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ON PROPOSITIONS

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Traditionally, propositions have been assigned at least three roles: meanings of sentences, objects of propositional attitudes, and bearers of truth values. We are not certain that there is any one sort of entity which can play all three roles; in particular it is not clear to us that the identity conditions satisfied by entities playing one of these roles must coincide with the identity conditions satisfied by entities playing another of these roles. In recent years it has become fashionable to construe propositions as functions of a certain sort, namely as functions taking truth values as values. We regard this view of propositions as a version of (at least) the traditional view of propositions as bearers of truth values; on this view, a proposition bears the value truth if and only if its value as a function is truth. Our aim is to specify what sort of functions propositions so viewed are, and in particular to specify identity conditions for propositions so viewed.

Before presenting our view, we state two reasons for identifying propositions with functions. First, those who object to propositions often do so on the grounds that their identity conditions are not clear. But the identity conditions for functions are clear: functions are identical if and only if they have the same values for the same arguments. So viewing propositions as functions tells us at least what sort of identity conditions to seek for propositions. Second, the identification of propositions with functions seems intuitively natural. In general, a function assigns an entity in its range to each entity in its domain. Propositions as bearers of truth values seem intuitively to fit this characterization; they select a truth value given how things are. Roughly and intuitively put then, a proposition is a function which assigns the value truth to a sentence if things are as the sentence says, and which assigns falsity otherwise. Thus the thesis that propositions are functions into truth values seems both theoretically well-motivated and intuitively plausible.

Usually those who identify propositions as bearers of truth values with functions take them as functions from possible worlds to truth values. Intuitively this is to identify a proposition with the function whose value is

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truth for all and only the possible worlds in which the proposition bears the value truth. However, it seems to us that this view yields wrong identity conditions for propositions. For since necessary truths are true in all possible worlds and since there is exactly one function whose value is truth for all possible worlds, this view has the consequence that there is exactly one necessarily true proposition. This consequence seems counter-intuitive to us; it seems clear to us that the prime number theorem and the law of quadratic reciprocity are distinct propositions which necessarily bear the value truth. Similarly, since necessarily equivalent propositions are true in exactly the same possible worlds and since for each set of possible worlds there is exactly one function whose value is truth for exactly the worlds in that set, the view in question has the consequence that necessarily equivalent propositions are identical. This consequence also seems counter-intuitive to us. For example, if p is a contingently true sentence and q is a necessarily true sentence, then p and the conjunction of p and q are necessarily equivalent contingently true sentences which will in general express distinct propositions. Thus, when we consider the modalities with which propositions bear truth values, it seems to us that the now classical construction of propositions as functions from possible worlds to truth values does not discriminate finely enough among propositions. Our aim is to make such finer discriminations possible.

To this end we return to the intuitive reason for identifying propositions with functions in the first place. A proposition is a function which assigns truth to a sentence if and only if things stand as the sentence says they stand. To formalize this intuition we introduce states of affairs or truth conditions as fixing what sentences say. We view states of affairs as segments of possible worlds, or, what comes to the same thing, we view possible worlds as divided up into states of affairs. A sentence is true in a world if and only if the state of affairs assigned to that sentence in that world obtains in that world. Sentences express the same proposition if and only if they are assigned the same truth conditions in all possible worlds. This is the basis for our finer discrimination among propositions, our more restrictive identity conditions for propositions. We also think our view fits better than the classical view with the following train of thought: When we contemplate some counterfactual situation, that is, when we suppose true some sentence not actually true, we do not seem to conjure up a whole possible world in which that sentence is true. Rather, we contemplate a state of affairs such that if it obtained our sentence would be true; precisely how things stand in the rest of any world in which the contemplated state of affairs obtains seems in general irrelevant so long as there is at least some full possible world in which that state of affairs obtains.

