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THE ADEQUACY OF MATERIAL DIALOGUE-GAMES

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The concept of a *material dialogue-game** is explained by P. Lorenzen, by K. Lorenz, and, from a somewhat different point of view, by K. J. J. Hintikka.[1] Whereas in *formal* dialogues the formulas uttered are meaningless schemata, *material* dialogues are carried through in an interpreted language: their sentences—at least the elementary ones—may have truth-values, and these truth-values have their bearing on the possibilities of winning or losing. Each of the three authors mentioned has asserted, at least implicitly that his game is *adequate* in the following sense: there exists a winning strategy for the proponent of a thesis, iff this thesis is true according to classical semantical theory.[2] K. Lorenz's proof of *Hauptsatz 1* can be reinterpreted to establish the adequacy of his *reine (faktische) Dialogspiele*.[3]

In this paper I will present a rather general definition of "*material dialogue-game*", though one limited to games in which all the elementary sentences are either true or false. This definition makes it possible to state and prove a theorem asserting the *adequacy* with respect to any two-valued model theory $\mathfrak{M}$ of all material dialogue-games that have three properties to be explained shortly: *local finiteness, regularity,* and *accordance in logical rules with the particular model theory under consideration.* These, to my opinion, are properties a reasonable material dialogue-game should have. The proof of the theorem is straightforward, once its key-concept—that of a *P-favorable position* in a game—has been defined. The adequacy of most known material dialogue-games follows as special cases of the theorem.

1 *A definition of 'material dialogue-game'* Material dialogues must be held in a *language*. In the following, let $\mathfrak{L}$ be some fixed language, with sentences, $A, B, C, \ldots$, some of them elementary. It is not required that

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$\mathfrak{L}$ be a language for sentential logic or for quantifier logic. Further, there must be *players*. We shall only consider games with two disputants, White(W) and Black(B). I will use 'P' as a variable over $\{W, B\}$, and denote P's adversary as $\overline{P}$.

A dialogue-game in $\mathfrak{L}$ shall be determined by its *positions*, and its *permitted moves*. These will now be treated in succession. A *position* x shall be a seven-tuple consisting of

(1): the *player*, P_x, whose turn it is at x.
(2): a *valuation*, v_x, of all the elementary sentences of $\mathfrak{L}$: For A elementary $v_x(A) = \mathbf{T}$ or $v_x(A) = \mathbf{F}$.
(3), (4): sets $\mathbf{A}(W)_x$ and $\mathbf{A}(B)_x$ of *assertions* already made by W and B before the current x was reached.
(5), (6): sets $\mathbf{D}(W)_x$ and $\mathbf{D}(B)_x$ of *defense sets*[4] of W and B at x.
(7): a *structural-rule function*, f_x, assigning natural numbers to assertions in $\mathbf{A}(W)_x$ and $\mathbf{A}(B)_x$, to defense sets in $\mathbf{D}(W)_x$ and $\mathbf{D}(B)_x$, and to elements of these latter sets (f_x may be empty or only partially defined).

By an *assertion* I mean a labelled sentence $\langle A, n\rangle$—where n is a natural number—; thus it makes sense to say that P has asserted A twice. Assertions $\langle A, n\rangle$ and $\langle B, m\rangle$ are *equiform* iff $A = B$. P's assertions represent possibilities of attack for $\overline{P}$. An attack usually provides the disputant whose assertion is attacked with some possible retorts; these constitute what I have called *defense sets*. A *defense set*, therefore, is defined to be a set of assertions. It is not necessary to introduce challenges, like "?" and "?n", as special components of the positions of the games, since their influence upon the situation is determined completely by the defense sets they introduce.[5] The *structural-rule functions* serve in formulating structural rules; more explanation will follow the definition of "material dialogue-game".

