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CONSISTENT, INDEPENDENT, AND DISTINCT PROPOSITIONS

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1. THE PROBLEM STATED. This is a sequel to [10] and acquaintance with it is presupposed. It was shown there that the existence postulate can be proved in certain non-regular systems. It followed, loosely speaking, that in those systems we had two consistent and independent and four distinct propositions. We now propose to construct a system in which there are denumerably many consistent, independent, and distinct propositions. Note that this is a propositional system. We, therefore, do not yet enter into the controversies surrounding quantified modal logic. At a future date we intend to add quantifiers to our system and we shall see what happens then. But we do claim that Lewis would have found our propositional system highly satisfactory. We now give precise definitions of some of the terms that will be used. The notation employed is that of [8].

2. DEFINITIONS. We first propose to define a proposition. We wish to say, roughly, that a wff B is a proposition if and only if every substitution-instance (**SI**) of B is equivalent to B. But it is necessary to proceed with caution. In the definitions that follow **P** is a propositional calculus which has, among its rules, the rule of substitution on variables. For the first two definitions **P** can be thought of as a propositional calculus in a very wide sense; the next two assume that it has, among its connectives $\sim$ and $\wedge$; the remaining ones have, in addition, the connective $\diamond$. These three connectives may be primitive or defined. Small letters stand for variables and capital letters denote formulas.

Definition 1. Let λ be a connective (primitive or defined) of **P** such that $\vdash_{\mathbf{P}} p \lambda p$; and further, the following rule (primitive or derived) is available in **P**: "If B results from A by substitution of $N(M)$ for $M(N)$ at one or more places in A (not necessarily for all occurrences of $M(N)$ in A), if $\vdash_{\mathbf{P}} M \lambda N$ and $\vdash_{\mathbf{P}} A$, then $\vdash_{\mathbf{P}} B$ [1, p. 101; with a slight variation]." Then λ is said to be an E-*connective of* **P**.

Comments. In the systems T, S4, and S5, both substitutivity of strict equivalents (**SSE**) and substitutivity of material equivalents (**SME**) are

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available. This is well-known. So both strict equivalence (**SE**) and material equivalence (**ME**) are E-connectives of these calculi. In S1, S2, and S3, **SSE** is available but not **SME**. This can be seen as follows. Supposing it were available.

Z1	$\sim\Diamond(p \wedge \sim p) \vee (p \wedge \sim p)$	[S1°]
Z2	$\Diamond(p \wedge \sim p) \supset (p \wedge \sim p)$	[*Z1*; S1°]
Z3	$(p \wedge \sim p) \supset \Diamond(p \wedge \sim p)$	[S1°]
Z4	$\Diamond(p \wedge \sim p) \equiv (p \wedge \sim p)$	[*Z2*; *Z3*; S1°]
Z5	$(p \wedge \sim p) \prec p$	[S1°]
Z6	$\Diamond(p \wedge \sim p) \prec p$	[*Z4*; *Z5*; **SME**]

But *Z6* is not a theorem of S1, S2, or S3. In fact, it is shown in [7] that addition of *Z6* to S1 or S2 gives us T, and to S3 gives us S4. So we have the incidental result that addition of **SME** to S1 or S2 gives us T, and to S3 gives us S4. It follows then that **SE** is an E-connective of S1, S2, and S3, but not **ME**. This shows a fundamental difference between S1, S2, S3; and T, S4, S5. Let us describe the former group as S-systems and the latter as T-systems. This agrees with the spirit of [8]. See pp. 145-146.

Definition 2. A wff B is said to be a *proposition of* **P** *with respect to the connective* λ, *where* λ *is an* E-*connective of* **P**, if and only if every **SI**, C, of B is such that $\vdash_{\mathbf{P}} B\lambda C$. When no ambiguity can result we shall omit mentioning: "with respect to the connective λ, where λ is an E-connective of **P**".

