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ON HAUBER'S STATEMENT OF HIS THEOREM

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Hauber's[1] theorem is one which is often referred to,[2] yet it seems that few people have actually seen Hauber's original work. Peano,[3] for example, cites Schröder, and Schröder in turn says that his source for the theorem is Venn.[4] In Scholz and Hasenjaeger,[5] reference is made to Hauber's book, but the section referred to is §293, where Hauber employs the theorem to prove several corollaries to a theorem from Euclid, rather than to §287, where Hauber gives the explicit statement of his theorem.

Hauber's book, *Scholae logico-mathematicae*, is devoted to showing a number of theorems and corollaries which can be proved using Euclid's fourth postulate. And it is from observing the method of proof which is used for many of the results that he comes to formulate his theorem. Hauber recognized its significance and specifically referred to it in his preface as an important contribution.[6]

Hauber's own statement of the theorem is as follows:

> Si genus aliquod dividatur in suas species duplici ratione, et singulis speciebus unius divisionis respondeant singulae species alterius ut attributa: vicissim etiam singulis speciebus alterius divisionis singulae species prioris ut attributa respondebunt.
>
> Ut si genus quoddam A dividatur primum in species b, c, ac deinde in species β, γ: ut Omne A sit aut b aut c, et rursus Omne A sit aut β aut γ; et praeterea, quae sint ex specie b, iis attribuatur β; quae ex specie c, iis γ; his igitur positis, vicissim, quae sunt ex specie β, iis attribuetur b; et quae ex specie γ, iis attribuetur c.
>
> Quod sic ostendetur, Si non iis A, quae sunt ex specie β, omnibus attribuatur b: cuicumque eorum non attribuitur, ei attribuetur c; quoniam Omne A est alterutrum, aut b aut c, per primam divisionem. Sed omnibus c etiam γ attribuitur per hypothesin. Ergo et omni β, quod non est b, attribuitur γ. Hoc vero fieri nequit: nulli enim β attribuitur γ, quia per hyp. nullum A est et β et γ, sed alterutrum eorum, per divisionem secundam. Non ergo iis A, quae sunt ex specie β, non omnibus attribuitur b: ergo omnibus attribuitur. Similiterque ostendetur, omnibus, quae sunt ex specie γ, etiam c attribui. Quod erat ostendendum.

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That is:

If some genus is divided into its species in two different ways, and if each species of one division belongs to a species of the other, then conversely each species of the other division belongs to a species of the one.

That is, if some genus A is divided first into the species b, c, and then into the species β, γ, in such a way that all A is either b or c, and also all A is either β or γ, and furthermore, those things which are of the species b belong to β, and those things which are of the species c belong to γ; if all this is the case, then conversely, those things which are of the species β belong to b, and those things which are of the species γ belong to c.

This can be shown as follows: If it is not the case that all those A which are of the species β belong to b, then those of them which do not belong to b will belong to c, since all A is one or the other—either b or c—according to the first division. But all c is γ by hypothesis. Therefore each β which is not b will belong to γ. This clearly cannot be the case, for no β belongs to γ, because according to the hypothesis no A is both β and γ, but rather one or the other of them, according to the second division. Therefore it is not the case that those A which are of the species β do not all belong to b; therefore they all belong. And similarly it can be shown that all those things which are of the species γ also belong to c. Which is what was to be shown.

Expressed symbolically, the theorem is as follows:

$$\begin{array}{llll} \textit{If} & A = b \cup c & \textit{where} & b \cap c = 0 \\ \textit{and} & A = \beta \cup \gamma & \textit{where} & \beta \cap \gamma = 0 \\ \textit{and if} & b \subset \beta & \textit{and} & c \subset \gamma, \\ \textit{then} & \beta \subset b & \textit{and} & \gamma \subset c. \end{array}$$

In comparing Hauber's formulation of the theorem with the others to be found in the literature, several differences may be noted. Hauber states his theorem for classes, yet in some books, such as Scholz & Hasenjaeger, it is presented as a theorem of the propositional calculus. Also, slight variations in the hypotheses may be noted. For Hauber, the two divisions of the basic class must each be both exhaustive and disjoint. While, for example, in Hilbert and Ackermann, the requirement that the first division be disjoint is dropped.[7] Still, such changes do not represent major alterations of the theorem; they only improve Hauber's statement and render it more elegant.

NOTES

1. The German mathematician Karl Friedrich Hauber was born at Schorndorf on May 18, 1775 and died at Stuttgart on September 5, 1851. He was known for his translations and commentaries on the works of Archimedes and Euclid. The title "Hauber's theorem" was given to the theorem in Chapter VII of *Scholae logico-mathematicae*, Stuttgart, 1829, by M. W. Drobisch in *Neue Darstellung der Logik*, Voss, Leipzig (1836), p. 162. Biographical references for Hauber are to be found in *Allgemeine Deutsche Biographie* and *La Grande Encyclopédie*.

2. Works referring to Hauber's theorem other than those mentioned here may be found by checking the bibliographies of *The Journal of Symbolic Logic.*

3. G. Peano, "Formola di logica matematica," *Rivista di matematica,* vol. I (1891), p. 184.

4. "Hauber's Schrift des Literaturverzeichnisses ist mir nicht zu Gesicht gekommen und halte ich mich bezüglich seines Satzes an die Angaben des Herr Venn p. 275." E. Schröder, *Vorlesungen über die Algebra der Logik,* vol. 2, pt. 1, Leipzig (1891), reprint, Chelsea, New York (1966), p. 285. The reference here is to J. Venn, *Symbolic Logic,* Macmillan, London (1881), p. 275.

5. H. Scholz and G. Hasenjaeger, *Grundzüge der mathematischen Logik,* Springer, Berlin (1961), p. 92.

6. "Inter quae theorema quoddam logicum se obtulit novum, quod equidem sciam; nec inelegans, et multiplicis in mathematica saltim doctrine usus; quod in capitis VIImi exordio proponentur; non indignum, ut spero, eorum cogitatione, qui in logicis contemplationibus aliquanto accuratius versari operae pretium ducunt." *Scholae logico-mathematicae,* Praefatio, p. IX. Several examples of recent instances in which the use of Hauber's theorem would have greatly simplified mathematical proofs are to be found in M. T. Alves, "A lei de Hauber demonstrada pela Álgebra de Boole," *Gazeta de matemática,* vol. 10, No. 41-42 (1949), pp. 17-19. For having translated this article and for his helpful comments on this manuscript, I must express my gratitude to Prof. Ignacio Angelelli.

7. "Wenn eine Klasse α in die Teilklassen β_1, β_2, β_3 zerfällt und andererseits auch in die Teilklassen γ_1, γ_2, γ_3 und wenn die Klassen γ_1, γ_2, γ_3 zueinander disjunkt sind, d.h. wenn zwei dieser Klassen keine gemeinsamen Elemente besitzen, wenn ferner die Klassen β_1, β_2, β_3 jeweils in den Klassen γ_1, γ_2, γ_3 enthalten sind, dann sind auch die Klassen γ_1, γ_2, γ_3 jeweils in den Klassen β_1, β_2, β_3 enthalten." D. Hilbert and W. Ackermann, *Grundzüge der theoretischen Logik,* 4th ed., Springer, Berlin (1959), p. 63.

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