A *move* is an ordered pair $\langle x, y\rangle$, where x and y are positions and $P_x \neq P_y$. Whereas the set of positions is the same for all dialogue-games in $\mathfrak{L}$, the set of *permitted moves*, $\mathbf{R}$, also called the *game relation*, may be different for different games. Each game has its rules, and its moves should conform to them. The rules of a material dialogue-game, which determine its permitted moves, are of two kinds: the *logical rules* (*allgemeine Spielregel*), which determine the kinds of attack and the relevant retorts that may occur in the game, and the *structural rules* (*spezielle Spielregel*), which determine when and how often these kinds of attack and these retorts may be used in a particular *tournament*[6] of the game.[7]

Without loss of generality we may suppose that for each natural number i and each complex sentence A of $\mathfrak{L}$ it makes sense to speak of an attack of the i-th kind on A. If there are only finitely many(k)kinds of attack possible (as in the case of languages for sentential logic), we can let the attack of the i-th kind coincide with the attack of the k-th kind, for $i > k$. Universal sentences make good examples of sentences that may be attacked in infinitely many ways. Let A be a complex sentence of $\mathfrak{L}$, then we shall denote by $\alpha_i(A)$ the set of sentences that must be simultaneously asserted in an attack on A of the i-th kind. In most known games $\alpha_i(A)$ is empty or contains at most one sentence, e.g., $\alpha_1(A \rightarrow B) = \{A\}$, $\alpha_1(A \vee B) = \emptyset$, $\alpha_1((\forall x)A(x)) = \emptyset$; an attack on a "Shefferstroke-sentence" $A|B$ involves two sentences: $\alpha_1(A|B) = \{A, B\}$. By $\delta_i(A)$ we shall denote the set of relevant retorts, from which a player may pick one if he is attacked by an attack of the i-th kind on A, e.g., $\delta_1(A \rightarrow B) = \{B\}$, $\delta_1(A \vee B) = \{A, B\}$, $\delta_i((\forall x)A(x)) = \{A(a_i)\}$ (a_i the i-th individual constant), $\delta_1(A|B) = \emptyset$. A *logical rule* may now be defined as a set $\{\langle\alpha_i, \delta_i\rangle\}_{i\in\omega}$ of pairs of functions, such that for each complex sentence A of $\mathfrak{L}$ and for each natural number i $\alpha_i(A)$ and $\delta_i(A)$ are (possibly empty) sets of sentences of $\mathfrak{L}$.

Let $\mathbf{L} = \{\langle\alpha_i, \delta_i\rangle\}_{i\in\omega}$ be a logical rule; we shall then say that the move $\langle x, y\rangle$ *conforms to* $\mathbf{L}$ *as an attack of the i-th kind on* the complex sentence A, iff for some $n\langle A, n\rangle \in \mathbf{A}(\overline{P}_x)_x$ and the only differences between x and y—except that, by definition of move, $P_y \neq P_x$—concern:

(1) the set of assertions of P_x; here assertions (each with a label not occurring in x) corresponding to all the sentences in $\alpha_i(A)$—if any—are added to $\mathbf{A}(P_x)_x$ in order to obtain $\mathbf{A}(P_x)_y$.
(2) the set of defense sets of $\overline{P}_x$; here exactly one defense set containing assertions (each with a label not occurring in x) corresponding to all the sentences—if any—in $\delta_i(A)$ is added to obtain $\mathbf{D}(\overline{P}_x)_y$.
(3) the structural rule function.

In addition to attacks on complex sentences conforming to the logical rule, we may have *attacks on elementary sentences* and *defense moves*. A move $\langle x, y\rangle$ is said to constitute an *attack on the elementary sentence A*, iff A is elementary, and for some $n\,\langle A, n\rangle \in \mathbf{A}(\overline{P}_x)_x$, and the only differences between x and y—except that $P_y \neq P_x$—concern:

(1): the set of defense sets of $\overline{P}_x$; here a defense set $\{\langle A, m\rangle\}$ (where m does not occur in x) is added to obtain $\mathbf{D}(\overline{P}_x)_y$.
(2): the structural function.

A *defense move* $\langle x, y\rangle$ consists in adding exactly one assertion (with a label not occurring in x) equiform to an element of a defense set in $\mathbf{D}(P_x)_x$ to $\mathbf{A}(P_x)_x$ in order to form $\mathbf{A}(P_x)_y$; further, the structural rule function may undergo some changes in this case as well. It is not excluded that a move belongs to several of these types at once.