Comments. The wffs $p \wedge \sim p$ and $p \vee \sim p$ are propositions of the Classical Propositional Calculus (**CPC**) w.r.t. **ME**. We show that $p \wedge \sim p$ is a proposition. Let $Q \wedge \sim Q$ be an **SI** of $p \wedge \sim p$. Then

Z1	$(p \wedge \sim p) \supset q$	[**CPC**]
Z2	$(p \wedge \sim p) \supset (Q \wedge \sim Q)$	[$Z1$, $q/Q \wedge \sim Q$]
Z3	$(Q \wedge \sim Q) \supset (p \wedge \sim p)$	[$Z1$, p/Q, $q/p \wedge \sim p$]
Z4	$(p \wedge \sim p) \equiv (Q \wedge \sim Q)$	[*Z2*; *Z3*; **CPC**]

Similarly, it can be seen that $p \vee \sim p$ is a proposition and also that both these wffs are propositions of the T-systems both w.r.t. **ME** and **SE**. But they are propositions of the S-systems w.r.t. **SE** but not w.r.t. **ME**. In these calculi it is meaningless (see definition above) to talk of propositions w.r.t. **ME** since **ME** is not an E-connective.

Now note that $\Diamond(p \wedge \sim p)$ is also a proposition w.r.t. **SE** of both S- and T-systems. Let $\Diamond(Q \wedge \sim Q)$ be an **SI** of $\Diamond(p \wedge \sim p)$. Then

Z1	$(p \wedge \sim p) \prec q$	[S1°]
Z2	$(p \wedge \sim p) \mathrel{≣} (Q \wedge \sim Q)$	[As above]
Z3	$\Diamond(p \wedge \sim p) \mathrel{≣} \Diamond(p \wedge \sim p)$	[S1°]
Z4	$\Diamond(p \wedge \sim p) \mathrel{≣} \Diamond(Q \wedge \sim Q)$	[*Z2*; *Z3*; S1°]

Similarly, it can be seen that any wff built from $p \wedge \sim p$ by prefixing any number of '$\sim$'s and '$\Diamond$'s in any order as well as conjunctions of such wffs are propositions w.r.t. **SE** of both T- and S-systems. More generally, of any system that contains S1°.

Definition 3. The propositions $P_1, P_2, \ldots, P_n$ (w.r.t. λ; of **P**) are said to be *distinct* (*w.r.t.* λ; *in* **P**) if and only if

$$\vdash_{\mathbf{P}} \sim(P_1 \lambda P_2) \wedge \sim(P_1 \lambda P_3) \wedge \ldots \wedge \sim(P_{n-1} \lambda P_n).$$

In this case, we say that $\{P_1, P_2, \ldots, P_n\}$ is a *distinct set of propositions* (*w.r.t.* λ; *in* **P**).

Comments. The propositions $p \wedge \sim p$ and $p \vee \sim p$ are distinct in **CPC** w.r.t. **ME** since

$$\vdash_{\mathbf{CPC}} \sim((p \wedge \sim p) \equiv (p \vee \sim p)).$$

Since every theorem of **CPC** is a theorem of the T-systems, they are also distinct in the T-systems w.r.t. **ME**. They are also distinct in both T-systems and S-systems w.r.t. **SE**.

Z1	$\Diamond(p \wedge \sim p) \vee \Diamond(p \vee \sim p)$	[S1]
Z2	$\sim[\sim\Diamond\{(p \wedge \sim p) \wedge (p \wedge \sim p)\} \wedge \sim\Diamond\{(p \vee \sim p) \wedge (p \vee \sim p)\}]$	[*Z1*; S1°]
Z3	$\sim[\{(p \wedge \sim p) \prec (p \vee \sim p)\} \wedge \{(p \vee \sim p) \prec (p \wedge \sim p)\}]$	[*Z2*; S1°]
Z4	$\sim\{(p \wedge \sim p) \equiv (p \vee \sim p)\}$	[*Z3*; S1°]

We have shown in [10] that the propositions (see comments to Definition 2) $\Diamond\Diamond(p \wedge \sim p)$, $\sim\Diamond\Diamond(p \wedge \sim p)$, $\Diamond(p \wedge \sim p)$, and $\sim\Diamond(p \wedge \sim p)$ are distinct in S6 w.r.t. **SE**.

