes of objects, yet we very naturally talk about these equivalence classes as if they were objects.

We work with equivalence classes of objects constantly, only we don't always recognize them as such. Let me give an example of our use of equivalence classes which will be illustrative. Let the objects of our universe of discourse be sentences—spoken or written—as occurring in space or time.

    (a) Identify ('=') then, is an equivalence relation generating equivalence classes which are just unit classes of individuals. We call these *sentence tokens*.

    (b) Often we go on to deal with equivalence classes with respect to the relation 'having the same linguistic (written or phonetic) structure as'. We call such equivalence classes *sentence types*.

    (c) As a next step we often take equivalence classes with respect to the relation 'having the same sense as'. We call such equivalence classes *propositions*.

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    (y) We could get to the point where we dealt with equivalence classes with respect to the relation 'necessarily having the same truth-value as'. I take it these equivalence classes are what Lewis deals with in strict implication.

    (z) Finally, we can work with equivalence classes with respect to the relation 'having the same truth-value as'. This would be the case with a two-valued truth functional logic.

Notice that what is happening as we proceed, is that we are treating of broader and broader equivalence classes of sentences until we finally end up with two huge equivalence classes of sentences characterized by true or false. Different views of entailment could be characterized by the equivalence classes of sentences with which they dealt. Between (c) and (y) we could place the equivalence classes of sentences which characterize the various multivalued, modal, and epistemic logics.

To return to causality, we can characterize equivalence classes with respect to the relation *E. By (31) we know that if these equivalence classes are different they have no members in common. We are therefore able to partition our essentially ordered causes into mutually exclusive classes of essentially ordered causes. This fits what the medievals mean by talking of species of causes. Each *E-equivalence class determines a species of cause.

Given the above, it is then possible to take these $*E$-equivalence classes as objects and to talk about one causing another. Take a medieval example: Consider a case of burning straw and ask what caused it. The medievals could mention something like John's putting a torch to the straw. But if we were to ask what causes straw to burn in general, we would not be given any particular cause, but something like, "the application of heat." That is, we are now dealing with a class of causes, a species of cause. Now the application of heat is able to bring about a great number of other effects besides the burning of straw. I take this as an important part of what the medievals mean when they say that the cause is higher than or superior to the effect. If we were then to seek the cause of the application of heat, we would be given a broader class of causes; the medievals would say a higher species of cause. They would also wish to argue that you cannot go on to infinity in tracing back these higher and higher species of causes.

As another dialectical argument, taken from Brown, let me suggest a modern notion to serve as a model for the medieval argument. Where we are dealing with equivalence classes of causes, interpret these as *explanations*. Then when the medievals talk about higher and higher species of causes, interpret them as talking about more and more wide-reaching explanations. For example, start with the presence of life on Earth. If asked for an explanation, we might propose the presence of certain amounts of oxygen in the atmosphere. (This is not of course, a complete explanation; but we can imagine situations where it would be suitable). In explanation of this we might appeal to the radiant energy of the sun. To explain this, we might move to some principles of physics on fusion. If the questioner were persistent and we were able, explanations might be pushed back to basic principles of physics. Were these in mathematical form, we might be required to explain the mathematical principles involved. We could retreat to broader and broader mathematical principles until we arrived at foundational notions. Then, if we were logicists, we might appeal to logic, giving more explanations until at last we got to something like the principle of contradiction. At this point we would refuse to give any further explanation.

We would never expect to run into a situation like the above. The medievals, however, would not find it strange, especially given their view of the hierarchical arrangement of the sciences. Where above I have spoken of logic, read 'metaphysics', and we have an account of tracing back the essentially ordered causes to a first principle, which we call 'God'.

The example helps to point up the ambiguity of the traditional notion of cause. It sometimes refers to an individual, but at other times it does not. In this latter case I propose 'explanation' as a suitable translation: Consider Aristotle's statement: "nor can the sources of movement form an endless series (man for instance being acted on by air, air by the sun, the sun by Strife, and so on without limit). Similarly the final causes cannot go on *ad infinitum* —walking for the sake of health, this for the sake of happiness, happiness for the sake of something else, and so one thing always for the sake of another."[22] We cannot make sense of this by talking about individuals. We must appeal to something like explanations, and the tradi-

tional denial of regress to infinity comes pretty close to the modern idea that explanations have to end sometime.

The notion of equivalence classes of causes allows us to propose another way of explicating Aristotle's remark that there would be infinite intermediates without allowing a regress to infinity. It might be possible to argue that the sense of this is that there cannot be an infinity of classes of causes, but that it would make no difference whether any one of these classes had a finite or an infinite number of members.

The present paper may be viewed as an attempt to show that the medieval argument for the existence of God does not collapse in ways frequently suggested. I do not think the medieval argument succeeds, but that is another topic. I take the notion of an essentially ordered cause as the most important notion of the arguments of Aquinas and Scotus. I have interpreted the medieval notion by means of Frege's ancestral relation, and have discussed some of the consequences of this interpretation.

In conclusion, the foregoing discussions confirm my belief that there is one basic principle underlying the perennial interest in arguments for the existence of God, namely: It ain't what you prove, it's the way that you prove it.

NOTES

1. Patterson Brown, "Infinite Causal Regression," *The Philosophical Review*, vol. 75 (1966), pp. 510-525.

2. They often cite *Physics* III, 7; 207a 32-207b 34.

3. II, 2; 994a 16; *cf.* the commentaries on the *Metaphysics*: of Aquinas, II, Lesson 3; of Scotus, II, c. 2.

4. *Op. cit.*, pp. 524-525.

5. E.g., W. Quine, *Methods of Logic* (New York, 1959), pp. 228 ff; R. Carnap, *Introduction to Symbolic Logic and its Applications* (New York, 1958), pp. 146 f.

6. The notion of heredity is important and widely used. In logic we seek to show that truth is hereditary with respect to our rules of inference. Substitutivity of identity states that *every* property is hereditary with respect to identity.

7. Numbers (12) and (13) have straightforward proofs.

8. Quine, *Mathematical Logic* (New York, 1951), p. 242.

9. Carnap, *op. cit.*, pp. 150-151.

10. B. Russell, *The Principles of Mathematics* (New York, 1938), pp. 123 and 315.

11. Quine, *Set Theory and its Logic* (Cambridge, Massachusetts, 1963), p. 104.

12. *Commentary on the Sentences*, I, d. 17, q. 8. I have used the reprint of the 1494-1496 Lyons edition.

13. *Cf.* Aristotle, *Physics*, 187b 7-10; Aquinas, *Summa Contra Gentiles*, I, Chapters 63 and 69.

14. This is obvious once one understands '*C xy*' as '$x \geq y$'.

15. Actually our two individuals would seem to have to be more than just siblings. They would have to be alike even to the point of coming out of their mother's womb side-by-side, else we could find a temporal property hereditary with respect to the parent relation which one has and the other does not.

16. *Summa Theologiae*, Ia, q. 4, a. 2, c.

17. *Op. cit.*, p. 516.

18. *Cf. Duns Scotus: Philosophical Writings*, translated by A. Wolter (Library of Liberal Arts, Bobbs-Merrill), p. 45. One must beware, for the hard-cover edition put out by Nelson and Sons does not have the same pagination.

19. *Summa Theologiae* Ia, q. 13, a. 5, ad 1; *Summa Contra Gentiles* II, Ch. 21.

20. Carnap, *op. cit.*, p. 136.

21. Logicians sometimes put this: It allows us to *contract* any model M to a normal model M', where the domain of M' is just the set of equivalence classes determined by the equivalence relation on M.

22. *Metaphysics*, 994a 6-10.

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