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# EXAMINATION OF THE AXIOMATIC FOUNDATIONS OF A THEORY OF CHANGE. I

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In 1934, Jan Salamucha published an article in which he was making a logical analysis of Thomas Aquinas' argument in his proof "Ex Motu" for the existence of God, [1], [2]. Here was an attempt to apply mathematical logic to a critical study of a classical problem in philosophy. As Professor Bolesław Sobociński pointed out in a biographical note, Salamucha's monograph was intended to show the important role that mathematical logic could play in studying complicated logical reasoning in the realm of philosophy, [3]. Some twenty years later, inspired by Salamucha's ideas, Johannes Bendiek published a historical study of the logical techniques used in the classical proofs for the existence of God, [4]. In recent years Francesca Rivetti-Barbò has published some extensive articles applying the tools of modern logic to Thomas Aquinas' proof "Ex Motu" for the existence of God, [5], [6]. It is our intention to evaluate the above-mentioned works with regard to our own study at the end of the present monograph.*

Our purpose here is to work out the axiomatization of the foundations of the theory of change which serves as the starting point and basis for the Thomistic proof "Ex Motu" for the existence of God. It is well known that Aquinas drew the elements of his theory of change from the Aristotelian system. We could thus consider our study as dealing also with the theory of change in the Greek Philosopher. We shall develop our subject in three parts followed by some appendices. The first part will present a formalization of the theory of change in question. In the second part we shall compare our formalization with some Thomistic and Aristotelian texts. The third part will deal with the proofs of the propositions which will appear in the formalization and with the proof of the consistency of the

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axiomatic system introduced in part one. Also in that last part we shall establish the independence of a certain set of axioms.

No one would expect to find already in Aquinas' or Aristotle's writings everything needed for a formalization. Although we were unable to find clear answers to certain questions relevant to our study in the texts of the two philosophers here concerned, it has been our aim to remain faithful to their philosophy. It is up to specialists in scholastic philosophy to decide in what measure we have succeeded in this.

§1. Introduction. The reasons for our choice of the word "change" instead of one of the words "movement, motion" to translate the Latin word "motus" is that we want to avoid eventual misunderstanding. In one's daily English usage, when speaking of movement or motion, one thinks first and almost exclusively of local movement or motion. The word "motus" has a much broader meaning in the Thomistic texts. It comprises all kinds of change. By the notion "motus", we shall mean a process in which one and the same subject in a temporal succession of phases exhibits different properties. Following Aristotle, Thomas Aquinas takes as the starting-point for his proof the empirical reality of change in the world. In our daily life we experience a manifold alteration and transformation in the inner world as well as in the outer world. We observe the alteration of landscape, the growth and the decay in vegetal and animal life, the constant transformation of human society, the progress in the arts and sciences, and so on. It is these empirically ascertainable facts which Aquinas takes as basis for his proof "Ex Motu".

St. Thomas' works are written in Latin, a natural language. For a study of the type which concerns us here, we need a so-called artificial or formalized language. The precise structure, explicitly given, of such a language is what we shall need. Let us explain briefly what is meant by: 1) a formalized language; 2) a calculus; 3) an axiomatic system; 4) consistency; and 5) independence.

1) A formalized language consists of a system of symbols together with a set of rules for their use. Such a language must be so specially devised as to enable us to express in it certain propositions of the theory under consideration. To this end, some definite symbols are selected to express the primitive notions (basic concepts) of the theory.

2) By a calculus we mean first the determination of the rules of formation governing the construction of expressions. These rules effectively determine which finite linear sequences of the given symbols constitute expressions of the formalized language. To a calculus also belongs a system of rules (relations on expressions) according to which certain logical connections between expressions can effectively be determined so that one expression can be said to be derivable from a set of expressions.

3) The formalization of a theory consists in the selection of some expressions of the theory in such a way that all the remaining propositions of the theory could be derived from the selected ones, called the axioms of the theory. The theory in question is then said to be represented by an axiom system.

4) Consistency is an obvious requisite of any axiom system. An axiom system is said to be inconsistent whenever an expression together with its negation are derivable within the axiom system, otherwise consistent. The consistency of a given axiom system could be established by constructing a model of the axiom system.