More precisely, our aim is to sketch a simple modification of possible worlds semantics equal in power to conventional ones but respecting the intuitions about identity conditions for propositions mentioned above. We construct sets $\mathscr{A}$ of truth conditions or states of affairs such that for each sentence φ in the languages L we consider and each possible world w, there is a unique a in $\mathcal{A}$ and w such that a obtains in w if and only if φ is true in w. Our construction justifies us in introducing a function f whose value for each φ and w is the required truth condition. We then require that sentences φ and ψ express the same proposition if and only if for all worlds w, $f(\varphi, w) = f(\psi, w)$. As shall emerge below, our identity condition for propositions allows considerable flexibility in constructing $\mathcal{A}$. At first blush it may seem odd that we permit a sentence to have distinct truth conditions in distinct possible worlds; perhaps it will be thought that we should assign truth conditions to sentences alone rather than to pairs consisting of a sentence and a world. But it seems to us that a universally quantified sentence of the form $(x)Fx$ should be true in a possible world w if and only if F is true of all the objects that are actual in w. Since different possible worlds need not have exactly the same populations, it seems preferable to us to permit a single sentence's truth condition to vary from world to world. We think similar remarks apply to sentences in which occur singular terms which are not rigid designators, and perhaps other sentences.

We now give a syntax for our first language **L**. **L** has a denumerable infinity of sentence letters

$$P_1, P_2, \ldots .$$

Atm is the set of all sentence letters of **L**. All sentence letters are sentences. If φ and ψ are sentences, so are $(\varphi \,\&\, \psi)$, $\sim\varphi$ and $\Box\varphi$. These are all the sentences of **L**. So much for syntax. Turning to semantics, we first sketch for comparison a conventional possible worlds semantics for **L**. Let $\mathcal{W}$ be a non-empty set; intuitively, $\mathcal{W}$ is a set of possible worlds. We take the numbers 0 and 1 to be the truth values truth and falsity respectively. Let V be a function from Atm $\times\, \mathcal{W}$ to $\{0, 1\}$; intuitively, $\mathrm{V}(P_i, w) = 0$ if and only if P_i is true in w. The pair $\mathfrak{U} = \langle \mathcal{W}, \mathrm{V} \rangle$ is a world structure. The truth of a sentence φ of **L** at a world w in a structure $\mathfrak{U}$ is defined by induction on the complexity of φ:

(1) P_i is true at w in $\mathfrak{U}$ if and only if $\mathrm{V}(P_i, w) = 0$;
(2) $(\varphi \,\&\, \psi)$ is true at w in $\mathfrak{U}$ if and only if φ is true at w in $\mathfrak{U}$ and ψ is true at w in $\mathfrak{U}$;
(3) $\sim\varphi$ is true at w in $\mathfrak{U}$ if and only if φ is not true at w in $\mathfrak{U}$;
(4) $\Box\varphi$ is true at w in $\mathfrak{U}$ if and only if for every $u \in \mathcal{W}$, φ is true at u in $\mathfrak{U}$.

A sentence φ of **L** is valid if and only if for every structure $\mathfrak{U}$ and world w, φ is true at w in $\mathfrak{U}$. As is well known, the valid sentences of **L** are exactly the sentences of **L** which are theorems of S5.

We now sketch an alternative semantics for **L**. As before, let $\mathcal{W}$ be a non-empty set; intuitively, $\mathcal{W}$ is still a set of possible worlds. We also assume a non-empty set $\mathcal{A}$; intuitively, $\mathcal{A}$ is a set of states of affairs. We think of all possible worlds as sharing the same states of affairs; different worlds are distinguished by which states of affairs obtain in them; some states of affairs obtain in all worlds, others obtain in none, and still others obtain in some but not in all worlds. Let f be a function from Atm $\times \mathcal{W}$ into $\mathcal{A}$; intuitively, $f(P_i, w)$ is the truth condition for P_i in w. Let g be a function from $\mathcal{A} \times \mathcal{W}$ into $\{0, 1\}$; intuitively, $g(a, w) = 0$ if and only if a obtains in w. A quadruple $\mathfrak{I} = \langle \mathcal{W}, \mathcal{A}, f, g \rangle$ is an interpretation: the truth of a sentence φ of L at a world w under an interpretation $\mathfrak{I}$ is defined by induction on the complexity of φ:

(1) P_i is true at w under $\mathfrak{I}$ if and only if $g(f(P_i, w), w) = 0$;
(2) $(\varphi \,\&\, \psi)$ is true at w under $\mathfrak{I}$ if and only if φ is true at w under $\mathfrak{I}$ and ψ is true at w under $\mathfrak{I}$;
(3) $\sim\varphi$ is true at w under $\mathfrak{I}$ if and only if φ is not true at w under $\mathfrak{I}$;
(4) $\Box\varphi$ is true at w under $\mathfrak{I}$ if and only if for each $u \in \mathcal{W}$, φ is true at u under $\mathfrak{I}$.

