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A NOTE CONCERNING A SOLE SUFFICIENT OPERATOR

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Let $E(k) = \{0,1, \ldots, k - 1\}$. In [3] Wesselkamper proves that a particular Markov operator S defined on $E(k)$ is complete with constants (a function is complete with constants if the set consisting of the function and all the constant functions is complete). In this short note we give an alternative proof of the completeness by showing that Rose's generalized condition disjunction [2] is easily defined using S and the constant functions. This conditioned disjunction is defined by $[x, y_0, \ldots, y_{k-1}] = y_i$ if $x = i$ and it was proved in [2] to be complete with constants. (Rose repeats the variable x at the end of the definition to maintain symmetry.)

$$S \text{ is defined by } Sxyz = \begin{cases} z, & \text{if } x = y \\ x, & \text{otherwise.} \end{cases}$$

We use three sets of definitions, as follows:

D1. $V_0x = S0Sx001$
$V_jx = S0Sjx01$, for each $j = 1,2, \ldots, k - 1$.

$$\text{Then } V_ix = \begin{cases} 1, & \text{if } x = i; \\ 0, & \text{otherwise.} \end{cases}$$

D2. $W_ixy = SV_ix1y$, for each i, $0 \leq i \leq k - 1$.

$$\text{Hence } W_ixy = \begin{cases} y, & \text{if } x = i, \\ 0, & \text{otherwise.} \end{cases}$$

D3. $Q_1xy_0y_1 = SW_0xy_00W_1xy_1$
$Q_{i+1}xy_0y_1 \ldots y_{i+1} = SQ_ixy_0 \ldots y_i0W_{i+1}xy_{i+1}$ for each $i = 1,2, \ldots, k - 2$.

Then $[x, y_0, \ldots, y_{k-1}] = Q_{k-1}xy_0 \ldots y_{k-1}$ so that S is complete with constants.

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[3] Wesselkamper, T. C., "A sole sufficient operator," *Notre Dame Journal of Formal Logic,* vol. XVI (1975), pp. 86-88.

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