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SUPERVALUATIONS AND TARSKI

NICHOLAS GRIFFIN

In 'Truth, Belief and Vagueness'[1] Kenton Machina gives the following argument which purports to show that use of supervaluations is inconsistent with Tarski's truth-schema:

(T) $\mathsf{T}(P)$ iff P.

Suppose (under the assumptions of a supervaluational semantics) that P lacks a truth-value, i.e., that

(1) $P = *$

Then it follows that

(2) $\mathsf{T}(P) = \mathbf{F}$

and thus, so Machina claims, that

(3) $P = \mathbf{F}$

which is inconsistent with (1).[2] Although Machina uses his argument to show the inadequacy of supervaluations only where truth-value gaps occur for reasons of vagueness,[3] it is clear that his argument, if valid, could be used when truth-value gaps occur for any reason. And thus, if valid, his argument would demonstrate the need of those who wish to use supervaluations to provide an alternative to Tarski's truth-schema.

However, Machina's argument is not valid. It rests upon an important ambiguity in the formulation of the truth-schema as (T), since no indication is given of how 'iff' is to be interpreted in (T). If allowance is to be made for wffs which are not true or false (either because they lack a value, or

1. *Journal of Philosophical Logic*, vol. 5 (1976), p. 51.

2. A parallel argument, where P takes a third value, is given by Susan Haack, *Deviant Logic* (Cambridge: Cambridge University Press, 1973), p. 68.

3. As in Kit Fine, "Vagueness, Truth and Logic," *Synthese*, vol. 30 (1975), pp. 265-300.

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take a different value) there are at least the following two ways of interpreting Machina's (T). The first interprets 'iff' as extended material equivalence:

(T1) $\mathbf{V}(\mathrm{T}(P)) = \mathbf{V}(P)$.

There is little to be said in favour of (T1), since, under it, the operator 'T' loses its characteristic feature of yielding only two-valued sentences for any given sentential argument. Moreover, although Machina could use (T1) to get from (2) to (3), he cannot use it to get from (1) to (2), since (T1) and (1) yield not (2) but

(4) $\mathbf{V}(\mathrm{T}(P)) = *$.

Alternatively, and more reasonably, (T) could be replaced by

(T2) $\mathbf{V}(\mathrm{T}(P)) = \begin{cases} \mathbf{T} \text{ if } \mathbf{V}(P) = \mathbf{T} \\ \mathbf{F} \text{ otherwise.} \end{cases}$

Using (T2) in Machina's argument we can get from (1) to (2), which yields by *modus tollens*, not Machina's (3), but

(5) $\sim(\mathbf{V}(P) = \mathbf{T})$.

It is clear how Machina's argument trades on the ambiguity of (T) between (T1) and (T2). The argument is valid only if (T2) is used to get from (1) to (2), and (T1) is used to get from (2) to (3). In general, there is little reason to think that Tarski's truth-schema (when appropriately formulated) is endangered either by the admission of truth-value gaps (contra Machina), or extra values (contra Haack).

McMaster University
Hamilton, Ontario, Canada