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NOTE ON ZEMAN'S MODAL SYSTEM S4.04

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In [3] Zeman constructed a new intermediate modal system, called S4.04, between S4 and S4.4 by adding the following formula:

L1 $\mathfrak{C}LMLpCpLp$

as a new axiom, to S4. It is self evident that **L1** is an immediate consequence of S4.4 and that its addition, as a new axiom, to S4.2 or any extension of the latter system yields S4.4. In this note I shall discuss the position of S4.04 among the modal systems which are intermediate between S4 and S4.4. An acquaintance with papers [1], [2], [3] and, especially, with the list of matrices published in [2] is presupposed.

For some purposes unconnected with the topic of this note S4.04 was investigated in [2], and in that paper using the matrices given there it was established that formula **L1** is such that:

(1) It is rejected by matrix $\mathfrak{M}9$ which verifies systems S4.2, S4.1 and S4.1.1. This means that S4.04 is a proper extension of S4, and that it is contained neither in S4.1 nor in S4.1.1.
(2) It is verified by $\mathfrak{M}11$ which falsifies S4.2. Besides, $\mathfrak{M}11$ falsifies the proper axioms of S4.1 and S4.1.1, namely **N1** for $p/10$: $\mathfrak{C}\mathfrak{C}\mathfrak{C}10L1010CML1010 = \mathfrak{C}\mathfrak{C}LC101210CM1210 = \mathfrak{C}\mathfrak{C}L310C410 = \mathfrak{C}LC12109 = \mathfrak{C}L19 = LC19 = L9 = 9$, and **M1** for $p/2$: $\mathfrak{C}\mathfrak{C}\mathfrak{C}2L2L2CML2L2 = \mathfrak{C}\mathfrak{C}LC21212CM1212 = \mathfrak{C}\mathfrak{C}L1112C412 = \mathfrak{C}LC12129 = LCL19 = LC19 = L9 = 9$. Hence, S4.04 is a proper subsystem of S4.4, and it contains neither S4.1 nor S4.1.1.
(3) Formulas **L1**, **N1** and **M1** are verified by $\mathfrak{M}5$ which rejects S4.2. Therefore, the addition of **L1**, as a new axiom, to S4.1 or to S4.1.1 generates no system which contains S4.2. This means that such systems would be proper subsystems of S4.4.

Thus, it follows from (1), (2) and (3) that there are, previously unknown, two new systems which are intermediate between S4 and S4.4, namely:

1 System S4.1.2 = {S4.1; **L1**} = {S4; **N1**; **L1**} which is a proper extension of the mutually unconnected systems S4.04 and S4.1.

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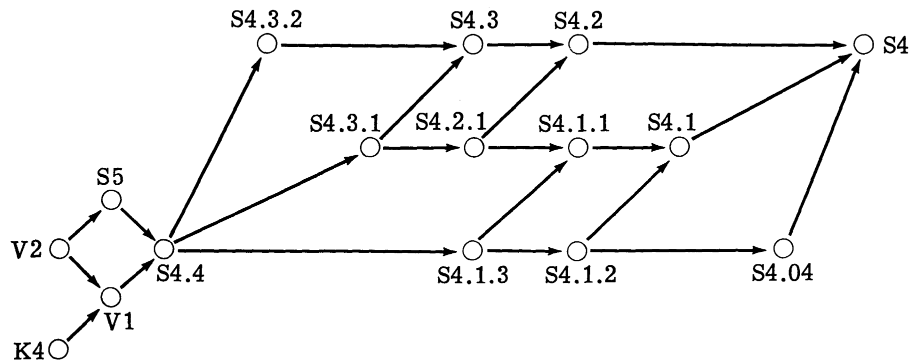
2 System S4.1.3 = {S4.1.1; **L1**} = {S4; **M1**; **L1**} which certainly is a proper extension of S4.1.1, and, most probably, also a proper extension of S4.1.2.

The open problem whether S4.1.2 is a proper subsystem of S4.1.3 is strictly connected with a still unsolved problem of long standing, viz. whether S4.1.1 is a proper extension of S4.1. It appears that the solutions of these two problems could be rather difficult.

The following updated diagram:



(In this diagram systems which are intermediate between S4.4 and S5, *cf.* [2], are omitted.)

visualizes the relations among the systems under consideration. An arrow which occurs between two systems indicates that a tail system is an extension of an edge system. In the literature we can find the proofs with an exception for S4.1.1 and for S4.1.3 that any other system represented in this diagram is a proper extension of its edge system.

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