Notre Dame Journal of Formal Logic
Volume XI, Number 2, April 1970

THE MISSING PREMISS

ALEX BLUM

Theorem: *For any set of sentences* $\{P_1, P_2, \ldots, P_n\}$ *and a sentence* Q *which is not deducible from* $P_1 \cdot P_2 \cdot \ldots \cdot P_n$:

(i) *There exists a sentence* P_Q, *such that*

(a) $P_1, P_2, \ldots, P_n, P_Q \vdash Q$;

and

(b) *If* P_{n+1} *is any sentence such that* $P_1, P_2, \ldots, P_n, P_{n+1} \vdash Q$ *then* $P_{n+1} \vdash P_Q$.

(ii) (a) *and* (b) *if and only if* $\vdash P_Q \equiv [(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q]$.

(iii) $Q \vdash P_Q$.

A sentence P will be called a 'weakest missing premiss' (or a 'wmp') if and only if $\vdash P \equiv P_Q$.

Proof:

I. (A) If P_{n+1} is any sentence such that $P_1, P_2, \ldots, P_n, P_{n+1} \vdash Q$ then:

1. $P_1 \cdot P_2 \cdot \ldots \cdot P_n \cdot P_{n+1} \vdash Q$
2. $P_{n+1} \cdot (P_1 \cdot P_2 \cdot \ldots \cdot P_n) \vdash Q$
3. $P_{n+1} \vdash (P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q$

(B) $P_1 \cdot P_2 \cdot \ldots \cdot P_n \cdot [(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q] \vdash Q$

Hence, there exists a sentence P_Q such that (a) and (b). And if $\vdash P_Q \equiv [(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q]$ then (a) and (b).

II. 1. If (a) then $P_Q \vdash (P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q$ (A)
2. If (b) then $(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q \vdash P_Q$ (B)
3. If (a) and (b) then $\vdash P_Q \supset [(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q]$ and $[(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q] \supset P_Q$
4. If (a) and (b) then $P_Q \equiv [(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q]$

Received November 22, 1968

III. Hence, there exists a sentence P_Q, such that (a) and (b). (a) and (b) if and only if $P_Q \equiv [(P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q]$. And since $Q \vdash (P_1 \cdot P_2 \cdot \ldots \cdot P_n) \supset Q$, $Q \vdash P_Q$.

I am indebted to Professor William Ruddish for strong encouragement.

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