5) An expression is said to be independent in an axiom system if it cannot be derived in the axiom system. An axiom system itself is called independent if every axiom $\theta$ of the axiom system **S** is independent in the axiom system **S'** obtained from **S** by the removal of $\theta$. The independence of a given axiom system can be shown by constructing models. The requirement of independence is often only a matter of elegance.

In formalizing a theory we need first to state the concepts pertaining to the theory. These belong to two groups: the primitive notions (undefined concepts) and the defined notions. The connections between the concepts are ascertained through axioms and propositions. Axioms are put forward without proofs, while the propositions must be derived in the axiom system.

§2. Symbolic Language. There is no single way to determine a symbolic language. It depends on the problem to be dealt with and on the objective in view. We shall now state clearly the symbolic language to be used hereafter.

1. We will use a two-sorted predicate calculus. In the usual symbolic languages it is customary to distinguish variables of different kinds. We divide them into types in such a way that variables of one of these types are classified as being of the lowest type. We then subdivide the class of the lowest type variables into two sorts which we will denote by "**a**" and "**b**". The reason for using here a two-sorted symbolic language is that we need to have at our disposal two separate classes of individuals. The variables of sort **a** will represent the "momentaneous subjects" and those of sort **b** will represent the "properties".

2. The following logical signs will be used throughout this paper.

| Sign | Usage | Meaning |
|---|---|---|
| $\neg$ | $\neg p$ | It is not the case that $p$. ("$p$" is a propositional variable) |
| $\rightarrow$ | $p \rightarrow q$ | If $p$ then $q$ |
| $\wedge$ | $p \wedge q$ | $p$ and $q$ |
| $\vee$ | $p \vee q$ | $p$ or $q$ (the nonexclusive "or") |
| $\leftrightarrow$ | $p \leftrightarrow q$ | $p$ if and only if $q$ |
| $\forall$ | $\forall x \theta$ | For all $x$, $\theta$ ("$\theta$" is a variable for expressions) |
| $\exists$ | $\exists x \theta$ | There is an $x$ such that $\theta$. |
| $=_{Df}$ | $D_1 =_{Df} D_2$ | "$D_1$" means the same thing as "$D_2$" ("$D_1$" denotes the definiendum and "$D_2$" the definiens.) |

3. In order to eliminate a great accumulation of parentheses, we establish the following rule for omitting parentheses. To the connective signs, we assign the following order of precedence "$\leftrightarrow$", "$\rightarrow$", "$\vee$", "$\wedge$", "$\neg$". Of these, each has greater precedence or greater scope than any listed to its right. It is also out of practical expediency that we agree to omit all universal quantifiers at the left of an axiom, a proposition and a definition, whenever the quantifier has the whole expression as its operand (scope).

4. We give now a synopsis of the individual variables of particular types along with a list of the primitive notions with their respective type. The meaning of these notions will of course be explained later.

| Type | Letters |
|---|---|
| **a** | Momentaneous subjects . . . $x, y, z, \ldots, x_1, y_1, \ldots$ |
| **b** | Properties . . . . . . . . $\alpha, \beta, \gamma, \ldots\ldots$ |

Types of the primitive notions

| | |
|---|---|
| $\sim$ | is of type (**a a**) |
| $<$ | is of type (**a a**) |
| **A** | is of type (**a b**) |
| **F** | is of type (**a b**) |
| **M** | is of type (**a a b**) |
| **B** | is of type (**a a b**) |
| $\dashv$ | is of type (**b b**) |

§3. Domain of momentaneous subjects. Following Aristotle, Aquinas makes various statements about change, the most important statement being: "If anything changes, it is changed by something else". It is, in this study, our principal aim to formalize the theory of change to such an extent as to provide a proof of the above statement.

To make change intelligible, we introduce first that which undergoes the change, the "subject". Such subjects could be a "table", a "block of marble", an "apple", and so on. These subjects are not taken as abstract entities, but rather as concrete beings with all their specific and individual characteristics. When speaking therefore of a subject, we do not refer to an arbitrary member of a particular species, but to this definite being, with its quantity, qualities, location, etc., uniquely determined.

