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AN ANSWER TO ARMSTRONG'S QUESTION ABOUT INCOMPLETENESS IN COPI

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In a recent article[1] Robert L. Armstrong raised a question about the proof presented by Irving M. Copi of the incompleteness of the first 19 rules of inference of Copi's method of deduction.[2] Copi's proof is a variation on a well-known technique introduced for axiomatic propositional logic by Bernays in 1918; in 1935 Huntington showed that the method could be extended to rules of inference.[3] This author has checked Copi's proof, and it is correct. Armstrong, on the other hand, presents formal proofs and accompanying arguments which cast doubt on Copi's proof. Armstrong's remarks are quite valuable, insofar as they reveal Copi's system to be less than transparent. But what are we to make of this situation? It can be resolved on the grounds of formal logic as follows.

Copi's collection of 19 rules is designed for systematizing the deduction of conclusions from premises. As it stands, however, the system cannot arrive at the *truth* of any statement whatsoever. This is because each of the first nine rules clearly requires initial premises, and each of the ten forms of the Rule of Replacement presupposes a premise or derived statement into which the replacement is made. But no premises, or axioms, are given *a priori* as a part of this formal system. Copi's 19 rules may be compared to the massive machinery of a new steel mill lying idle, waiting for the opening-day arrival of raw material. The spirit of formalization precludes after-the-fact "changing the rules of the game," such as the introduction of additional premises not initially provided for in a

1. Robert L. Armstrong, "A question about completeness," *Notre Dame Journal of Formal Logic*, vol. 17 (1976), pp. 295-296.

2. Irving M. Copi, *Symbolic Logic*, 4th edition, The Macmillan Co., New York (1973), pp. 47-50.

3. Alonzo Church, *Introduction to Mathematical Logic I*, Princeton University Press, Princeton (1956), p. 163.

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formal system. Armstrong's several formal proofs, which do start from enthematic premises, do not take place within the formal system of the 19 rules. In particular, Armstrong uses the Replacement Rule 19 (Tautology) as if it were an axiom, a use that is not permitted under the "rules of the game" set up by Copi.

The addition of Copi's Rule of Conditional Proof provides the raw material needed to get the mills rolling. This rule may be formulated as

$$\text{If}\quad \begin{array}{c} P \\ A \\ \therefore B \end{array} \quad \text{is valid, so is} \quad \begin{array}{c} P \\ \therefore A \supset B \end{array}$$

It allows one to assert premise-free as valid—in other words, as true—that $p \cdot q \supset p$ and $p \supset p \vee q$, on the basis of Rule 7 (Simplification) and Rule 9 (Addition). Instantiating p for q, and using Rule 19 (Tautology Replacement), we derive $p \supset p$ and then by Rule 16 (Implication) $\sim p \vee p$, the Principle of Excluded Middle.[4]

It is of interest to note that, although vindicated up to this point, Copi does err when he claims that the Rule of Indirect Proof, when added to the 19 rules and the Rule of Conditional Proof, "serves to strengthen our proof apparatus still further."[5] The Rule of Indirect Proof may be formulated as

$$\text{If}\quad \begin{array}{c} P \\ \sim C \\ \therefore \sim A \wedge A \end{array} \quad \text{is valid, so is} \quad \begin{array}{c} P \\ \therefore C \end{array}$$

Suppose that the first argument is valid, and let P be given as premise of the second. Applying the Rule of Conditional Proof twice to the first argument permits us to assert the validity of $P \supset (\sim C \supset (\sim A \wedge A))$. Then Modus Ponens, based on the premise P, yields $C \supset (\sim A \wedge A)$, which may be converted successively to $\sim\sim C \vee (\sim A \wedge A)$ (Rule 16—Implication), $C \vee (\sim A \wedge \sim\sim A)$ (Rule 14—Double Negation), $C \vee \sim (A \vee \sim A)$ (Rule 10—De Morgan), $\sim (A \vee \sim A) \vee C$ (Rule 11—Commutation), and $\sim (\sim A \vee A) \vee C$ (Commutation again). But we proved the Principle of Excluded Middle above, using only the 19 rules plus the Rule of Conditional Proof; so $\sim A \vee A$, and by Rule 14 (Double Negation), $\sim\sim (\sim A \vee A)$. For the final step, apply Rule 4 (Disjunctive Syllogism), to $\sim (\sim A \vee A) \vee C$ and $\sim\sim (\sim A \vee A)$, arriving at C.

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4. Armstrong's remarks in defense of using the Principle of Excluded Middle as an enthematic premise require comment. Intuitionists, both philosophers and mathematicians, will likely be disgruntled at his characterization of the Principle of Excluded Middle as a proposition, "that everyone can be expected to accept as true," and for which "there can be no logical objection to introducing it as an additional premise in order to construct a formal proof." Some will regret that Copi's book also makes no mention of philosophical objections to the use of this Principle.

5. *Symbolic Logic*, p. 55.

We have shown that the Rule of Indirect Proof is a derived rule in the system of the 19 rules plus the Rule of Conditional Proof.

It is curious that the version of the Method of Deduction given in Copi's *Introduction to Logic*, 3rd (1968) and 4th (1972) editions, differs substantially from the one discussed here, that of *Symbolic Logic*, 4th edition (1973). The treatment in *Introduction to Logic* completely omits mention of the Rules of Conditional Proof and Indirect Proof, and Absorption replaces Destructive Dilemma as Rule 6 of the 19 rules. The preface to the 2nd edition of *Introduction to Logic* includes the remark: "And the rules in the present edition constitute a *complete* set, in sharp and significant contrast to the incomplete set of rules given in the first edition."[6] As Copi noted in *Symbolic Logic*, Absorption cannot be derived from the 19 rules discussed here.[7]

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6. Irving M. Copi, *Introduction to Logic*, 3rd edition, The Macmillan Company, New York (1968), p. vii. The emphasis is Copi's.

7. *Symbolic Logic*, 3rd ed., p. 58.