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NECESSITAS CONSEQUENTIS IN A SINGLETON POSSIBLE WORLD

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In [1], M. J. Cresswell adopts a Kripke semantics according to which "a one-world model is a model in which Lp and p have the same truth-value and in which $CpLp$ is true." In [5], I proved that not only is the system $M'(T')$, with $CpLp$ as a thesis, formally consistent, but it does not collapse into classical sentential calculus. Now I wish to show that there is a sense of "possible world," closely allied to that of Kripke, in which Lp and p do not necessarily have the same truth-value and in which $CpLp$ is contingent.

In Kripke [3], R is idle in a normal model structure $\langle G, K, R\rangle$ where $K = \{G\}$. That is, R fails to distinguish between Kripke [3] and Kripke [2]. Now, in Kripke [2], a possible world is a truth-value assignment to every atomic subformula of a wff α. We depart from Kripke in this—that, for us, a possible world is not a truth-value assignment to atomic variables. It is a set of such assignments. Following Massey [4], we understand by a plenary set Ω a set of partial and complete truth-tables for a wff α such that any truth-value assignment Σ to the variables of α is represented in some member of Ω.

We let a member of Ω represent a possible world. That is, we let a partial or complete truth-table for a wff α represent a set of truth-value assignments for a wff α. The semantics for 'L' are then stipulated, not across possible worlds but within them as in Massey [4].

L_1

α	$L\alpha$
t	t

L_2

α	$L\alpha$
f	f

L_3

α	$L\alpha$
t	f
f	f

Now, consider the following plenary set of truth-tables for $CpLp$.

T_1

P	$CpLp$
t	t t

T_2

P	$CpLp$
f	t f

T_3

P	$CpLp$
t	f f
f	t f

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Hence, the wff $CpLp$ has three candidates for $K = \{G\}$ represented respectively by T_1, T_2, and T_3. T_1 and T_2 satisfy Cresswell's claim. $K = \{T_3\}$ does not. For with T_3 as the one and only possible world, p and $CpLp$ are contingent and Lp false.

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