To the characteristics of a concrete subject belongs the sequence of the successive moments of its existence. If we consider a precise point of time at which a subject exists, we speak then of a "momentaneous subject" whereby this particular point of time is included. Beyond that it would be possible to refer to the class of the momentaneous subjects which are "genidentical" (the term "genidentical" was first introduced by K. Lewin in [7].), i.e. one and the same subject at all the moments of its temporal existence. It is important to point out here that in our study, we shall always consider a subject as one whole in a similar way as for example physicists speak of a closed system of energy.

Latin letters $x, y, z, \ldots$ will be used to denote individual variables for momentaneous subjects.

There obviously are certain relations between momentaneous subjects. The simultaneity of two momentaneous subjects is such a relation which we introduce now as our first primitive notion.

**Pn 3.1.** $x \sim y$: *x and y are simultaneous, i.e. the momentaneous subjects represented by the variables x and y coincide in time.*

The relation "$\sim$" is evidently an equivalence relation, i.e. it is reflexive, symmetric and transitive. This fact we state in our first axioms.

A3.1.1 $x \sim x$
A3.1.2 $x \sim y \rightarrow y \sim x$
A3.1.3 $x \sim y \wedge y \sim z \rightarrow x \sim z$

In considering one and the same subject at two different points of time, we are dealing with two nonidentical momentaneous subjects. They are of course related since they belong to the same concrete subject. We introduce here our second primitive notion which we denote by "$<$".

**Pn 3.2.** $x < y$: *x and y are genidentical and x is earlier in time than y.*

What properties does this relation have? It is not difficult to convince ourselves that the relation "$<$" is irreflexive, asymmetric, transitive and dense. If one holds that there is a smallest interval of time in the physical world, a modification of the axiom 3.2.4 below would be necessary. However, it would not impeach our results in any essential way. The next axioms will formulate these formal properties.

A3.2.1 $\neg x < x$
A3.2.2 $x < y \rightarrow \neg y < x$
A3.2.3 $x < y \wedge y < z \rightarrow x < z$
A3.2.4 $x < z \rightarrow \exists y (x < y < z)$

($x < y < z$ stands for $x < y \wedge y < z$. It is a mere abbreviation. We shall use such abbreviations when it is convenient.)

Our next axiom states that there could be no pair of momentaneous subjects for which both of the preceding relations could hold.

A3.3 $x < y \rightarrow \neg x \sim y$

We shall need later on to put in parallel the points of time of genidentical momentaneous subjects with the points of time of other genidentical momentaneous subjects. This is what the following axiom formulates.

A3.4 $x_0 < x_1 < x_2 \wedge y_0 < y_2 \wedge x_0 \sim y_0 \wedge x_2 \sim y_2 \rightarrow \exists y_1 (y_0 < y_1 < y_2 \wedge x_1 \sim y_1)$

The following defined notion shall serve as an abbreviation.

D3.1 $x \leqslant y =_{Df} x < y \vee x = y$.

The relation "$<$" includes a time ordering. We introduce now a defined relation which leaves out the time ordering (any order in time). We shall call it the relation of "genidentity". (This relation, as it is used here, was first introduced by H. Hermes in [8]).

D3.2 $\mathbf{G}xy =_{Df} x \leqslant y \vee y < x$

($\mathbf{G}xy$ means that $x$ and $y$ are genidentical.)

The next two additional axioms express how momentaneous subjects are related. When each of two momentaneous subjects $x_1$ and $x_2$ stands in the relation "<" with one and the same third momentaneous subject $x_3$, either $x_1$ and $x_2$ are identical or they stand in the relation "<" and thus are genidentical.

A3.5 $x_1 < y \wedge x_2 < y \rightarrow \mathbf{G}x_1x_2$
A3.6 $x < y_1 \wedge x < y_2 \rightarrow \mathbf{G}y_1y_2$

Using D3.2, it can be shown that the relation of genidentity is an equivalence relation. An additional proposition is introduced for abbreviation purposes.

S3.1 $x < y \rightarrow \mathbf{G}xy$
S3.2.1 $\mathbf{G}xx$
S3.2.2 $\mathbf{G}xy \rightarrow \mathbf{G}yx$
S3.2.3 $\mathbf{G}xy \wedge \mathbf{G}yz \rightarrow \mathbf{G}xz$

Since the relation of genidentity is an equivalence relation, the field of "**G**" could be divided into mutually exclusive classes that would satisfy the two following conditions: a) **G** holds for each pair of individuals in any one of these classes; and b) if an individual in one of these classes bears the relation "**G**" to another individual, then this second individual belongs to the same class as the first individual. We could thus define the notion of subject as being the class of genidentical momentaneous subjects.

