

THE SQUARE OF OPPOSITION

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1 *Aristotle's View* In the *Organon* Aristotle recognized that a statement could be denied in more than one way. The place of the Greek οὐ, the Latin *non*, and the English *not* in a statement determines how it is to be denied. As it turns out, none of these modes of denial is rendered by the modern logician's notion of propositional negation. Indeed, for Aristotle only two sorts of things can be denied: (i) terms themselves (e.g., from 'man' we get 'not-man' and from 'ill' we get 'not-ill')¹; and (ii) predicates can be denied of subjects. The negation of a statement is taken in the sense of (ii) rather than the modern propositional sense, for he says that "we mean by negation a statement denying one thing of another"².

In the statement 'Socrates is ill', 'is ill' is the *predicate* and 'ill' is the *predicate term*. We negate the statement by denying the predicate of the subject. And we deny a predicate by attaching the denial operator 'not' to it (rather than by attaching the denial operator to the predicate term). So the negation of 'Socrates is ill' is 'Socrates is not ill', i.e., the denial of ill to Socrates. If, however, we attach the denial operator to the predicate *term*, we get 'Socrates is not-ill', a statement which is in fact an affirmation of not-ill to Socrates. Given some affirmative statement and the two modes of denial, we can generate what Aristotle called "the four".

Supposing, I mean, the verb 'is' to be added to 'just' or 'not-just', we shall have two affirmative judgements; supposing that 'is not' is added, we then have two negative judgements. Together these make up the four. This the subjoined examples makes clear:—

| Affirmations | Negations |
|-----------------|----------------------------------|
| Man is just | Man is not just |
| Man is not-just | Man is not not-just ³ |

1. See *On Interpretations*, Chapter II.

2. *Ibid.*, 17a26. All quotations, unless specified otherwise, are from the Loeb translation.

3. *Ibid.*, 19b24-30.

The logical relation between a statement and its negation is clear—they are contradictories. If two statements are contradictories they must always have opposite truth-values. “Affirmations and negations are opposed, it is patent, in none of the ways upon which we have already touched. It is here, and here only, indeed, that one opposite needs must be true, while the other must always be false.”⁴

But what is the logical relation between two affirmations which differ only in that the predicate term of one is the denial of the predicate term of the other? What logical relation holds between ‘Socrates is ill’ and ‘Socrates is not-ill’; between ‘Every man is just’ and ‘Every man is not-just’; between ‘Not-man is just’ and ‘Not-man is not-just’? Aristotle was interested almost exclusively in the logical features of statements which are “universal in character”.⁵ Statements have either individual subjects (e.g., ‘Socrates is ill’) or universal subjects. Statements having universal subjects may be universal in character (e.g., ‘All men are just’, ‘Every man is just’) or nonuniversal in character (e.g., ‘Man is just’).⁶ Aristotle tells us that ‘Every man is just’ and ‘No man is just’ are *contraries*, but he fails to make clear to us the role of ‘no’ in ‘No man is just’. It does not mean ‘Not-man is just’, nor ‘Not every man is just’, nor ‘Every man is not just’. (If it meant the latter, then ‘No man is just’ would be the contradictory rather than merely the contrary of ‘Every man is just’.) I suggest that ‘No man is just’ is an affirmative statement of universal character, whose predicate term is denied. Thus ‘No man is just’ can be read as ‘Every man is not-just’.⁷ Chapter XIV of *On Interpretations* is meant to settle this issue for us, but it does not entirely succeed. What we learn is that contrary statements have “contrary senses”. But the last sentence of the book does give us an important clue.

While two propositions that are true can together be truly asserted, two contrary propositions must predicate contrary qualities, and these in the selfsame subject can never together inhere.⁸

This, I believe, is a key passage in understanding Aristotle here.

One of the things claimed in the passage quoted above is that two statements will be contraries whenever they predicate (i.e., affirm) contrary

4. *Categories*, 13b1–3.

5. *On Interpretations*, 17b3.

6. *Ibid.*, 17b1–14.

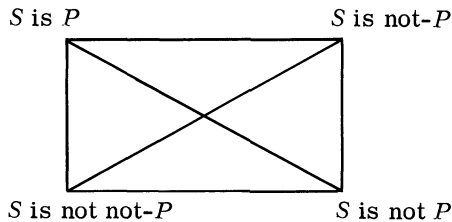
7. For an argument against this view see M. Thompson, “On Aristotle’s square of opposition,” *Philosophical Review*, vol. LXII (1953), pp. 251–265. For an argument in favor of my thesis see F. Sommers, “On a Fregean dogma,” *Problems in the Philosophy of Mathematics*, North-Holland Publishing Co., Amsterdam (1967). Here and elsewhere I have profited immeasurably from the study of Professor Sommers’ published work.

8. *On Interpretations*, 24b7–10.

qualities of their common subject. Now clearly, 'Every man is just' and 'Every man is not just' do not do this. The first affirms and the second denies the quality (just) of the subject. But in 'Every man is just' and 'Every man is not-just' some quality is being affirmed in each case of a common subject, and the two qualities are clearly contrary (just and not-just). So, if just and not-just are contrary qualities, and if two statements are contrary whenever they affirm contrary qualities of a common subject, then the proper contrary of any affirmative statement⁹ (e.g., 'Every man is just') is a statement which differs from it only in that its predicate term (the term marking the quality being affirmed or denied of the subject) is denied (e.g., 'Every man is not-just'). Thus, 'Socrates is ill' and 'Socrates is not-ill'; 'Man is just' and 'Man is not-just'; and 'Not-man is just' and 'Not-man is not-just' are all contrary pairs.¹⁰

The logical relation of contrariety is not the relation of contradiction.¹¹ Of two contradictories one must be true, the other false. But, as suggested in the final sentence of *On Interpretations*, two contraries can never both be true. This is so because contrary qualities "in the selfsame subject can never together inhere."¹² Nothing, however, prohibits the possibility of both of two contraries being false.