Definition 4. An infinite set of propositions (w.r.t. λ; of **P**) is said to be a *distinct set of propositions* (*w.r.t.* λ; *in* **P**) if and only if every finite subset of the infinite set is a distinct set of propositions (w.r.t. λ; in **P**).

Comments. None yet. See section 3.

Definition 5. The propositions $P_1, P_2, \ldots, P_n$ (w.r.t. λ; of **P**) are said to be *consistent* (*w.r.t.* λ; *in* **P**) if and only if

(1) $\vdash_{\mathbf{P}} \Diamond(P_1 \wedge \ldots \wedge P_n)$.

In this case, we say that $\{P_1, P_2, \ldots, P_n\}$ is a *consistent set of propositions* (*w.r.t.* λ; *in* **P**).

Comments. Suppose now that **P** contains S1°. Then it is easy to see that (1) is equivalent to

$$\vdash_{\mathbf{P}} \sim\{(P_1 \wedge \ldots \wedge P_{r-1} \wedge P_{r+1} \wedge \ldots \wedge P_n) \prec \sim P_r\}\ [r = 1, 2, \ldots, n].$$

And this, of course, is in accord with our intuitive notion of consistency.

Definition 6. An infinite set of propositions (w.r.t. λ; of **P**) is said to be a *consistent set of propositions* (*w.r.t.* λ; *in* **P**) if and only if every finite subset of the infinite set is a consistent set of propositions (w.r.t. λ; in **P**).

Definition 7. The propositions $P_1, P_2, \ldots, P_n$ (w.r.t. λ; of **P**) are said to be *independent* (*w.r.t.* λ; *in* **P**) if and only if

(2) $\vdash_{\mathbf{P}} \Diamond(P_1 \wedge \ldots \wedge P_{r-1} \wedge \sim P_r \wedge P_{r+1} \wedge \ldots \wedge P_n)\ [r = 1, 2, \ldots, n]$.

In this case, we say that $\{P_1, P_2, \ldots, P_n\}$ is an *independent set of propositions* (*w.r.t.* λ; *in* **P**).

Comments. Again let **P** contain S1°. Then (2) is equivalent to

$$\vdash_{\mathbf{P}} \sim\{(P_1 \wedge \ldots \wedge P_{r-1} \wedge P_{r+1} \wedge \ldots \wedge P_n) \dashv P_r\} \; [r = 1, 2, \ldots, n],$$

which again is in accord with our intuitive notion of independence.

Definition 8. An infinite set of propositions (w.r.t. λ; of **P**) is said to be an *independent set of propositions* (*w.r.t.* λ; *in* **P**) if and only if every finite subset of the infinite set is an independent set of propositions (w.r.t. λ; in **P**).

3. MODAL SYSTEM S10. The following theorem is implicit in [10].

Theorem 1. $\mathfrak{M} = \langle M, D, \cap, -, \mathrm{P}\rangle$ *is a* σ*-regular* S6*-matrix if and only if*

(A) $\langle M, \cap, -, \mathrm{P}\rangle$ *is a weak modal algebra*;
(B) D *is an additive ideal of* M;
(C) $x = 0$ *if and only if* $-\mathrm{P}(x) \in D$;
(D) $x \leq \mathrm{P}x$;
(E) $\mathrm{PP}0 \in D$;
(F) $\mathrm{P}0 \leq \mathrm{P}x$.

We now show that

Theorem 2. $\mathfrak{M} = \langle M, D, \cap, -, \mathrm{P}\rangle$ *is a* σ*-regular* S6*-matrix if and only if*

(A) $\langle M, \cap, -, \mathrm{P}\rangle$ *is a modal algebra*;
(B) D *is an additive ideal of* M;
(C) $x = 0$ *if and only if* $-\mathrm{P}(x) \in D$;
(D) $x \leq \mathrm{P}x$;
(E) $\mathrm{PP}0 \in D$.