The following five propositions are trivial. The reason for introducing them is that they will allow us to shorten certain proofs.

S3.3 $\mathbf{G}xy \wedge x \sim y \rightarrow x = y$
S3.4 $x \sim y \wedge \neg x = y \rightarrow \neg \mathbf{G}xy$
S3.5 $\neg \mathbf{G}xy \wedge \mathbf{G}yz \rightarrow \neg \mathbf{G}xz$
S3.6 $x_1 < y \wedge x_2 < y \wedge x_1 \sim x_2 \rightarrow x_1 = x_2$
S3.7 $x < y_1 \wedge x < y_2 \wedge y_1 \sim y_2 \rightarrow y_1 = y_2$

§4. Domain of properties. Primitive notion "**A**" (actual). After dealing with the bearer of change, i.e. that which undergoes the change, we turn to what we agree to call "properties". A property is that which a momentaneous subject has or could have. Depending on whether a momentaneous subject has or could have a certain property, we shall speak respectively of an "actual" or "possible" property of this momentaneous subject. Let us clarify what is meant here by giving an example. A man who at this very moment is sitting, has the actual property of the sitting position. He could as well be standing up or lying down; because of this the standing up or lying down positions are not actual but possible properties of this momentaneous man.

Greek letters $\alpha, \beta, \gamma, \ldots$ will be used to denote the individual variables for properties.

In the scholastic philosophy there are hardly any notions that have such an important role as the pair "act" and "potency". It plays a decisive role in explaining such problems as coming to be and duplication of species. This pair of notions have been used with various shades of meaning. A formalized language requires however an exact determination of notions. For this we shall in the future speak of the pair "in actu" and "in potentia", respectively "actual" and "potential".

Before giving the definition of change, it is necessary to elucidate the notion "in actu". We already explained what was meant by an actual property (determination). The primitive notion "in actu" will express that a momentaneous subject possesses a determination as an actual determination.

Pn4.1 $\mathsf{A}x\alpha$: *x is actual with regard to* $\alpha$.

For Aquinas as well as for Aristotle no concrete subject could exist without some determination. A subject always exists as an individual, i.e. with one or more of the characteristics referred to above. The next axiom states this fact.

A4.1 $\exists\alpha \mathsf{A}x\alpha$

§5. Primitive notion "F" (capable of). Defined notions "P" and "V" (potential and change). The notions of subject and property provide us with the basis for the definition of change. We want, however, to introduce an additional primitive notion, namely "capacity", and a defined notion "in potentia" (being-in-potency).

It would be a mistake to consider the notion "in potentia" as implying a mere denial of being. It is our intention to prevent such a misunderstanding. In the scholastic philosophy the notion "in potentia" seems to be taken as a primitive notion. We have chosen instead to introduce first the primitive notion "capacity" and to lead back to it the notion "in potentia". When we say that a certain property is a possible determination of a momentaneous subject, the justification for our saying so must reside in some relationship. It is this relationship between momentaneous subject and property that we want to express by the notion "capacity".

Pn5.1 $\mathsf{F}x\alpha$: *x is capable of* $\alpha$.

Undoubtedly a momentaneous subject which possesses a particular property, is capable of this property. Moreover, we seem to be justified in stating that a subject which at a given point in time is capable of a certain determination, was also at earlier moments of its existence capable of this determination. We so obtain the following two axioms.

A5.1 $\mathsf{A}x\alpha \rightarrow \mathsf{F}x\alpha$

A5.2 $x_1 < x_2 \wedge \mathsf{F}x_2\alpha \rightarrow \mathsf{F}x_1\alpha$

The notion "in potentia" can now be defined.

D5.1 $\mathbf{P}x\alpha =_{Df} \mathbf{F}x\alpha \wedge \neg \mathbf{A}x\alpha$

$\mathbf{P}x\alpha$ could read: $x$ is potential in $\alpha$, i.e. $x$ is capable of having the property $\alpha$, but at the particular moment of its existence, $x$ does not possess this property. A few propositions could now be derived.

