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A NOTE ON THE TRUTH-TABLE FOR $p \supset q$

ROBERT J. FARRELL

A didactic problem which inevitably confronts the teacher of elementary symbolic logic is the justification of the truth-table for $p \supset q$. Every teacher has encountered the inquisitive, contentious student who refuses to admit $\mathsf{F} \supset \mathsf{T}$ and $\mathsf{F} \supset \mathsf{F}$ are true. The justifications commonly found in textbooks, for one reason or another, all fall short of satisfying this student.

For instance, Jan Łukasiewicz, in order to justify the truth-table, made the following use of the obviously true proposition, If x is divisible by 9, then x is divisible by 3:

> This implication is true for all the values of the numerical variable x. Hence on substituting $x/16$ we should obtain a true sentence. The substitution yields:
>
> If 16 is divisible by 9, then 16 is divisible by 3.
>
> We have thus obtained an implication with a false antecedent and a false consequent. In view of such examples we agree $C00 = 1$, i.e., that *an implication with a false antecedent and a false consequent is true*. By substituting $x/15$ we obtain:
>
> If 15 is divisible by 9, then 15 is divisible by 3.
>
> Now the antecedent is false and the consequent true. We therefore agree that $C01 = 1$, i.e., that *an implication with a false antecedent and a true consequent is true*.
>
> By substituting $x/18$ we obtain an implication with a true antecedent and a true consequent:
>
> If 18 is divisible by 9, then 18 is divisible by 3.
>
> Consequently, we agree that $C11 = 1$, i.e., that *an implication with a true antecedent and a true consequent is true*.[1]

Unfortunately, the contentious student can employ an analogous argument to make a prima facie case for evaluating $\mathsf{F} \supset \mathsf{F}$ and $\mathsf{F} \supset \mathsf{T}$ (as well as $\mathsf{T} \supset \mathsf{F}$) as false. Consider the obviously false proposition: If Fig. *ABCD* is a square, it has only three sides. Now consider the following three figures:

1. Jan Łukasiewicz, *Elements of Mathematical Logic*, Pergamon Press, New York (1963), p. 26.

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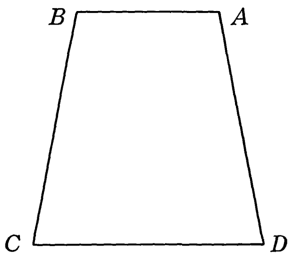

Fig. 1

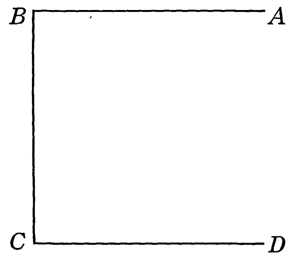

Fig. 2

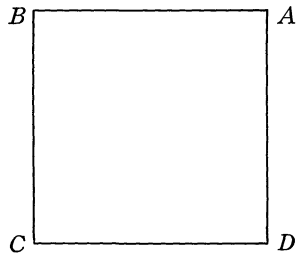

Fig. 3

Fig. 1 gives $F \supset F$, Fig. 2 gives $F \supset T$, and Fig. 3 gives $T \supset F$. Following the same line of argumentation used by Łukasiewicz, we should say these implications are false. That is, just as Łukasiewicz's obviously true proposition makes it appear that $p \supset q$ is false only if the antecedent is true and the consequent false, our obviously false proposition makes it appear that $p \supset q$ is true only if the antecedent is true and the consequent true.

Of course, to allow ourselves to be convinced by this prima facie argument would be disastrous for truth-functional logic. Such obviously valid argument forms as $(p \cdot q) \supset p$ would cease to be tautologies. Moreover, the distinction between implication and conjunction would vanish. Considerations of this latter sort provide the basis for Richard Purtill's justification of the accepted truth-table.[2] Granted that the values of $T \supset T$ and $T \supset F$ are non-controversial, there are only four possibilities for completing the truth-table.