Using the vocabulary explained above we define a *material dialogue-game* as an ordered pair

$$\mathbf{G} = \langle \mathbf{L_G}, \mathbf{R_G}\rangle, \text{ such that}$$

(1) $\mathbf{L_G}$ is a logical rule.
(2) $\mathbf{R_G}$ is a set of moves.
(3) if $\langle x, y\rangle \in \mathbf{R_G}$, then either

(i) $\langle x, y\rangle$ is an attack on a complex sentence comforming to $\mathbf{L}_{\mathbf{G}}$,

or

(ii) $\langle x, y\rangle$ is an attack on an elementary sentence,

or

(iii) $\langle x, y\rangle$ is a defense move.

Positions not in the domain of $\mathbf{R}_{\mathbf{G}}$ will be called *end positions* of **G**. If x is an end position of **G** and a tournament of **G** ends at x, then P_x will be said to have *lost* the tournament and $\overline{P}_x$ will be said to have *won* the tournament. We do not admit draws. There is no further loss of generality: all games we are interested in can be brought into this form, if necessary by introducing some dummy moves.

Of course, given a dialogue-game **G**, a move may fall under one of (3), (i)-(iii) and yet fail to be a permitted move of **G**; any further restrictions put on $\mathbf{R}_{\mathbf{G}}$ may be said to belong to the structural rule. Such restrictions can be formulated in terms of the numbers assigned to the assertions and defense sets by the structural rule functions. For instance, if you want to allow three attacks on each assertion, and no more, the number $f_x(\langle A, n\rangle)$ may indicate how many attacks are still allowed; this number should be three at the introduction of $\langle A, n\rangle$ in the tournament and go down by one each time an attack is made on $\langle A, n\rangle$; $f_x(\langle A, n\rangle) = 0$ may indicate that the assertion is "dead", that is, that it can no longer be used. Or again, if you want the game to be over as soon as B asserts a true elementary sentence, all you have to do is this: consider the moves $\langle x, y\rangle$ that consist of the positing of a true elementary sentence by B, and permit only those that have a structural rule function f_y such that $f_y(Q) = 0$ for all assertions and defense sets Q in y, where 0 indicates that Q is dead. It should be noticed that the valuation of the elementary sentences of $\mathfrak{L}$ remains fixed during each tournament in **G**: words should not change their meaning in the course of a discussion.

2 *Conditions a reasonable material dialogue-game should fulfill* The definition of material dialogue-games given above is rather wide, and it is not to be expected that all games conforming to it be adequate with respect to a certain given model theory, or even that they be intuitively acceptable in any sense whatever. I will now discuss the conditions that dialogue-games must fulfill in order to be called reasonable.

First, the structural rules should not be too liberal. It seems reasonable not to allow any disputant to let the discussion drag on without an ending; therefore, stipulations to prevent this should be part of the structural rule. If all the tournaments of a game end after a finite number of moves, the game is called *locally finite*.[8] Thus, the first condition a reasonable dialogue-game must fulfill is that it be locally finite.

Second, the structural rules should not be too stringent. All problems statable in the language should be discussable. The structural rule should not prevent a dialogue on a certain problem to get started at all. By a *problem* I here mean an ordered quadruple $\langle L, R, \mathsf{v}, P\rangle$, where L stands for a set of sentences to be defended by B and R for a set of sentences to be defended by W, and where v is a valuation of the elementary sentences and P is the player making the first move. Let a position x be, by definition, a *starting position* of a material dialogue-game **G**, iff

(1) $\mathbf{D}(\mathrm{W})_x = \mathbf{D}(\mathrm{B})_x = \emptyset$
(2) if $\langle A, n\rangle \in \mathbf{A}(\overline{P}_x)_x$, and it is not the case both that A is elementary and that $\mathsf{v}_x(A) = \mathbf{T}$, then the structural rule of **G** permits P_x to attack A in the next move. If A is complex, attacks of any kind provided by $\mathbf{L_G}$ are permitted.