S5.1 $\mathbf{G}x_1x_2 \wedge \mathbf{A}x_1\alpha \wedge \mathbf{P}x_2\alpha \rightarrow \neg x_1 \sim x_2$
S5.2 $\mathbf{A}x\alpha \wedge \mathbf{P}x\beta \rightarrow \neg \alpha = \beta$
S5.3 $\mathbf{P}x\alpha \rightarrow \exists\beta(\mathbf{A}x\beta \wedge \neg \alpha = \beta)$
S5.4 $x_1 < x_2 \wedge \neg \mathbf{A}x_1\alpha \wedge \mathbf{A}x_2\alpha \rightarrow \mathbf{P}x_1\alpha$

In everyday language, S5.1 says that it is not possible for two genidentical momentaneous subjects to exist simultaneously when one is actual and the other potential with regard to the same property. The proposition S5.2 states that for one and the same momentaneous subject to be actual and potential is only possible with regard to different determinations.

We have now all the required assumptions for the explicit definition of change. As it was mentioned before, by change we mean a process in the course of which one and the same subject shows at different points in time distinct properties. One and the same subject is observed at different moments of its existence. Suppose that at the later moment this subject shows a determination which it did not have at the earlier moment. We then say that this subject has undergone a change. Three things are to be distinguished: the initial state (before the change), the final state (after the change), and that which undergoes the change, the subject. An apple which first was green, has become red; the apple has undergone a change, yet it remains the same apple.

A change could also be described as a process of the transition from being-in-potency to being-in-act with regard to a certain determination. Let us consider the example of the formation of a statue from a block of marble. In its initial state the marble block contains merely the capacity to become a statue. In its final state, the marble block has actually taken the form of a statue. The transition from the possible determination - marble statue - to the actual determination - marble statue - constitutes the change of the marble block. The marble is that which undergoes the change, the subject. It is worth noting here that one could say that a subject is in a state of change with regard to a certain determination only when that particular change could be conceived as one continuous process of actualization. This would not be the case when dealing with instantaneous changes. Were we to formalize the definition of change as one continuous process of actualization, we would need a new primitive concept. We want to avoid using such a notion which, we think, would unnecessarily complicate our formalization. We shall show in an appendix how such a formalization could be obtained.

In our present formalization we consider the essence of a change to be the end-result, i.e. the acquisition by one and the same subject of a new determination.

D5.2 $\mathbf{V}y_1y_2\alpha =_{Df} y_1 < y_2 \wedge \mathbf{A}y_2\alpha \wedge \forall y(y_1 \leqslant y < y_2 \rightarrow \neg \mathbf{A}y\alpha)$

$\mathsf{V}y_1y_2\alpha$ reads: The subject to which belong the momentaneous subjects represented by the variables $y_1$ and $y_2$, has undergone a change with regard to the determination $\alpha$ during the time interval determined by $y_1$ and $y_2$, and this change came to an end at the point in time belonging to $y_2$.

It should be understood that in our definition of change we have avoided any reference either to an active or to a passive component. We want to consider here change as neutral. The following propositions could now be derived.

S5.5 $\mathsf{V}y_1y_2\alpha \rightarrow \mathsf{G}y_1y_2$
S5.6 $\mathsf{V}y_0y_2\alpha \wedge \mathsf{V}y_1y_2\alpha \rightarrow \mathsf{G}y_0y_1$
S5.7 $\mathsf{V}y_0y_2\alpha \wedge y_0 < y_1 < y_2 \rightarrow \mathsf{V}y_1y_2\alpha$
S5.8 $\mathsf{V}y_1y_2\alpha \rightarrow \mathsf{P}y_1\alpha$

The proposition S5.5 follows directly from the definition of change. It expresses the genidentity of the two momentaneous subjects which in the change with regard to a definite determination come into consideration. In other words, in a change there must exist a definite subject which is considered at two distinct moments of its existence. The numerical unity of the bearer of a change is stated in the proposition S5.6. The proposition S5.7 declares that when the three requirements of D5.2 are satisfied in the case of two momentaneous subjects $y_0$ and $y_2$, these requirements are also satisfied by the momentaneous subject $y_2$ and each momentaneous subject $y_1$ which is genidentical with $y_2$ and which exists at any point of the time interval determined by $y_0$ and $y_2$. In other words, the bearer of a change with regard to a certain determination remains in potency towards that determination as long as the change has not come to its end.

