232
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AN EARLY FIFTEENTH CENTURY DISCUSSION OF INFINITE SETS

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In the opening years of the fifteenth century, or perhaps a little earlier, John Dorp[1] wrote a commentary on Buridan's *Compendium Totius Logicae*;[2] and it is here that one finds a discussion of infinite sets which is not only quite unexpected[3] but which suggests that other thinkers of that period were interested in the same topic.

The question of infinite sets arose in the context of the theory of reference. Medieval logicians assumed that affirmative sentences were true only if the subject and object terms had reference, but this assumption conflicted with their intuitions about such sentences as "I imagine a chimera" and "The word 'chimera' refers to a chimera". These sentences seem to be true, but "chimera" cannot refer to actual or possible chimeras, since a chimera is an impossible object, just as a round square is an impossible object. The question then arose of how such sentences were to be treated, and one obvious answer was to postulate a class of imaginary objects which included impossible objects and to which reference could be made in intentional contexts.[4] In his discussion of this answer, Dorp presented several arguments against the claim that one could refer to impossible objects. The third argument is as follows:

> Let the imagination portray the whole line of imaginable objects, just as they are imagined by mathematicians. Then this line of imaginables either merges with the line of possibles or it does not. In the first case we have what we need. In the second case we do not, for the line of the possibles is infinite, there being an infinite number of possibles. If, therefore, the line of imaginables goes beyond the possibles, one infinite will be greater than another, which is contrary to what Aristotle said in Book III of *Physica* and whose opposite he declared both there and in Book I of *De Caelo*.[5]

In other words, if we postulate a set of imaginary objects it will either correspond to or be greater than the set of possible objects. In the first case there is no problem. In the second case we arrive at a falsehood, and should reject the claim that impossible objects can be spoken of.

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Dorp answered this objection as follows:

> Let the whole line of imaginables be admitted to the imagination. Further, let it be said that it is not the same as the line of possibles but contains the line of possibles and also some imaginary impossibles. And when it is inferred that one infinite is greater than another, some do not feel this to be unsuitable. On behalf of this view, it should be noted that one thing can go beyond another in two ways. In one way one thing goes beyond the other in that it has a determinate comparative relationship to it, and thus the one which goes beyond is greater than the other. In the other way one thing goes beyond another to which it has no determinate comparative relationship, and then it is not necessary that one should be greater than the other. And thus it is said that the whole line of imaginables can be admitted to the imagination, and that it goes beyond the whole line of possibles. However, there is no determinate comparative relationship between these lines, and thus one is not greater than the other.[6]

In other words, to say that a set A is larger than another set B can be to say that, when a one-to-one correspondence is established between the members of A and the members of B, some members of A will be left over. No infinite set is larger than another in this sense, so Aristotle was right. However, an infinite set A can be larger than another infinite set B in the sense that it contains all the members of B and some additional members. This claim can be shown to be consistent with what Aristotle said in two ways. One can argue, though Dorp does not, that there is at least one case in which the members of an infinite set A, such as the set of natural numbers, can be put into one-to-one correspondence with the members of an infinite set B, such as the set of even numbers, even though B does not contain all the members that A does. Thus A is shown not to be larger than B in the first sense. One can also argue, and this is what Dorp does, that there is at least one case in which the members of an infinite set A, such as the set of real numbers, simply cannot be correlated with the members of an infinite set B, such as the set of natural numbers, so that, although A contains more members than B, there is no way of showing that A is larger than B in the first sense, any more than there is a way of showing that it is not.

Historians of logic must always be wary of taking isolated passages out of context and reading modern developments into them. However, in the case of Dorp there do seem to be good grounds for claiming that he was aware of something describable as a non-denumerably infinite set. It is a great pity that he does not give us more detail about the reasoning that lay behind his assertions, but it is to be hoped that further research into late fourteenth and early fifteenth century mathematics will reveal it to us.

NOTES

1. Dorp received his M.A. from the University of Paris in 1393 and he was last heard of at the University of Cologne in 1418. The dates of his birth and death are not known.

2. Johannes Buridanus, *Compendium Totius Logicae*, Venedig (1499). Facsimile edition: Frankfurt/Main, Minerva G.m.b.H. (1965). This edition contains Dorp's commentary.

3. For another medieval reference to infinite sets, see I. Thomas, "A 12th century paradox of the infinite," *The Journal of Symbolic Logic*, vol. 23 (1958), pp. 133-134.

4. For further discussion and references, see E. J. Ashworth, "Chimeras and Imaginary Objects: A Study in the Post-Medieval Theory of Signification" [in progress].

5. Buridanus, *op. cit.*, sign. I5. "Tertio sic. signetur ad ymaginationem tota linea ymaginabilium sicut ymaginantur mathematici. tunc vel ista linea ymaginabilium vel transit in lineam possibilium vel non. si primum habetur intentum non secundum: quia linea possibili est infinita ex quo infinita sunt possibilia. si ergo linea ymaginabilium excederet ultra possibilia unum infinitum esset alio maius: quod est contra aristotelem iii. physicarum. et cuius oppositum declaratur ibidem et etiam primo celi."

6. *Ibid.* "Ad tertiam admittitur ad ymaginationem tota linea ymaginabilium. ulterius dicitur quod non est eadem linea possibilium. sed continet lineam possibilium et cum hos aliqua ymaginabilia impossibilia. et cum infertur: tunc unum infinitum esset alio maius aliqui non reputant esse inconveniens. Pro isto nota quod unum potest alterum excedere dupliciter. Uno modo unum excedit alterum in hoc quod habeat certam proportionem ad illud. et sic illud quod excedit altero maius est. Alio modo unum excedit alterum ad quod non habet aliquam certam proportionem: et tunc non oportet quod unum sit altero maius. Et sic dicitur quod licit ad ymaginationem tota linea ymaginabilium et excedat totam lineam possibilium. tamen inter illas lineas non est proportio certa. et sic una non est alia maior."

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