We now prove that our alternative semantics is equal in strength to the conventional semantics. First we show that for any structure $\mathfrak{U} = \langle \mathcal{W}, \mathsf{V} \rangle$, there is an interpretation $\mathfrak{I}$ such that for each $w \in \mathcal{W}$ and each positive integer i, P_i is true at w in $\mathfrak{U}$ if and only if P_i is true at w under $\mathfrak{I}$. Keep $\mathcal{W}$ fixed. Let $\mathcal{A}$ be the set of positive integers, let $f(P_i, w) = i$ and let $g(i, w) = \mathsf{V}(P_i, w)$. Then $\mathfrak{I} = \langle \mathcal{W}, \mathcal{A}, f, g \rangle$ is the required interpretation. Conversely, for each interpretation $\mathfrak{I} = \langle \mathcal{W}, \mathcal{A}, f, g \rangle$ there is a structure $\mathfrak{U}$ such that for each $w \in \mathcal{W}$ and each positive integer i, P_i is true at w under $\mathfrak{I}$ if and only if P_i is true at w in $\mathfrak{U}$. Again keep $\mathcal{W}$ fixed and let $\mathsf{V}(P_i, w) = g(f(P_i, w), w)$. Then $\mathfrak{U} = \langle \mathcal{W}, \mathsf{V} \rangle$ is the required structure. By a trivial induction it follows that for each structure $\mathfrak{U}$ there is an interpretation $\mathfrak{I}$ such that for any $w \in \mathcal{W}$ and any sentence φ of L, φ is true at w in $\mathfrak{U}$ if and only if φ is true at w under $\mathfrak{I}$. Equally trivially, for each interpretation $\mathfrak{I}$ there is a structure $\mathfrak{U}$ such that for any $w \in \mathcal{W}$ and any sentence φ of L, φ is true at w under $\mathfrak{I}$ if and only if φ is true at w in $\mathfrak{U}$. Thus a sentence φ of L is valid (or a theorem of S5) just in case for each interpretation $\mathfrak{I}$ and world $w \in \mathcal{W}$, φ is true at w under $\mathfrak{I}$. This completes our proof that our alternative semantics is as strong as the conventional one. It is obvious that this result can be extended to conventional semantics for L with alternativeness relations.

Returning now to propositions, we must extend f so that $f(\varphi, w)$ is defined for each sentence φ of L. In effect we have already assumed that for each sentence letter P_i and each w there is a unique $a \in \mathcal{A}$ such that a obtains in w if and only if P_i is true in w. To extend f we extend our assumption. Thus, suppose that:

(1) for every $a \in \mathcal{A}$ and $w \in \mathcal{W}$ there is a unique $b \in \mathcal{A}$ such that b obtains in w if and only if a does not obtain in w;
(2) for every a and b in $\mathcal{A}$ and $w \in \mathcal{W}$ there is a unique $c \in \mathcal{A}$ such that c obtains in w if and only if a obtains in w and b obtains in w;
(3) for every $a \in \mathcal{A}$ and $w \in \mathcal{W}$ there is a unique $b \in \mathcal{A}$ such that b obtains in w if and only if for every $u \in \mathcal{W}$, a obtains in u.