§6. Primitive notion "**M**" (Mover-Moved). In our definition of change no mention was made of any cause. If we analyze the whole reality of an arbitrary change, it would appear meaningful to look for an explanation of the coming to be. For example, if we observe that a thermometer which had a temperature of 15°C, indicates now a temperature of 20°C, or that an electron has jumped from one orbit to another, we are likely to ask why, i.e. to look for the causes of the respective changes. In one case the subject-causing-the-change (which we shall call the mover) might be warm air, in the other a quantum of energy. It is of course possible that more than one subject might be the immediate cause of one change. In such cases, we want however to speak of one subject-causing-the-change in a way similar to the combining of forces in mechanics into a unique resulting force. For a subject, the acquiring of a new property, a new determination, can be rightly looked upon as an effect, a result of some sort. The following questions then come to mind. Why has this result taken place? What is at the origin of this effect? How could we express the relationship between the cause of the apparition of a new determination and the bearer of a change? We need to introduce an additional primitive notion which will formulate the relation between mover and moved, i.e. between the subject causing the change and the subject undergoing the change.

**Pn6.1** $\mathsf{M}xy\alpha$: *x changes y to α, i.e. the subject represented by x moves the subject represented by y to α, namely in the time interval determined by x and y.*

It is important to remark that the possibility or impossibility of the genidentity of the subjects represented by $x$ and $y$ remains an open question. This primitive notion 6.1 will be henceforth referred to as the relationship "mover-moved". We so introduce explicitly the active component (mover) along with the passive component (moved).

It would seem obvious that for two subjects to stand in the relation "mover-moved" they must exist simultaneously. This requirement is stated in the next two axioms.

A6.1 $\mathsf{M}xy\alpha \rightarrow \exists x_1(x_1 \sim y \wedge x < x_1)$
A6.2 $\mathsf{M}xy\alpha \rightarrow \exists y_0(x \sim y_0 \wedge y_0 < y)$

These two axioms together with the fact that the relation "<" is dense allow the inference that a subject which exists at two distinct moments, exists also in the time interval determined by them. We can now derive some additional propositions.

S6.1 $\mathsf{M}xy\alpha \rightarrow \neg x \sim y$
S6.2 $\mathsf{M}xy\alpha \wedge y_0 < y_1 < y \wedge y_0 \sim x \rightarrow \exists x_1(x_1 \sim y_1 \wedge x < x_1)$
S6.3 $\mathsf{M}xy\alpha \wedge x < x_1 < x_2 \wedge x_2 \sim y \rightarrow \exists y_1(x_1 \sim y_1 \wedge y_1 < y)$

With regard to the relation "M", we want to specify the moment at which the change is completed. For this, we use two axioms.

A6.3 $\mathsf{M}xy\alpha \rightarrow \mathsf{A}y\alpha$
A6.4 $\mathsf{M}xy\alpha \wedge x \sim y_0 \leqslant y_1 < y \rightarrow \neg \mathsf{A}y_1\alpha$

($x \sim y_0 \leqslant y_1 < y$ stands for $x \sim y_0 \wedge y_0 \leqslant y_1 \wedge y_1 < y$. It is a mere abbreviation.)

In section 5, the proposition S5.6 stated one of the requirements for the unity of a change. An additional requirement which we bring in through an axiom, is the numerical unity of the mover.

A6.5 $\mathsf{M}x_1y\alpha \wedge \mathsf{M}x_2y\alpha \rightarrow \mathsf{G}x_1x_2$

We now obtain the following three propositions the meaning of which is clear.

S6.4 $\mathsf{M}xy\alpha \rightarrow \exists y_0(x \sim y_0 \wedge \mathsf{V}y_0y\alpha)$
S6.5 $\mathsf{M}xy_2\alpha \wedge x \sim y_1 < y_2 \rightarrow \mathsf{V}y_1y_2\alpha$
S6.6 $\mathsf{M}xy\alpha \wedge \mathsf{A}x\alpha \rightarrow \neg \mathsf{G}xy$

Up to now no answer has been given to the question as to whether one and the same subject with regard to one and the same determination could be at the same time mover and moved in a change. The proposition S6.6 gives the answer for the case where the mover is actually in possession of the determination.

