SYLLOGISTIC AND NON-SYLLOGISTIC ASPECTS OF THE
COMPARATIVE ARGUMENT

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The proposition 'X is less than Y' possesses a duality of meaning which makes for confusion within the comparative argument when the purpose is to distinguish the syllogistic from the non-syllogistic forms. This is evidenced by textbook writers who list comparative arguments under the caption 'irregular arguments' without mention of the fact that such arguments may as often be regular and syllogistic as irregular and non-syllogistic.

Consider the above proposition 'X is less than Y'.

(1) It may be taken to mean that X is an object or a mangitude which as such is less than the object or mangitude Y. Thus, this hill is less than (smaller than) this mountain or four inches is less than five inches.

Or,

(2) 'X is less than Y' may be taken to mean that X belongs to the class of all objects less than Y. The latter is as legitimate as the former though seldom, if ever, meant.

Because, as far as we know, Aristotle did not consider the matter of more-than and less-than and the later logicians, Noah K. Davis, in particular, refer to 'Formal Logic' by Augustus De Morgan as if standard in dealing with 'more than' and 'less than' arguments, we shall use the latter's form which is that of 'less-than' instead of the more usual 'greater than' relation of subject to predicate. De Morgan uses the following example*:

$$X \text{ is less than } Y$$
$$Y \text{ is less than } Z$$
$$\text{Therefore,} \quad X \text{ is less than } Z$$

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*Formal Logic, Augustus De Morgan, pp. 20-22.

He then turns this into the following:

> X is (a magnitude less than Y)
> Y is (a mangitude less than Z)
> Therefore, X is (a magnitude less than Z)

It is clear here that De Morgan had in mind meaning (1) above. X, Y, and Z are discrete points on a scale of mangitudes, and as such constitute for him the terms of the argument. But a study of his further analysis reveals that he is at least dimly aware of meaning (2). He says:

> When we say that X is less than Y, we say that if X were applied to Y every part of X would match a part of Y and there would be parts of Y remaining over. But when we say 'Every X is Y,' meaning the premise of a common syllogism, we say that every instance of X is an instance of Y without saying anything as to whether there are or are not instances of Y still left, after those which are also X are taken away. If then, we wish to write an ordinary syllogism in a manner which will correspond with 'X is less than Y, Y is less than Z, therefore, X is less than Z,' We must introduce more definite amount of assertion than was made in the preceding forms. Thus,
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> > Every X is Y, and there are Ys which are not Xs
> > Every Y is Z, and there are Zs which are not Ys
> > Therefore, Every X is Z, and there are Zs which are not Xs
> >
> > The Ys contain all the Xs and more
> > The Zs contain all the Ys and more
> > The Zs contain all the Xs and more
>
> This form of argument is called *a fortiori* argument because the premises are more than sufficient to prove the conclusion, and the event of the conclusion is thereby greater than its mere form would indicate.

De Morgan's nearest approach to meaning (2) exists in his observation that the Ys contain all the Xs and more, the Zs contain all the Ys and more, without saying as to whether there are instances of Y still left, after those which are also X are taken away or instances of Z still left after those which are also Y are taken away. This is an objection only when the terms are viewed as specific magnitudes and the conclusion demands the amount of difference between the subject and the predicate. The implication is that when *less than* and other comparative expressions such as *west of, whiter than*, etc., are involved, the reasoning is of necessity non-syllogistic.

Because De Morgan would not object to the proposition, All men are mortals, as the premise of a syllogism on the ground that there are mortals which are not men still left after those which are men have been taken away, we can only conclude that he had magnitudes of amount in mind. Thus,

> X is six inches less than Y
> Y is four inches less than Z
> Therefore, X is ten inches less than Z

This reasoning is as logical as is the syllogistic argument, but it is non syllogistic. Aside from the existence of four terms, the conclusion does not follow as an immediate inference. The amounts of 'less than' are specified and, hence, computation necessarily must take place between the premises and the conclusion. When the predicate terms are viewed as amounts of magnitude the reasoning is non-syllogistic.

But, if we consider X to be a member of the class of all objects 'less than Y' (meaning 2) and 'less than Y' a member of the class of all objects 'less than Z', we obtain a syllogistic argument. Thus,

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X is less-than-Y

Less-than-Y is less-than-Z

Therefore, X is less-than-Z

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When 'is' is taken as the copulative, thought is immediately directed upon what is less than the object named rather than to the object itself. Thus, Y does not appear as a term which is greater than X though by inference it obviously is greater than X. 'Less than Y' and Y are different objects, one general and the other particular.

In conclusion, we may say that De Morgan's error lay in holding that the proposition 'X is less than Y' must be viewed as having the relation 'less than' to Y, overlooking the possible view that X may be conceived as having the property of belonging to the class of all objects 'less than Y'. That the proposition itself tends to produce a mind-set favorable to the former is readily observed when in the instance of 'greater than' under meaning (2) we attempt to produce a corresponding image of the argument by means of Euler's Circles. For example, in saying 'six is greater than five' the unsuspecting person has the tendency to balk at placing the mark of six within the circle representing 'greater than five', though six is but one member of an infinite class of objects all of which are greater than five. However, our thought-block has less potency when such comparatives as 'west of', 'whiter than', etc., are involved. This, we may suppose, is for the reason that the idea of magnitude is further removed from the center of attention than is the case in the instance of terms involving digit qualifiers.

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