All reasonable material dialogue-games should provide starting positions for all problems, i.e., they must fulfill the following condition:

(1) For any problem $\langle L, R, \mathsf{v}, P\rangle$ there exists a starting position x of **G**, such that

(a) $A \in L (A \in R)$ iff there is a n such that $\langle A, n\rangle \in \mathbf{A}(\mathrm{B})_x (\langle A, n\rangle \in \mathbf{A}(\mathrm{W})_x)$
(b) $\mathsf{v}_x = \mathsf{v}$
(c) $P_x = P$

Such a position x will be called a *starting position in* **G** *for* $\langle L, R, \mathsf{v}, P\rangle$.

There is another respect in which the structural rule must not be too stringent: disputants should have a right of immediate response. If one of P's assertions has been attacked, P should be allowed to produce a relevant retort in the next move, and if $\overline{P}$ makes an assertion, P should be allowed to attack that assertion in the next move. However in some situations this right of immediate response should be cancelled in order not to clash with the condition of local finiteness. Hence, we put the following two conditions on a reasonable game **G**.

(2) If $\langle x, y\rangle \in \mathbf{R_G}$ and this move introduces a new defense set containing an assertion $\langle A, n\rangle$ into the set of defense sets of P_y, and it is not the case both that A is elementary and that $\mathsf{v}_x(A) = \mathbf{F}$, then P_y may defend himself in the next move by an assertion $\langle A, m\rangle$ (where m is a lable not in y).
(3) If $\langle x, y\rangle \in \mathbf{R_G}$ and this move introduces a new assertion $\langle A, n\rangle$ into the set of assertions of P_x, and it is not the case both that A is elementary and that $\mathsf{v}_x(A) = \mathbf{T}$, then P_y may attack A in the next move; in case A is complex, P_y may use any kind of attack provided by the logical rule $\mathbf{L_G}$.

A game in which the structural rule is not too stringent, that is, a game fulfilling conditions (1), (2), and (3), will be called *regular*.[9]

Thirdly, the *logical rule* should be in accord with a choice of logical constants in $\mathfrak{L}$ and with the meanings of these logical constants.

These constants and their meanings are given by a *model theory* $\mathfrak{M}$. There may be several such model theories for $\mathfrak{L}$. Each model theory provides models M, N, . . . based on interpretations of the non-logical constants of $\mathfrak{L}$. The internal structures of model theories and of models do not concern us here, but it will be assumed that we are dealing with *two-valued model theories*, i.e., that with each model M of a theory $\mathfrak{M}$ there is associated a valuation v_M of *all* the sentences of $\mathfrak{L}$, assigning truth or falsity to them:

$$v_M(A) = \mathbf{T} \text{ or } v_M(A) = \mathbf{F}.$$

A reasonable logical rule should, for each false sentence, provide a mode of attack that is both honest and ruthless, i.e., such that only true assertions need to be made in the attack and such that all permitted retorts are false; for a true sentence the logical rule should not provide such a mode of attack. A logical rule meeting this condition will be said to be *in accord with* the model theory concerned. More precisely, a logical rule $\mathbf{L} = \{\langle \alpha_i, \delta_i \rangle\}_{i \in \omega}$ is said to be *in accord with* a model theory $\mathfrak{M}$, iff for every model M of $\mathfrak{M}$ and for every complex sentence A of $\mathfrak{L}$:

(1) if $v_M(A) = \mathbf{F}$, then there is a natural number i and a pair $\langle \alpha_i, \delta_i \rangle \in \mathbf{L}$ such that

(a) for all $B \in \alpha_i(A)$: $v_M(B) = \mathbf{T}$
(b) for all $B \in \delta_i(A)$: $v_M(B) = \mathbf{F}$

(2) if $v_M(A) = \mathbf{T}$ and i is a natural number and $\langle \alpha_i, \delta_i \rangle \in \mathbf{L}$, then either there is a $B \in \alpha_i(A)$ such that $v_M(B) = \mathbf{F}$, or there is a $B \in \delta_i(A)$ such that $v_M(B) = \mathbf{T}$.