A third requirement for the numerical unity of a change is the numerical unity of the time interval during which the change takes place. We introduce it with the following axiom.

A6.6 $\mathbf{M}xy\alpha \wedge x < x_1 \sim y_1 < y \rightarrow \mathbf{M}x_1y\alpha$

On the basis of this axiom, we can deduce a proposition complementary to S6.5.

S6.7 $\mathbf{M}xy_2\alpha \wedge x < x_1 \sim y_1 < y_2 \rightarrow \mathbf{V}y_1y_2\alpha$

It is through the introduction of the primitive notion "**M**" that we were able to express the relationship between mover and moved in a change. However, this relationship has been formulated in one direction, namely from mover to moved. We need an additional axiom for the formulation of this relationship in the direction from moved to mover.

A6.7 $\mathbf{V}y_1y_2\alpha \rightarrow \exists x \exists y(x \sim y \wedge y_1 \leqslant y < y_2 \wedge \mathbf{M}xy_2\alpha)$

We can now obtain a proposition stating the relation between mover and moved in the form of an equivalence.

S6.8 $\mathbf{V}y_0y_2\alpha \leftrightarrow \exists x_1 \exists y_1(x_1 \sim y_1 \wedge y_0 \leqslant y_1 < y_2 \wedge \mathbf{M}x_1y_2\alpha \wedge \forall y(y_0 \leqslant y < y_2 \rightarrow \neg \mathbf{A}y\alpha))$

§7. Primitive notion "**B**" (Valuation). In section 5 the notion of capacity was referred to as a relationship between property and momentaneous subject. Although the primitive concept "in actu" was understood as actual determination of a momentaneous subject, it would have been correct to speak again of a relationship between property and momentaneous subject. When we introduced the primitive notion "**M**", we interpreted it as the relation "mover-moved" with regard to a determination. It would appear then that the relation "mover-moved" always includes a relationship between the subject moved and a property as well as a relationship between the subject mover and the property in question.

It is now our intention to gather in a single concept the various relationships between properties and momentaneous subjects so as to allow us to value the relationships existing between many momentaneous subjects and a same determination. It is therefore a question of assessing the participation of momentaneous subjects in one and the same property so as to be able to refer to a momentaneous subject as having a greater share in a determination than another momentaneous subject with regard to the same determination. As an illustration of this, suppose that we consider a group of three persons and that we want to establish a comparison among them with reference to their command of the English language. Let us say that the situation is as follows: one has English as mother tongue; the second can make himself well understood in English although he is not as yet in full command of the English language; and finally the third is about to start learning the English language. We could consider the first person as being "in actu" with respect to the determination "the mastery of the English language", the second as being in the process of acquiring this mastery and the third as being "in potentia", that is, as having the capacity to reach this mastery of English. In assessing their respective share (participation) in the command of the English language, we could rightly say that the first has a greater share than the second, and the latter a greater participation than the third. The following primitive notion will serve to establish the comparison referred to above.

Pn7.1 $\mathsf{B}xy\alpha$: *x has at most as large a share in α as y.*

It is evident that this relation "B" is reflexive and transitive. Let us then formulate this in two axioms.

A7.1 $\mathsf{B}xx\alpha$
A7.2 $\mathsf{B}xy\alpha \wedge \mathsf{B}yz\alpha \rightarrow \mathsf{B}xz\alpha$

The relation "B" is also a smaller-equal-than relationship. To simplify our use of the preceding relation, we introduce now two additional concepts.

D7.1 $\mathsf{W}xy\alpha =_{Df} \mathsf{B}xy\alpha \wedge \neg \mathsf{B}yx\alpha$
D7.2 $\mathsf{I}xy\alpha =_{Df} \mathsf{B}xy\alpha \wedge \mathsf{B}yx\alpha$

The relation "W" is irreflexive, asymmetric and transitive as the following propositions formulate it.

S7.1 $\neg \mathsf{W}xx\alpha$
S7.2 $\mathsf{W}xy\alpha \rightarrow \neg \mathsf{W}yx\alpha$
S7.3 $\mathsf{W}xy\alpha \wedge \mathsf{W}yz\alpha \rightarrow \mathsf{W}xz\alpha$

Let us consider now a momentaneous subject $x$ which actually possesses a certain determination $\alpha$ and a momentaneous subject $y$ which is capable of possessing $\alpha$ but does not as yet possess the determination $\alpha$. We should be justified to say that $y$ has a smaller participation in $\alpha$ than $x$. We formulate this in an axiom.