Proof. First suppose that $\mathfrak{M}$ is a σ-regular S6-matrix. Then by Theorem 1 above and Definitions II.2, II.19 [8], it remains to show that for $x, y \in M$, $\mathrm{P}(x \cup y) = \mathrm{P}x \cup \mathrm{P}y$. By 44.4 [3], $\mathrm{P}(x \cup y) \Leftrightarrow (\mathrm{P}x \cup \mathrm{P}y) \in D$. By Definition II.14 (iv) [8], $\mathrm{P}(x \cup y) = \mathrm{P}x \cup \mathrm{P}y$. Conversely, suppose $\mathfrak{M}$ satisfies conditions (A)-(E). By Theorem 1 above and Theorem II.1 [8], it remains to show that $\mathrm{P}0 \leq \mathrm{P}x$. From $\mathrm{P}(x \cup y) = \mathrm{P}x \cup \mathrm{P}y$ we get $\mathrm{P}x = \mathrm{P}x \cup \mathrm{P}0$ whence $\mathrm{P}0 \leq \mathrm{P}x$.

Now consider the following matrix $\mathfrak{M}$. Let $K = \{2, 3, 4, \ldots\}$. Let M be the class of all subsets of K. Let D be the class of all subsets x of K such that $2 \in x$. If x and y are any subsets of K, let $x \cap y$ be the intersection of x and y. If x is any subset of K, let $-x$ be the complement of x with respect to K. We define a unary operation $\mathrm{P}x$ for x any subset of K as follows. If x is the null set, or a set which contains just one member, then

$$\begin{aligned} \mathrm{P}\Lambda &= \{3\} \\ \mathrm{P}\{2\} &= \{2, 3\} \\ \mathrm{P}\{3\} &= \{2, 3, 5, 6, \ldots\} \\ \mathrm{P}\{4\} &= \{2, 3, 4, 6, \ldots\} \end{aligned}$$

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$$\mathsf{P}\{n\} = \{2,\ 3,\ .\ .\ .\ ,\ n,\ n+2,\ .\ .\ .\}\ (n \geq 3).$$

If x is any other set, we define $\mathsf{P}x$ to be the union of all sets $\mathsf{P}y$ where y is a subset of x which contains but one member. Our matrix $\mathfrak{M} = \langle M, D, \cap, -, \mathsf{P}\rangle$. By Theorem 2, $\mathfrak{M}$ is a σ-regular S6-matrix.

Our system S10 is as follows. Take the formulation of S6 given in [3]. Add to it as axioms every formula not already a theorem of S6 and which is verified by this matrix. This is S10. We are, of course, saying that S10 is that system which is formulated by taking the four Lewis rules as primitive, and which has $\mathfrak{M}$ as a σ-regular characteristic matrix. If the reader feels deprived I cannot help it. I have been unable so far to find a set of axioms. We can, however, note a few things. S10 is stronger than S6. $\mathfrak{M}$ verifies $\Diamond\sim\Diamond\Diamond(p \wedge \sim p)$ but Group I [6] verifies S6 and falsifies it. It may be noted that this is the axiom for S3.1 [9]. S10 is independent of S3 (and hence of S7). $\mathfrak{M}$ falsifies the formula $\Diamond(\Diamond p \wedge \sim\Diamond(p \wedge \sim p)) \prec \Diamond p$ when $p = \{3\}$ and this is the axiom for S3 [9]. And **SE** is an E-connective of S10 (by definition). But not **ME**. If it were, then, as in comments to Definition 1, we would be able to derive $\Diamond(p \wedge \sim p) \prec p$ which is falsified by $\mathfrak{M}$. We shall speak of propositions of S10. It will be tacit that they are with respect to **SE**.