This concludes my discussion of the properties a reasonable material dialogue-game should have; the adequacy theorem can now be formulated.

3 *The adequacy theorem and its proof*

Adequacy theorem *Let* **G** *be a material dialogue-game (in a language* $\mathfrak{L}$*) that is both locally finite and regular. Let* $\mathfrak{M}$ *be a two-valued model theory for* $\mathfrak{L}$*, such that* $\mathbf{L}_\mathbf{G}$ *is in accord with* $\mathfrak{M}$*. If* M *is a model of* $\mathfrak{M}$ *and* x *is a starting position of* **G** *for* $\langle L, R, v, P \rangle$*, such that* v *and* v_M *agree on elementary sentences of* $\mathfrak{L}$*, then*

(a) *if for all assertions* $\langle A, n \rangle \in \mathbf{A}(P)_x$ *it holds that* $v_M(A) = \mathbf{T}$*, and if for some assertion* $\langle A, m \rangle \in \mathbf{A}(\overline{P})_x$ *it holds that* $v_M(A) = \mathbf{F}$*, then there is a winning strategy for* P *in* x*.*
(b) *if for all assertions* $\langle A, n \rangle \in \mathbf{A}(\overline{P})_x$ *it holds that* $v_M(A) = \mathbf{T}$*, then there is a winning strategy for* $\overline{P}$ *in* x*.*
(c) *if* $L = \emptyset$*, and* $R = \{A\}$*, and* $P =$ B *(in this case* B *may be called opponent and* W *proponent of the thesis* A *under the empty set of assumptions), then there is a winning strategy for* W *in* x *iff* $v_M(A) = \mathbf{T}$ *(otherwise, if* $v_M(A) = \mathbf{F}$*, there is a winning strategy for* B *in* x*).*

Proof: Part (c) of the theorem expresses what may be called "simple adequacy", and follows from (a) and (b) and the fact that not both W and B can have a winning strategy in the same position. To prove (a) and (b) we need to define the concept of a *P-favorable position.* It will be obvious from this definition (see below) that the positions described under (a) and (b) are P- and $\overline{P}$-favorable (with respect to M) respectively: condition (1) of the definition is fulfilled by hypothesis, and condition (3) is trivial, since $\bigcup(\mathbf{D}(P)_x) = \bigcup(\mathbf{D}(\overline{P})_x) = \emptyset$, as is condition (4) in the case of (b), since $P_x \neq \overline{P}$; condition (2) is fulfilled in virtue of the text under (a) and (b) above, condition (4)—in the case of (a)—is fulfilled because it is given that for some assertion $\langle A, m\rangle \in A(\overline{P})_x$ it holds that $v_M(A) = \mathbf{F}$ and because attacks on this assertion are permitted, since x is a starting position. The theorem follows then from the lemma, stated below, that a player P has a winning strategy for any P-favorable-position. Q.E.D.

Definition of P-favorable (with respect to M and **G**): A position x will be said to be *P-favorable with respect to a two-valued model* M *and a material dialogue-game* **G**, iff

(1) v_M and v_x agree on elementary sentences.
(2) if $\langle A, n\rangle \in \mathbf{A}(P)_x$, then $v_M(A) = \mathbf{T}$.
(3) if $\langle A, n\rangle \in \bigcup(\mathbf{D}(\overline{P})_x)$, then $v_M(A) = \mathbf{F}$.
(4) if $P_x = P$, then either there is an $\langle A, n\rangle \in \mathbf{A}(\overline{P})_x$ such that $v_M(A) = \mathbf{F}$ and attacks of all kinds on A are permitted by the structural rule of **G** as P's next move, or there is an $\langle A, n\rangle \in \bigcup(\mathbf{D}(P)_x)$, such that $v_M(A) = \mathbf{T}$ and defense by means of an assertion $\langle A, m\rangle$ is permitted by the structural rule of **G** as P's next move.

