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In Memoriam

CAREW ARTHUR MEREDITH
(1904-1976)

DAVID MEREDITH

Carew Arthur Meredith was born on July 28, 1904. His father, Arthur Carew Meredith, was a Dublin barrister and King's Counsel, whose opinion, years later, was sought by De Valera on legal matters concerning the Irish Free State. His mother, Jessica, née Twemlow, was one of the first women in Ireland to obtain a degree—she and eight others were dubbed "the nine muses." With two sisters, Meredith grew up in Dublin, attending Irish schools until he won a scholarship to Winchester in England. From Winchester he proceeded to Trinity College, Cambridge, where he studied mathematics. He obtained First Class Honours in Part I of the Tripos in 1923, First Class Honours with Distinction in Part II in 1924, and took his B.A. in 1925. An interest in mathematical logic met with small encouragement from Littlewood, who remarked "I don't see what you can get out of it," but Meredith gained some knowledge of the field in general and of the work of the Polish logicians in particular.

After graduating, Meredith married Sybil Clark and settled down to what was to be for many years his occupation: that of university coach or "grinder." He worked assiduously on his own—often going to bed at nine and rising at three, to have the freshest hours of the day for his own use—but it is not clear what he was working at during this period: no papers were published and no notes survive.

Until 1939, the Merediths lived in England but Carew Meredith was a convinced pacifist, and with the coming of the war he and his wife removed themselves to Ireland—the English authorities declining an offer to have the Meredith views directly explained to them. In Ireland Meredith continued to coach, shedding a fairly heavy load only slowly, after he was appointed lecturer in mathematics at Trinity College in 1943. This post he retained until he retired in 1964. After retirement, the Merediths spent a few years in Cornwall, England, but finally returned to Ireland, where Carew Meredith died on March 31, 1976.

The climactic event to wh innocents in logic, and to the rambunctious pre-medical students who for many years brought Meredith the

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unenviable reputation of least heard lecturer in Dublin) must have seemed a completely unremarkable existence, was the Irish advent, after the war, of Jan Łukasiewicz. Brought to Ireland with other scientists, through the agency of De Valera, Łukasiewicz was appointed professor at the Royal Irish Academy, where he lectured on mathematical logic. Meredith attended these lectures from 1947 on, and became keenly interested in the Łukasiewicz' detachment operation, for which—as he himself once phrased it—he "seemed to have some aptitude." Modest though this self-appraisal sounds, it accurately pinpoints Meredith's particular strength in logic. Few logicians can have had so deep and intimate a knowledge of the highways and byways of the propositional calculus, and none—save perhaps M. Wajsberg—has announced so remarkable a succession of axiomatics. For propositional logic—the area of the Łukasiewicz' detachment—became, and remained, Meredith's strongest and most abiding preoccupation. Beginning with the proof in 1951 that $C\delta\delta 0\delta p$ is the shortest axiom of the extended propositional calculus, Meredith continued, for nearly twenty years, to pour out for the world of logic a profligacy of axiomatic results. His work forms a direct continuation of the work of the Polish logicians represented in the 1930 Łukasiewicz and Tarski "Untersuchungen über den Aussagenkalkül" (so much so, in fact, that Łukasiewicz, in annotating this paper for republication in 1956, could not resist appending some of Meredith's results in slightly anachronistic footnotes) and he in person became almost a logicians' advisory bureau. Many of the results that were prepared for publication by A. N. Prior in the sixties had been discovered as Meredith attempted to respond to queries from logical colleagues. He found the shortest known single axioms for two-valued propositional calculus in several different primitive bases. He gave a proof of the sufficiency of *CCCpqrCsCCqCrtCqt* as a single axiom for Hilbert's Positive Implicational Logic, which, twenty-one years after its publication, could be shortened by just two detachments; and when Prior posed to him the problem of axiomatizing the implicational fragment of Lewis' S5, he produced the single axiom *CCCCCttpqCrsCCspCuCrp*. With the exception of two excursions into syllogistic, all of his publications are in propositional logic, and all bear witness to his "aptitude" in detachment.

In light of his particular bent, it is not surprising that Meredith's most widely used innovation in logic is his so-called "condensed detachment." This is usually presented simply as a convenient abbreviative device: since for any ordered pair of propositional theses, among the non-null results of doing a detachment with the first thesis, or some substitution in it, as major premiss, and the second thesis, or some substitution in it, as minor premiss, there is a unique thesis of which all other theses obtainable from the pair by substitution and detachment are substitution instances, the laborious and space consuming business of specifying operations under the substitution rule can be dispensed with in favor simply of writing '**D***mn*' for this unique thesis, where m and n are the numbers given the premisses. Thus, for instance, while given

1. *CCpCqrCqCpr*
2. *Cpp*

Łukasiewicz would write

$$1\ p/Cpq,\ q/p,\ r/q * C2\ p/Cpq - 3$$

3. *CpCCpqq*

Meredith would simply write

D12 = 3. *CpCCpqq*

Since **D**12 is unique it can of course be used, without being shown, in further **D**-expressions—to the joy of editors and to the distraction of readers.

For Meredith, however, **D** was not simply a convenient way of abbreviating proofs. He saw it as an operator on classes of formulae, and noted such properties of it as the following:

I. $\mathbf{D}\wedge a = \mathbf{D}a\wedge = \wedge$
II. $\mathbf{D}\vee a = \vee$, unless $a = \wedge$
III. Where X = *CpCpp*, $\mathbf{DD}\mathrm{X}ab = a \cap b$

A problem that particularly intrigued him was the problem of the **D**-derivability of the "weak" theses of a system—that is, those theses which are substitution instances of some other thesis that is either shorter or of the same length but with a greater number of distinct variables. A case in point is the **D**-derivability in Positive Logic of X in property III above. This is a substitution instance both of *CpCqp* and of *CpCqq*, but the following **D**-derivation apparently eluded Meredith.

	1. *CCqrCCpqCpr*
	2. *CCpCqrCqCpr*
	3. *CCpCpqCpq*
	4. *CpCqp*
DD1**D**131	= 5. *CCpCqrCCqpCqr*
D1**DD**15**D**12	= 6. *CCpCqCrCstCpCCsqCsCrt*
DD1**DD**1611	= 7. *CCpCqrCCsCtpCCqsCqCtr*
D37	= 8. *CCpCqpCCqpCqCqp*
D84	= 9. *CCqpCqCqp*
D9**D**34	= X. *CpCpp*

In some systems there are weak theses which are definitely not **D**-derivable. No **D**-derivation exists, for instance, for *CCppCpp* in the system whose three axioms are the first two Positive Logic axioms just given together with *Cpp*. But whether because of dissatisfaction with his investigations or, as is more likely, because of lack of interest on the part of his logical friends, Meredith never published anything on **D** *qua* detachment operator, and it has entered the literature simply as a convenient means of abbreviation.

The attitude expressed in the remark of Littlewood, quoted earlier, is not uncommon: few non-logicians ever see what the logician "can get out of it." But the joy and passion with which Meredith "did" logic was, to fellow logicians at least, sufficient reason for the doing of it. He did logic whenever time and opportunity presented themselves, and he did it on whatever materials came to hand: in a pub, his favored pint of porter within reach, he would use the inside of cigarette packs to write proofs for logical colleagues. Many of these will remember him as a brilliant logician, a generous teacher, and a delightful friend.

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