A7.3 $\mathsf{P}x\alpha \wedge \mathsf{A}y\alpha \rightarrow \mathsf{W}xy\alpha$

If any two momentaneous subjects happen both to be actually in possession of a certain determination, we find ourselves justified to say that they have equal share in $\alpha$.

A7.4 $\mathsf{A}x\alpha \wedge \mathsf{A}y\alpha \rightarrow \mathsf{I}xy\alpha$

In the hypothesis that a subject represented by $x$ brings a subject $y$ into the determination $\alpha$ during the time interval determined by $x$ and $y$, and this without any outside influence, one would seem justified to formulate the following axiom which states that the momentaneous subject $x$ has at least as large a participation in the determination $\alpha$ as any momentaneous subject $z$ which actually possesses the determination $\alpha$.

A7.5 $\mathsf{M}xy\alpha \wedge \mathsf{A}z\alpha \rightarrow \mathsf{B}zx\alpha$

We then obtain the proposition:

S7.4 $\mathsf{M}xy\alpha \wedge \mathsf{P}z\alpha \rightarrow \mathsf{W}zx\alpha$

We should recall here that, in setting up the domain of momentaneous subjects, we emphasized that each momentaneous subject has to be considered as a whole. With this in mind, we can see that the proposition S7.4

implies the impossibility for a momentaneous subject to be mover with respect to a determination $\alpha$ and at the same time to be also in a state of potency with regard to this same determination. It is important to remark that we have now reached the nucleus of our study. Whenever a momentaneous subject is mover with regard to a determination $\alpha$, the following two claims hold: first, there exists a relationship between this momentaneous subject and the determination $\alpha$; secondly, the momentaneous subject has at least as large a participation in $\alpha$ as any momentaneous subject which is actually in possession of $\alpha$. A fundamental question must be settled here. Whenever a momentaneous subject stands as mover with regard to a determination $\alpha$, is it required that it be itself in actual possession of the determination $\alpha$? If this were the case, then it would of course be impossible for this momentaneous subject to be at the same time also in a state of potency with respect to the determination in question since being-in-act and being-in-potency with regard to the one and same determination at the same time would be contradictory.

S7.5 $\mathsf{M}xy\alpha \rightarrow \neg \mathsf{P}x\alpha$

It is now possible to give a satisfactory answer to the question whether a momentaneous subject could be at the same time mover and moved.

S7.6 $\mathsf{M}xy\alpha \rightarrow \neg \mathsf{G}xy$

This proposition can be generalized to cover a time interval as follows:

S7.7 $\mathsf{M}x_1y_1\alpha \wedge x_1 < x \sim y < y_1 \rightarrow \neg \mathsf{G}xy$

In proposition S7.4 is included that whenever two subjects stand in the relation "mover-moved" with respect to a determination $\alpha$, the subject-moved possesses a smaller share into $\alpha$ than the subject-mover during the complete time interval.

S7.8 $\mathsf{M}x_1y_1\alpha \wedge x_1 \leqslant x \sim y < y_1 \rightarrow \mathsf{W}yx\alpha$

The present formalization was aimed at the deduction of the proposition: "If anything changes, it is changed by something else". This goal is reached with the following proposition:

S7.9 $\mathsf{V}y_1y_2\alpha \rightarrow \exists x \exists y(x \sim y \wedge y_1 \leqslant y < y_2 \wedge \mathsf{M}xy_2\alpha \wedge \neg \mathsf{G}xy_2)$

Before closing this first part we add some remarks concerning our axiomatization.

1. The axiomatization presented above is not the only possible one. It is certainly possible to establish a different axiomatization using the same assumptions and reaching the same goal. Although different, the two axiomatizations would then be equivalent.

2. In setting up our formalization we used six primitive concepts. The other notions were explicitly defined from them. In any formalization, it is desirable to keep the number of primitive notions down. We think that a reduction of the number of primitive notions would have been achieved only at the expense of clarity.

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*(To be continued)*.

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