It is then evident that f can be extended in such a way that for any sentence φ of L and any $w \in \mathcal{W}$, $f(\varphi, w)$ is the unique element of $\mathcal{A}$ which obtains in w if and only if φ is true in w. For each sentence φ of L, let f_φ: $\mathcal{W} \rightarrow \mathcal{A}$ be defined thus: for all $w \in \mathcal{W}$, $f_\varphi(w) = f(\varphi, w)$. For each sentence φ of L, let g_φ: $\mathcal{A} \times \mathcal{W} \rightarrow \{0, 1\}$ be defined thus: for any $a \in \mathcal{A}$ and $w \in \mathcal{W}$, $g_\varphi(a, w) = 0$ if and only if $g(a, w) = 0$ and $f_\varphi(w) = a$; otherwise, $g_\varphi(a, w) = 1$. Then let $\mathsf{Prop}(\varphi)$, the proposition determined by a sentence φ, be g_φ. Thus $\mathsf{Prop}(\varphi) = \mathsf{Prop}(\psi)$ if and only if for all $w \in \mathcal{W}, f(\varphi, w) = f(\psi, w)$, as was promised above. Let π be the set of all functions from $\mathcal{A} \times \mathcal{W}$ into $\{0, 1\}$; π is the set of all propositions. Clearly, for any sentence φ of L, $\mathsf{Prop}(\varphi) \in \pi$, as seems desirable.

It should be apparent that our construction of propositions is independent of the particular ways $\mathcal{A}$ might be constructed. To illustrate this fact and to take some of the sting out of the apparent ontological excesses of our assumptions about $\mathcal{A}$, we give two examples. Suppose every positive integer greater than one is in $\mathcal{A}$ and that whenever a and b are in $\mathcal{A}$ so are $\langle a, b\rangle$, $\langle 0, a\rangle$, and $\langle 1, a\rangle$. These being the only elements of $\mathcal{A}$, $\mathcal{A}$ is ontologically respectable. Suppose we have two possible worlds, w_1 and w_2. For each positive integer n and $k = 1, 2$ let $f(P_n, w_k) = (n \times k) + 1$. Let

$$f((\varphi \,\&\, \psi), w_1) = \langle f(\varphi, w_1), f(\psi, w_1)\rangle, f((\varphi \,\&\, \psi), w_2) = \langle f(\varphi, w_2), f(\psi, w_2)\rangle,$$
$$f({\sim}\varphi, w_1) = \langle 0, f(\varphi, w_1)\rangle, f({\sim}\varphi, w_2) = \langle 0, f(\varphi, w_2)\rangle, f(\Box\varphi, w_1) = \langle 1, f(\varphi, w_1)\rangle$$
and $f(\Box\varphi, w_2) = \langle 1, f(\varphi, w_2)\rangle$.

If n is a positive integer greater than one, let $g(n, w_2) = 0$ if and only if n is odd and let $g(n, w_2) = 0$ if and only if n is even. For $k = 1, 2$ and any pair $\langle a, b\rangle \in \mathcal{A}$ with $a \neq 1$ and $a \neq 0$, $g(\langle a, b\rangle, w_k) = 0$ if and only if $g(a, w_k) = g(b, w_k) = 0$; if $a = 0$, $g(\langle a, b\rangle, w_k) = 0$ if and only if $g(b, w_k) = 1$; if $a = 1$, $g(\langle a, b\rangle, w_k) = 0$ if and only if $g(b, w_1) = g(b, w_2) = 0$. It is then clear that ${\sim}({\sim}P_1 \,\&\, {\sim}P_2)$ and ${\sim}({\sim}P_2 \,\&\, {\sim}P_3)$ are true in both worlds though they determine different propositions. Likewise, P_1 and P_3 are true in the same worlds though they also determine different propositions. Of course, in this example distinct sentences of L always determine distinct propositions. In this respect and in our use of indices to satisfy the constraints on $\mathcal{A}$, we discriminate propositions about as finely as they would be using Carnap's device of intensional isomorphism.

We next give an example which discriminates propositions less finely than above but more finely than does the classical view of propositions as functions from possible worlds to truth values. To give this example we extend our construction to quantified languages. Our new language L contains an infinity of variables: $x_1, x_2, \ldots$; and of constants $a_1, a_2, \ldots$; and for each positive n and j a predicate F_j^n. We shall write the predicate F_1^1 as A; we intend this predicate to be interpreted as "is actual". If F_j^n is a predicate and $y_1, \ldots, y_n$ are variables, $F_j^n(y_1, \ldots, y_n)$ is a formula. If φ and ψ are formulae and y is a variable, then $(\varphi \,\&\, \psi)$, ${\sim}\varphi$, $\Box\varphi$, and $(y)\varphi$ are formulae. These are the only formulae. Note that no constants occur in a formula. A sentence of L is the result of replacing all free occurrences of variables in a formula by constants (where distinct occurrences of the same variable are replaced by the same constant) or a formula in which no variable occurs free; in the second case, the sentence is said to be constant free. If $\varphi(b_1, \ldots, b_k)$ is a sentence containing exactly the constants $b_1, \ldots, b_k$, then its corresponding formula $\varphi(x_1, \ldots, x_k)$ is got thus: rewrite the bound variables in $\varphi(b_1, \ldots, b_k)$ from left to right using the earliest of $x_{k+1}, x_{k+2}, \ldots$ compatible with avoiding collisions of bound variables and then substitute $x_1, \ldots, x_k$ for $b_1, \ldots, b_k$ respectively in alphabetical order.