Lemma *Let* **G**, $\mathfrak{M}$, *and* M *fulfill the conditions of the Theorem. If x is P-favorable with respect to* M *and* **G**, *then there is a winning strategy for P in x.*

Proof: The set of P-favorable positions with respect to M constitutes a *pseudo-cycle* for P,[10] that is: once a tournament has moved into a P-favorable position (with respect to M), P can keep it that way and $\overline{P}$ cannot make the situation not P-favorable. The proof of this proceeds by cases:

A: $P_x = P$

A1: There is an assertion $\langle A, n\rangle \in \mathbf{A}(\overline{P})_x$, such that $v_M(A) = \mathbf{F}$ and attacks by P on A are permitted.

A1.1: A is elementary: If P attacks A in a move $\langle x, y\rangle$, then y will be P-favorable, since only a set $\{\langle A, m\rangle\}$ will have been added to $\mathbf{D}(\overline{P}_x)_x$ in order to form $\mathbf{D}(\overline{P}_x)_y$; condition (3) of the definition of "P-favorable" will be satisfied, for $v_M(A) = \mathbf{F}$; condition (4) will be satisfied trivially.

A1.2: A is complex. Since $\mathbf{L_G} = \{\langle \alpha_i, \delta_i\rangle\}_{i\in\omega}$ is in accord with $\mathfrak{M}$, and M is a model of $\mathfrak{M}$, there is a permitted attack move $\langle x, y\rangle$ and a natural number i, such that $\langle x, y\rangle$ conforms to $\mathbf{L_G}$ as an attack of the i-th kind on A, and such that for all $B \in \alpha_i(A)$: $v_M(B) = \mathbf{T}$ and for all $B \in \delta_i(A)$: $v_M(B) = \mathbf{F}$. Such a y is P-favorable again.

A2: There is no such assertion. Then, by condition (4), there must be an $\langle A, n\rangle \in \bigcup(\mathbf{D}(P_x)$ such that $v_M(A) = \mathbf{T}$ and a defense move $\langle x, y\rangle$ is permitted consisting of adding an assertion $\langle A, m\rangle$ to $\mathbf{A}(P_x)_x$. Obviously, y is P-favorable again.

B: $P_x = \overline{P}$

B1: $\overline{P}$ cannot move and loses the tournament.

B2: $\overline{P}$ can move. Say he moves by $\langle x, y\rangle$. Then $\langle x, y\rangle$ must be an attack on a true sentence or a defense move using a false sentence. It is trivial that y fulfills the conditions (1) through (3) of the definition of "P-favorable".

B2.1: $\langle x, y\rangle$ is an attack on an elementary sentence A. $v_M(A) = \mathbf{T}$. This will give P a defense set $\{\langle A, m\rangle\}$; since **G** is *regular* he may use this defense set in his next move, hence condition (4) is fulfilled.

B2.2: $\langle x, y\rangle$ is an attack on a complex sentence A. $v_M(A) = \mathbf{T}$. Since this attack must be conforming to the logical rule, it will consist of adding assertions corresponding to elements of a set $\alpha_i(A)$ to $\mathbf{A}(P_x)_x$ and adding a defense set containing assertions corresponding to the elements of a set $\delta_i(A)$ to $\mathbf{D}(\overline{P}_x)_x$. $\mathbf{L_G}$ is in accord with $\mathfrak{M}$, hence either there is a $B \epsilon \alpha_i(A)$ such that $v_M(B) = \mathbf{F}$, and P may (by regularity) attack the corresponding assertion in the next move, or there is a $B \epsilon \delta_i(A)$ such that $v_M(B) = \mathbf{T}$, and P may (by regularity) use B in a defense move. In either case condition (4) has been fulfilled by y.

B2.3: $\langle x, y\rangle$ is a defense move using a false sentence. Then $\overline{P}$ has to add a false assertion to $\mathbf{A}(\overline{P})_x$ in order to obtain $\mathbf{A}(\overline{P})_y$. By regularity, P may attack that assertion in the next move, hence y fulfills condition (4) in this case as well.