Consider now the following denumerable list of formulas:

P_1	$\Diamond\Diamond(p \wedge \sim p)$
P_2	$\Diamond\sim\Diamond\Diamond(p \wedge \sim p)$
P_3	$\Diamond\sim\Diamond\sim\Diamond\Diamond(p \wedge \sim p)$
*	*
*	*
*	*
P_n	$(\Diamond\sim)^{n-1}\Diamond\Diamond(p \wedge \sim p)$

where $(\Diamond\sim)^{n-1}$ has the obvious meaning. By comments to Definition 2, each of these formulas is a proposition of S10. We shall now show that they are consistent, independent, and distinct in S10. The method of proof is completely straightforward. We simply check that our matrix verifies the required formulas. Before we proceed let us note that each of the formulas (which again are propositions) $\Diamond(p \wedge \sim p)$, $\sim\Diamond\Diamond(p \wedge \sim p)$, $\sim\Diamond\sim\Diamond\Diamond(p \wedge \sim p)$, . . . is such that every way of evaluating it on the basis of $\mathfrak{M}$, using $\cap$, $-$, P in place of $\wedge$, $\sim$, $\Diamond$ leads to the same element of M, viz., to $\{3\}$, $\{4\}$, $\{5\}$, . . . respectively. Also note that the propositions P_1, P_2, . . . correspond similarly to the elements (of M) $\{2, 3, 5, 6, \ldots\}$, $\{2, 3, 4, 6, \ldots\}$, ...

Now observe that $\mathfrak{M}$ verifies each of the propositions P_1, P_2, ... Hence they are all theorems of S10. It follows then that

$$\vdash_{S10} \Diamond(P_i \wedge \ldots \wedge P_j)$$

where $\{P_i, \ldots, P_j\}$ is any finite subset of $\{P_1, P_2, \ldots\}$. By Definitions 5 and 6, $\{P_1, P_2, \ldots\}$ is a consistent set of propositions.

Next notice that the sets $\{2, 3, 5, 6, \ldots\}$, $\{2, 3, 4, 6, \ldots\}$, ... are such that the complement of each is contained in every other. And if $\{n\}$ is the complement of any, then $\mathrm{P}\{n\}$ is designated. From these remarks it follows that if $\{P_{i_1}, P_{i_2}, \ldots, P_{i_n}\}$ is any finite subset of $\{P_1, P_2, \ldots\}$, then

$$\vdash_{S10} \Diamond (P_{i_1} \wedge \ldots \wedge P_{i_{r-1}} \wedge \sim P_{i_r} \wedge P_{i_{r+1}} \wedge \ldots \wedge P_{i_n}) \ [r = 1, 2, \ldots, n].$$

By Definitions 7 and 8, $\{P_1, P_2, \ldots\}$ is an independent set of propositions.

Lastly, take any two different (not yet distinct) propositions P_i and P_j from the set $\{P_1, P_2, \ldots\}$, i.e., $i \neq j$. By the previous paragraph,

$$\vdash_{S10} \Diamond (P_i \wedge \sim P_j).$$

By S1°,

$$\vdash_{S10} \Diamond (P_i \wedge \sim P_j) \vee \Diamond (P_j \wedge \sim P_i).$$

Again by S1°,

$$\vdash_{S10} \sim \{\sim \Diamond (P_i \wedge \sim P_j) \wedge \sim \Diamond (P_j \wedge \sim P_i)\}.$$

By S1° again,

$$\vdash_{S10} \sim (P_i \equiv P_j),$$

so that P_i and P_j are distinct. It follows by Definitions 3 and 4 that $\{P_1, P_2, \ldots\}$ is a distinct set of propositions.

We make a final observation. We claim that S10 is such that no finite σ-regular matrix will verify it. For all n we have

$$\vdash_{S10} \sim (P_1 \equiv P_2) \wedge \sim (P_1 \equiv P_3) \wedge \ldots \wedge \sim (P_{n-1} \equiv P_n).$$