Turning to semantics, let $\mathcal{W}$ be a non-empty set; $\mathcal{W}$ is still the set of possible worlds. Let $\mathbf{D}$ be a non-empty set; intuitively, $\mathbf{D}$ is the set of all possible individuals (naturally, all actual individuals are possible). We think of all worlds as sharing the same possible individuals; that is, an individual is possible in one world only if it is possible in all. Let $\mathfrak{a}$ be a function from $\mathcal{W}$ to the power set of $\mathbf{D}$; intuitively, $\mathfrak{a}(w)$ is the set of possible individuals which are actual in w. We should probably wish that $\mathbf{D} = \bigcup_{u \in \mathcal{W}} \mathfrak{a}(w)$. If we wish to adopt a counterpart theory, we should require that for distinct w and u in $\mathcal{W}$, $\mathfrak{a}(w)$ and $\mathfrak{a}(u)$ are disjoint; otherwise we can let them meet. We make the second choice. Let Σ be the set of all sequences of elements of $\mathbf{D}$, that is, the set of all functions from positive integers to $\mathbf{D}$. As shall emerge, this definition represents a choice to quantify over possible individuals. We regard this choice as an evil necessary to smoothing the truth definition; we shall use the predicate $\mathbf{A}$ and the sets $\mathfrak{a}(w)$ to remedy this evil. Let $\mathfrak{i}$ be a function such that for each constant a_i, each predicate F_j^n other than $\mathbf{A}$ and each w, $\mathfrak{i}(a_i, w) \in \mathbf{D}$ and $\mathfrak{i}(F_j^n, w) \subseteq \mathbf{D}^n$; we require that $\mathfrak{i}(\mathbf{A}, w) = \mathfrak{a}(w)$ for all w. Let $\mathcal{A}$ be the set of all $n + 1$-tuples $\langle z_1, \ldots, z_n, S\rangle$ such that $z_1, \ldots, z_n \in \mathbf{D}$ and $S \subseteq \Sigma$. Clearly $\mathcal{A}$ is no more ontologically suspect than $\mathcal{W}$ and $\mathbf{D}$ unless one is backward enough to have scruples about elementary set theory; as it were, $\mathcal{A}$ is a logical construction from $\mathcal{W}$ and $\mathbf{D}$. It is also clear that our ontology of states of affairs can be accepted by those who object to possible but not actual worlds and to possible but not actual individuals simply by letting $\mathcal{W}$ be the unit set of the actual world and letting $\mathbf{D}$ be the set of all actual individuals (pretending that the latter set exists—this is a problem we all share).

To relate the syntax and semantics of $\mathbf{L}$, we define an auxiliary function $\mathbf{E}$. Given $\mathfrak{i}$ and a $w \in \mathcal{W}$, $\mathbf{E}$ assigns each formula of $\mathbf{L}$ a subset of Σ as its extension. $\mathbf{E}$ is defined by induction on the complexity of a formula:

(1a) For predicates F_j^n other than $\mathbf{A}$, $\mathbf{E}(F_j^n(x_{k_1}, \ldots, x_{k_n}), \mathfrak{i}, w) = \{\sigma \in \Sigma \mid \langle\sigma(k_1), \ldots, \sigma(k_n)\rangle \in \mathfrak{i}(F_j^n, w)\}$;

(1b) $\mathbf{E}(\mathbf{A}(x_k), \mathfrak{i}, w) = \{\sigma \in \Sigma \mid \sigma(k) \in \sigma(w)\}$;