Thus the set of P-favorable positions with respect to M constitutes a pseudo-cycle for P. Moreover, in virtue of condition (4), this set does not contain any end positions z such that $P_z = P$, hence no end positions with loss for P. Since **G** is *locally finite* it follows that there is a winning strategy for P in every position of the set. (Indeed this strategy has been described, implicity, in this proof and boils down to attacking falsehoods and telling the truth). Q.E.D.

4 *Final remarks* The material games in [3] and [5], referred to in footnote 1, are special cases to which the adequacy theorem applies. The same holds for the material games in [4], if they are modified as follows:

(1) For the opponent a rule for winning the game should be instituted that is exactly analogous to that already present for the proponent.[11]
(2) In all games (except the *streng-konstruktive* one) *Wiederholungs-schranke*, present in the first edition of [4], should be restored.

Since *regularity* is a rather weak condition to be set upon the structural rules, we may conclude that the particular form of these rules is largely irrelevant for the existence of winning strategies in material dialogue-games—this in contradistinction to the situation in formal dialogue-games. There seems to be no smooth connection between the material and the formal games.[12]

It remains an open problem if, and how, the adequacy theorem can be extended to cover many-valued models.[13]

NOTES

1. P. Lorenzen in Kamlah and Lorenzen [4], Ch. VII, esp. p. 221 (Ch. VII is Lorenzen's); K. Lorenz in [5] (*faktische Dialogspiele*, p. X); K. J. J. Hintikka in [3].

2. P. Lorenzen in [4], p. 219: "Bei Beschräkung auf Junktoren und auf wahrheitsdefinite Primaussagen gilt darüber hinaus, dasz jede tautologisch-wahre Aussage stets konstruktiv dialogisch verteidigbar ist". K. Lorenz in [5], p. 44 (quoted in note (3)), also in [6], p. 92. K. J. J. Hintikka in [3], pp. 68, 69: "The following observation has struck me as being especially suggestive: There is a very close connection between the concept of a truth-value of a sentence and the game-theoretical concept of the value of the correlated game. If I have a winning strategy the value of the game is the payoff of winning, i.e., the "value" of winning the game. This is also precisely the case in which the sentence is true. Hence the payoff of winning as a value of the game can be identified with the truth-value "true" of the sentence, and correspondingly for falsity".

3. Lorenz in [5], p. 40 ff. On p. 44 Lorenz remarks: "Der Halbformalismus Ω_r ist unter dem Namen semantischer Halbformalismus bereits bekannt. Er definiert nichts anderes als die übliche klassische Zuordnung der beide Wahrheitswerte w_r und f_r zu logisch zusammengesetzten Aussagen C auf Grund ihrer Zuordnung zu den direkten Teilaussagen von C". *Hauptsatz 1* says that there is a winning strategy for the proponent (opponent) of a sentence A in a *reines Dialogspiel mit entscheidbarer Basis* if $\prec A$ ($A \prec$) is provable in Ω_r. Thus the existence of winning strategies is connected with provability in Ω_r, and Ω_r is connected with semantics.

4. It is possible to define dialogue-games with either assertions or sets only and to consider an assertion as a special kind of set or the other way around. This has been done by Drieschner in [2].

5. This has been remarked by F. van Dun in [8], p. 107.

6. Stegmüller, [7], p. 84: "We distinguish between a *game* as a type and a 'concrete performance' of the game or a *tournament*". My use of the words "game" and "tournament" agrees with Stegmüller's.

7. For structural vs. logical rules cf. Lorenz [5], p. 15, 20, and Stegmüller [7] p. 85 ff.

8. Locally finite = *localement fini* = *partienendlich*. *Cf.* Berge [1] p. 24.

9. My use of the word "regular" partially conforms to that of K. Lorenz in [5], p. 20.

10. *Cf.* Berge [1], p. 20.

11. Lorenzen in [4], p. 213.

12. Lorenz [5], p. XI: "Die reine Logik ist leer". That is: Lorenz's simplest type of material dialogue-games (*reine Dialogspiele*) does not lead in any straightforward way to equally simple formal games.

13. (*Added 1977*); Connections between dialogue-theory and many-valued logics have been studied by R. Giles, e.g. in [9], esp. p. 411.

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