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A REMARK ON NOTE ON DUALITY

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Chandler Works and Wolfgang Yourgrau ([1], p. 284) write:

"Let P be a compound proposition whose truth value is a function of the truth values of the undecomposed mutually independent propositions, $p_1, p_2, \ldots, p_k, \ldots, p_m, \ldots$. We represent the truth column for P by $f(P) = (a_1, a_2, \ldots, a_k, \ldots, a_n)$, where $a_k = 0$ or $a_k = 1$ and $n = 2^m$. Similarly, to another compound proposition, say Q, corresponds the numerical function $f(Q) = (b_1, b_1, \ldots, b_k, \ldots, b_n)$".

From this they conclude:

"Hence, $P \equiv Q$, if and only if $f(P) = f(Q)$, i.e. if and only if $a_k = b_k$ $(k = 1, 2, \ldots, n)$".

But this conclusion does not follow because of the following reasons:

(1) Two compound propositions may be equivalent, even though they may *not* have the *same number* of 'undecomposed mutually independent propositions'. Thus, for example, $p \equiv: p \supset q .\supset p$. Here $f(p) = (1,0)$ and $f(p \supset q .\supset p) = (1,1,0,0)$; hence $f(p) \neq f(p \supset q .\supset p)$, yet $p \equiv: p \supset q .\supset p$.

(2) Two compound propositions having the same *number* of 'undecomposed mutually independent propositions' may not be equivalent, even though their 'numerical functions' are identical. Take, for example, the two compound propositions, $p \supset q$ and $r \supset s$. Here $f(p \supset q) = (1,0,1,1)$, and $f(r \supset s) = (1,0,1,1)$, so that $f(p \supset q) = f(r \supset s)$, yet $p \supset q .\not\equiv. r \supset s$.

Thus the conclusion of the authors is not true generally, hence theorem (2), as it stands, is not proved, for the proof used the 'logical equivalence' of '$P \equiv Q$' and '$f(P) = f(Q)$' where P and Q are any two propositions. However, a special case of the theorem can be proved:

(2*) *If P and Q contain exactly the same independent propositions, then $P \equiv Q$ if and only if $P^{\mathrm{d}} \equiv Q^{\mathrm{d}}$*

for as the authors themselves have stated "P^{d} also depends on the *same* independent propositions as P" (italics ours).

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The authors have used theorem (2) in the proofs of theorems (8), (9), (10), (11), (12), (13) and have recommended its use in the proof of theorem (19); hence these proofs are wrong. But it is enough to use theorem (2*) to prove these theorems as they are concerned with the same P and Q, and their duals.

REFERENCE

[1] Works, Chandler and Wolfgang Yourgrau, "Note on duality in propositional calculus," *Notre Dame Journal of Formal Logic*, vol. IX (1968), pp. 284-288.

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