(2) $\mathbf{E}((\varphi \mathbin{\&} \psi), \mathfrak{i}, w) = \mathbf{E}(\varphi, \mathfrak{i}, w) \cap \mathbf{E}(\psi, \mathfrak{i}, w)$;

(3) $\mathbf{E}(\Box \varphi, \mathfrak{i}, w) = \bigcap_{u \in \mathcal{W}} \mathbf{E}(\varphi, \mathfrak{i}, u)$;

(4) $\mathbf{E}(\sim \varphi, \mathfrak{i}, w) = \Sigma - \mathbf{E}(\varphi, \mathfrak{i}, w)$;

(5) $\mathbf{E}((x_k)\varphi, \mathfrak{i}, w) = \{\sigma \in \Sigma \mid (\forall \tau \in \Sigma)(\forall j \neq k)\,[\tau(j) = \sigma(j) \;.\!\supset\!.\; \tau \in \mathbf{E}(\varphi, \mathfrak{i}, w)]\}$.

The truth condition function f can now be defined explicitly. Given i, a $w \in \mathcal{W}$ and a constant free sentence φ of $\mathbf{L}$, $f(\varphi, i, w) = \mathbf{E}(\varphi, i, w)$. For a sentence $\varphi(a_{k_1}, \ldots, a_{k_n})$ with constants, $f(\varphi(a_{k_1}, \ldots, a_{k_n}), i, w)$ is $\langle i(a_{k_1}, w), \ldots, i(a_{k_n}, w), \mathbf{E}(\varphi(x_1, \ldots, x_n), i, w)\rangle$ where $\varphi(x_1, \ldots, x_n)$ is the formula corresponding to $\varphi(a_{k_1}, \ldots, a_{k_n})$. Note that for any sentence φ of $\mathbf{L}$, and i and any $w \in \mathcal{W}$, $f(\varphi, i, w) \in \mathcal{A}$ and that $\mathcal{A}$ and f satisfy the extended assumptions above. The function g: $\mathcal{A} \times \mathcal{W} \rightarrow \{0, 1\}$ is defined thus: for any $\langle z_1, \ldots, z_n, S\rangle \in \mathcal{A}$ and any $w \in \mathcal{W}$, if $n > 0$ $g(\langle z_1, \ldots, z_n, S\rangle, w) = 0$ if and only if for some $\sigma \in S$ and each $j = 1, \ldots, n$, $\sigma(j) = z_j$; if $n = 0$, $g(S, w) = 0$ if and only if S is non-empty; otherwise, $g(\langle z_1, \ldots, z_n, S\rangle, w) = 1$. In the interesting cases, this means that if φ is a constant free sentence of $\mathbf{L}$, $g(f(\varphi, i, w), w) = 0$ if and only if $\mathbf{E}(\varphi, i, w)$ is non-empty, and for a sentence $\varphi(a_{k_1}, \ldots, a_{k_n})$ with constants, $g(f(\varphi(a_{k_1}, \ldots, a_{k_n}), i, w), w) = 0$ if and only if for some $\sigma \in \mathbf{E}(\varphi(x_1, \ldots, x_n), i, w)$ and each $j = 1, \ldots, n$, $\sigma(j) = i(a_k\,, w)$. We can now take an octuple $\langle \mathcal{W}, \mathbf{D}, \sigma, i, \mathcal{A}, \mathbf{E}, f, g\rangle$ as an interpretation of our quantificational language $\mathbf{L}$.

It is not difficult to show that for each constant free sentence φ of $\mathbf{L}$, if $\mathbf{E}(\varphi, i, w)$ is non-empty, it is Σ. This result, usual for truth definitions, assures that our truth function g is well behaved. It was for this reason that we quantified over all of $\mathbf{D}$. But above we argued that a sentence should be allowed to have different truth conditions in different possible worlds on the grounds that a universally quantified sentence of the form $(x)Fx$ should be true at a possible world w if and only if the predicate F is true of all individuals actual in w. To recover and respect this intuition, we introduce two definitional abbreviations; let

$$(\forall . x)\varphi$$

be short for

$$(x)(\mathbf{A}(x) \rightarrow \varphi)$$

and let

$$(\exists . x)\varphi$$

be short for

$$(\exists x)(\mathbf{A}(x) \;\&\; \varphi)$$

(using conventional short hand). It seems to us that if we then say that a sentence of the form $(x)Fx$ is true at a world w if and only if for a suitable i and a corresponding expression φ of $\mathbf{L}$, $g(f((\forall . x)\varphi, i, w), w) = 0$, then we have respected the above intuition as well as can be expected.