⊃	T	F
T	T	F
F	F	F

Table 1

⊃	T	F
T	T	F
F	T	F

Table 2

⊃	T	F
T	T	F
F	F	T

Table 3

⊃	T	F
T	T	F
F	T	T

Table 4

Table 1 is unacceptable because it confounds implication with conjunction. Table 2 makes the value of the implication depend entirely on the value of the consequent. As Purtill puts it, the best English equivalent of such a connective would not be "if p then q," but rather something like "q whether or not p." Table 3 confounds implication with equivalence. That leaves only Table 4. However, while this process of elimination shows that no other truth-table is reasonable, it does not apply the accepted table to ordinary discourse in such a way as to positively justify it to the skeptical student.

I. M. Copi attempts just such a positive justification.

> Under what circumstances should we agree that the conditional statement:
>
> If blue litmus paper is placed in this solution, then the litmus paper will turn red.

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2. Richard L. Purtill, *Logic for Philosophers*, Harper & Row, New York (1971), pp. 11-12.

> is false? . . . It is important to realize that this conditional does not assert any blue litmus paper is actually placed in the solution, or that any litmus paper actually turns red. It asserts merely that if blue litmus paper is placed in the solution, then the litmus paper will turn red. It is proved false in case blue litmus paper is actually placed in the solution and does not turn red. The acid test, so to speak, of the falsehood of a conditional statement is available when its antecedent is true, for if its consequent is false while its antecedent is true, the conditional itself is thereby proved false.[3]

Copi makes it clear that a conditional statement does not assert that its antecedent or its consequent is true. He thus makes it clear that a conditional *may* be true even though one or both of its components is false. And certainly there is no difficulty convincing the student that a conditional statement is false if its antecedent is true and its consequent false. What Copi does not make clear, however, is why a conditional should be considered false *if and only if* that combination of truth-values is the case. We still require some persuasive, common language illustration to convince the contentious student that a conditional statement is true in case any one of the truth-value combinations $\mathsf{T} \supset \mathsf{T}$, $\mathsf{F} \supset \mathsf{F}$, or $\mathsf{F} \supset \mathsf{T}$ is the case. The following remarks provide such an illustration.

Consider two people, Jack and Max, in a room with a covered birdcage. Jack, the owner of the cage, tells Max (who does not know its contents) that the cage contains a raven named Charlie. Max then asserts a conditional statement, of whose truth he is convinced, namely:

If Charlie is a raven, Charlie is black.

Now, under what conditions could Max reasonably continue to maintain that his conditional statement is true, after the cover is removed? He could do so as long as the contents do not disconfirm it. And we would normally regard it disconfirmed only in case Charlie turns out to be a non-black raven. That is, in none of the following cases is it disconfirmed: Charlie is a raven and is black, Charlie is not a raven and is not black, Charlie is not a raven and is black. Hence, in none of these cases would Max have reason to abandon his conditional statement; or, in any of them, he could reasonably continue to assert it as true.

This illustration is valuable in that it provides the student with an example of a conditional the truth-values of whose components are clearly undetermined at the time it is asserted. Moreover, it makes it clear that a conditional can be reasonably asserted as true when any one of these situations is the case: the antecedent and consequent are both true, the antecedent and consequent are both false, or the antecedent is false and the consequent true. Thus I propose to introduce the truth-table for $p \supset q$ by means of such an illustration, and to define $p \supset q$ thus:

$$(p \supset q) =_{df} [(p \cdot q) \vee (\sim p \cdot \sim q) \vee (\sim p \cdot q)]$$

3. Irving M. Copi, *Introduction to Logic,* Third Edition, The Macmillan Company, New York (1968), p. 225.

This definition will of course give the desired truth-table, and will introduce what we mean to assert by $p \supset q$ in terms of the altogether non-controversial functors for negation, conjunction, and disjunction. I believe this definition to be preferable for didactic purposes to the simpler and more common definition:

$$(p \supset q) =_{df} \sim(p \cdot \sim q)$$

because it defines $p \supset q$ in terms of what we positively do mean to assert by the implication, rather than in terms of what we mean to deny by it. That is, with the attendant illustration, it makes clear to the student what conditions are compatible with the reasonable maintainance of a statement of the form $p \supset q$.

Walsh College
Canton, Ohio