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ON SOME OPEN QUESTIONS OF B. SOBOCIŃSKI

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In Sobociński's [2] and [3] several questions are left open, among them

(1) Is K1.1 a proper extension of K1?
(2) Is K2.1 a proper extension of K2?
(3) Is K3.1 a proper extension of K3?
(4) Does there exist a system intermediate between S4.4 and S5?

With the aid of the matrices

$\mathfrak{M}$

C	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*N*
**1*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*8*
2	*1*	*1*	*3*	*3*	*5*	*5*	*7*	*7*	*7*
3	*1*	*2*	*1*	*2*	*5*	*6*	*5*	*6*	*6*
4	*1*	*1*	*1*	*1*	*5*	*5*	*5*	*5*	*5*
5	*1*	*2*	*3*	*4*	*1*	*2*	*3*	*4*	*4*
6	*1*	*1*	*3*	*3*	*1*	*1*	*3*	*3*	*3*
7	*1*	*2*	*1*	*2*	*1*	*2*	*1*	*2*	*2*
8	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*

$\mathfrak{M}'$

p	*M*	*L*
**1*	*1*	*1*
2	*1*	*4*
3	*1*	*4*
4	*1*	*4*
5	*5*	*8*
6	*5*	*8*
7	*5*	*8*
8	*8*	*8*

$\mathfrak{M}''$

p	*M*	*L*
**1*	*1*	*1*
2	*1*	*8*
3	*1*	*8*
4	*4*	*8*
5	*1*	*5*
6	*1*	*8*
7	*1*	*8*
8	*8*	*8*

all four questions are here answered in the affirmative, a familiarity with [2] and [3] being presupposed.

Ad (1)-(3). Matrices $\mathfrak{M}$ and $\mathfrak{M}'$ verify K1, K2 and K3 but falsify *CLCLCpLppp* for *p/3*: *CLCLC3L333* = *CLCLC3433* = *CLC433* = *C13* = *3*.

Ad (4). We exhibit such a system and show it to be Halldén-incomplete in the sense of [1], i.e., to contain wffs α and β having one variable each and no variable in common and such that $A\alpha\beta$, but neither α nor β, is a thesis.

Consider the system S4.7 obtained by adding *ALCMpLMpLCLMqMLq* as an axiom to S4.4. Matrices $\mathfrak{M}$1 and $\mathfrak{M}$2 of [2] verify S4.7 but falsify the S5 thesis *LCMpLMp*, while $\mathfrak{M}$1 and $\mathfrak{M}$3 verify S4.7 but falsify *LCLMqMLq*. S4.7 is thus a Halldén-incomplete extension of S4.4 and is properly

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contained in S5; that it is also a proper extension is shown by the fact that $\mathfrak{M}$ and $\mathfrak{M}''$ verify S4.4 but falsify $ALCMpLMpLCLMqMLq$ for $p/4$ and $q/6$: $ALCM4LM4LCLM6ML6 = ALC48LC18 = AL5L8 = CN58 = C48 = 5$.

REFERENCES

[1] Halldén, S., "On the semantic non-completeness of certain Lewis calculi," *The Journal of Symbolic Logic*, vol. 16 (1951), pp. 127-129.

[2] Sobociński, B., "Modal system S4.4," *Notre Dame Journal of Formal Logic*, vol. 5 (1964), pp. 305-312.

[3] Sobociński, B., "Family *K* of the non-Lewis modal systems," *Notre Dame Journal of Formal Logic*, vol. 5 (1964), pp. 313-318.

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