So for any σ-regular S10-matrix, $-(\mathrm{PP0} \Leftrightarrow \mathrm{P} - \mathrm{PP0}) \cap -(\mathrm{PP0} \Leftrightarrow \mathrm{P} - \mathrm{P} - \mathrm{PP0}) \cap \ldots \cap -\{(\mathrm{P}-)^{n-2}\mathrm{PP0} \Leftrightarrow (\mathrm{P}-)^{n-1}\mathrm{PP0}\} \epsilon D$. By Definition II.14 (ii) [8], Theorem III.6 [9], and Boolean algebra, we have that $-(\mathrm{PP0} \Leftrightarrow \mathrm{P} - \mathrm{PP0}) \epsilon D$, $-(\mathrm{PP0} \Leftrightarrow \mathrm{P} - \mathrm{P} - \mathrm{PP0}) \epsilon D, \ldots, -\{(\mathrm{P}-)^{n-2}\mathrm{PP0} \Leftrightarrow (\mathrm{P}-)^{n-1}\mathrm{PP0}\} \epsilon D$. Now if $-x \epsilon D$, $x \notin D$; for if $x \epsilon D$, then by Definition II.14 (iii) [8], $x \cap -x \epsilon D$, i.e., $0 \epsilon D$, which contradicts Theorem III.8 (D) [8]. So $\mathrm{PP0} \Leftrightarrow \mathrm{P} - \mathrm{PP0} \notin D$, $\mathrm{PP0} \Leftrightarrow \mathrm{P} - \mathrm{P} - \mathrm{PP0} \notin D$, $\ldots$, $(\mathrm{P}-)^{n-2}\mathrm{PP0} \Leftrightarrow (\mathrm{P}-)^{n-1}\mathrm{PP0} \notin D$. But if $x \Leftrightarrow y \notin D$, $x \neq y$; for, if $x = y$, then $(x \Leftrightarrow y) = (x \Leftrightarrow x) \epsilon D$. So $\mathrm{PP0} \neq \mathrm{P} - \mathrm{PP0}$, $\mathrm{PP0} \neq \mathrm{P} - \mathrm{P} - \mathrm{PP0}, \ldots, (\mathrm{P}-)^{n-2}\mathrm{PP0} \neq (\mathrm{P}-)^{n-1}\mathrm{PP0}$. In other words, the elements $\mathrm{PP0}$, $\mathrm{P} - \mathrm{PP0}, \ldots, (\mathrm{P}-)^{n-1}\mathrm{PP0}$ of the matrix are all distinct (in a mundane sense; not, of course, in the sense of Definition 3). And this is true for all n.

There are a number of places where Lewis complains about Boolean algebras, e.g., "Boolean algebra is a rather unsatisfactory form for any calculus of logic [5, p. 30]". But there is nothing wrong with Boolean algebras as such. Our matrix $\mathfrak{M}$ is a Boolean algebra. It seems to me that what Lewis "intends to assert" can be clarified in the light of future developments. The postulates of material implication are such that the two-element Boolean algebra (we think of it as a matrix) is *characteristic* for the system. To him this was very objectionable. His systems S1-S5 do not have finite characteristic matrices [2]. This was a significant step forward. But a defect remained. The two-element matrix *verified* S1-S5. This is remedied in the non-regular systems S6-S8 since no two-element matrix will verify them (this is easily seen by noting that there are four propositions of these systems distinct in them and using the argument of the previous paragraph.) But there are four-element matrices which do so. In this paper we have taken yet another step. We have produced a system which is an extension of S2 independent of S3—and S2 was the system that Lewis favored—such that no finite matrix will even verify it. Having said this, let us make a qualification. Our discussion has been restricted to σ-regular matrices. It would have been nice to prove in complete generality that no finite model will verify the system. But it is well-known that such a general assertion is almost impossible to prove. All kinds of structures each of which has the perfect right to be called a model keep sprouting. In view of this, it does not seem too objectionable that we have limited our treatment to σ-regular matrices. One more thing. The reader may feel uncomfortable that S10 is defined in terms of an infinite matrix (non-denumerable, at that); then it is shown that no finite σ-regular matrix will verify it, and this fact is paraded as a virtue of the system. But there is no cause for alarm. Non-denumerable characteristic matrices have been exhibited for quite a few modal systems [4, pp. 207-208], but they do not share this latter property with S10.

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