Turning again to propositions, suppose that the function i has been fixed. For each sentence φ of $\mathbf{L}$, let f_φ: $\mathcal{W} \rightarrow \mathcal{A}$ be defined thus: for any $w \in \mathcal{W}$, $f_\varphi(w) = f(\varphi, i, w)$. Let g_φ: $\mathcal{A} \times \mathcal{W} \rightarrow \{0, 1\}$ be defined thus: for any $a \in \mathcal{A}$ and $w \in \mathcal{W}$, $g_\varphi(a, w) = 0$ if and only if $g(a, w) = 0$ and $f_\varphi(w) = a$; otherwise $g_\varphi(a, w) = 1$. As before let $\mathrm{Prop}(\varphi)$, the proposition determined by φ, be g_φ. Then $\mathrm{Prop}(\varphi) = \mathrm{Prop}(\psi)$ if and only if for all $w \in \mathcal{W}$, $f(\varphi, i, w) = f(\psi, i, w)$. Let π be the set of all functions from $\mathcal{A} \times \mathcal{W}$ into $\{0, 1\}$. π is the set of all propositions. For any sentence φ of $\mathbf{L}$, $\mathsf{Prop}(\varphi) \in \pi$. Thus on our view propositions are functions from states of affairs and possible worlds to truth values, while on the classical view they are functions from possible worlds to truth values. But if we are to assign propositions to sentences on the classical view, then in a way that view is a special case of ours. To see this, let $\mathcal{A}$ be $\mathcal{W}$, let f_φ: $\mathcal{W} \to \mathcal{A}$ be the identity function $f_\varphi(w) = w$ and let g_φ: $\mathcal{A} \times \mathcal{W} \to \{0, 1\}$ be defined thus: for any $a \in \mathcal{A}$ and $w \in \mathcal{W}$, $g_\varphi(a, w) = 0$ if φ is true in w; otherwise, $g_\varphi(a, w) = 1$. Let $\mathsf{Prop}(\varphi)$ be g_φ and let π be the set of all functions from $\mathcal{A} \times \mathcal{W}$ into $\{0, 1\}$. Then for any φ, $\mathsf{Prop}(\varphi) \in \pi$. As it were, on the classical view each world is just one big truth condition and all sentences have the same truth conditions. We think our construction of propositions allows for a more intuitive individuation of propositions than does the classical construction, while retaining the classical insight that propositions are functions into truth values. We prefer our construction because we think that different sentences often have different truth conditions.

We close with an appeal to authority. At 4.022 in the *Tractatus*, Wittgenstein writes: 'A proposition *shows* how things stand *if* it is true. And it *says that* they do so stand.' We think that our construction of the proposition expressed by a sentence φ in two stages, f_φ and g_φ, accords well with Wittgenstein's dictum. For $g_\varphi(a, w) = 0$ if and only if $f_\varphi(w) = a$ and $g(a, w) = 0$; one could think of the first conjunct as showing how things stand in w if φ is true in w and of the second conjunct as saying that they do so stand. It should also be clear that our construction of $\mathcal{A}$ for the quantificational language $\mathbf{L}$ has something of a logical atomist hue about it, though perhaps of a more Russellian shade than Wittgensteinian. (The differences between these logical atomist authorities serve to emphasize that our construction of propositions is independent of the particular ways states of affairs themselves might be constructed.) To conclude in a logical atomist tone of voice, we could say: The world is the totality of actual states of affairs. States of affairs may be complexes composed of n objects and an n-ary relation. But whatever states may be, the function of a sentence is to pick out a state of affairs and say that it obtains. This function can be identified with the proposition expressed by the sentence, since sentences which say the same thing perform the same function and express the same proposition.

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