To return to "the four" statements, we can now see that for any affirmative statement, three other statements can be generated from it by (1) denying its predicate term, (2) denying its predicate, and (3) both (1) and (2). The result of (1) is an affirmation which is the contrary of the original statement. The result of (2) is the contradictory of the contrary of the original. These relations can be displayed on the following "square of opposition" (where P is the predicate term and S , the subject term, may be individual or universal, and the character of the statement may or may not be universal).



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9. We need not be bound by Aristotle's exclusive interest in universally characterized statements; an interest, based upon his view that all scientific principles would be so characterized.
10. In *Metaphysics*, 1055b18 (Ross' translation) we find, "For every contrariety involves, as one of its terms, a privation . . ."
11. *Ibid.*, 1055a37-1055b3.
12. Sommers has called this "the law of incompatibility" and has correctly distinguished it from what is called "the law of contradiction". See "Predicability," *Philosophy in America*, ed. by M. Black, Cornell University Press, Ithaca, N.Y. (1965), p. 273. This law is also clearly stated in *Categories*, 14a10-14.

I suggest that this is the most general square of opposition and that the traditional square of four categoricals (statements with universal subjects, universally characterized) is merely a special, but important and interesting, subcase of this square.

2 *Contrariety* In *On Interpretations* Aristotle says that “contraries belong to those things that within the same class differ most”.¹³ As examples of contrary qualities he gives: justice-injustice, black-white,¹⁴ and ill-well.¹⁵ But he says that red and yellow are not contrary qualities.¹⁶ In *Metaphysics* he says, however, “The primary contrariety is that between a positive state and privation.”¹⁷

Aristotle’s view of contrariety is tangled, but not impossible to straighten out. If a thing fails to have some property naturally, then while it is correct to deny that predicate of that thing (e.g., ‘A stone is not sighted (has not sight)’), it is not correct to say that that predicate is privative to that subject (viz. ‘A stone is sightless (is not-sighted, has not-sight)’).¹⁸ It would seem that any predicate term, *P*, and its denial, not-*P*, would be primary contraries since they indicate the primary contrariety between a positive state (being *P*) and a privative state (being not-*P*). Let us call the terms ‘*P*’ and ‘not-*P*’ *logical contrary terms*. But what of black and white and red and yellow?

Surely, if a thing is black it is in a state of privation with respect to white. So ‘*S* is black’ implies ‘*S* is not-white’. But the converse of this does not hold since a thing may be neither black nor white (e.g., red). Yet these same things can be said of red, yellow, etc. ‘*S* is red’ implies, but is not implied by, ‘*S* is not-white’. The reason Aristotle treats black and white as contraries but not the other colors is that he wants to think of contraries as those things which “differ most”. He thinks of all colors as arranged on a scale from black to white so that black and white are the ones which are most different, the farthest apart. It will not harm Aristotle’s logical insights, however, to abandon this view of contrariety strictly in terms of things which are most different.

Let us say that any two qualities are contrary (e.g., red-yellow, red-black, black-white, heavy-light, round-square, square-triangular, in Boston-in London) if and only if they are incompatible, cannot inhere together in the same subject.¹⁹ We will call such predicate term pairs *contraries*. Given that the terms *A*, *B*, *C*, *D*, . . . are all mutual contraries

13. *On Interpretations*, 23b23-24. See also *Metaphysics*, 1018a25-31.

14. *Categories*, 10b13-15.

15. *Ibid.*, 14a10-12.

16. *On Interpretations*, 10b18-19.

17. *Metaphysics*, 1055a34.

18. See *Categories*, 12a26-12b5.

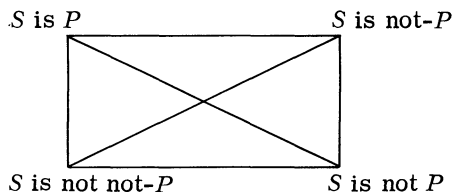
19. *On Interpretations*, 24b7-10.

(e.g., different color terms), we can say that with respect to any one of them (e.g., A) the affirmation of any of the others to a subject would imply the affirmation of its logical contrary (not- A) to that subject. Moreover, since the affirmation of the logical contrary (not- A) implies the affirmation of one of the (nonlogical) contraries (B or C or D or . . .), we can define the logical contrary of a predicate term as the disjunction of all of its nonlogical contraries. In other words, given any term and the set of all terms contrary to it, its logical contrary will be equivalent to the disjunction of all the members of that set. If A is any term and B, C, D, \dots its contraries, then a subject is not- A if and only if it is B or C or D or . . .

We can see now why a stone cannot correctly be said to be in a state of privation with respect to sight. For, to say that this is so would be to affirm not-sighted of it, which implies that some term contrary to 'sighted' (e.g., 'blind') would be truly affirmed to it. Yet a stone is neither blind nor sighted. Likewise, it is incorrect to say that a number, say 2, is privative with respect to some color, say red, since to do so would be to affirm not-red of 2, which would imply the affirmation of some other color to 2! If 2 is not-red then it is blue or green or black or white or pink or . . . But since no color term can be affirmed truly of 2, we can say that '2 is red' and '2 is not-red' are both false.

Two statements are contrary if and only if they affirm contrary predicate terms (e.g., 'red' and 'white') of a common subject. Two statements are logical contraries if and only if they affirm logically contrary predicate terms (e.g., 'red' and 'not-red') of a common subject. Any contrary of a statement will imply the logical contrary of that statement. It follows that whatever holds of the logical contrary of a statement will hold of any of its contraries. In what follows, then, we need only talk of the logical contrary, rather than the contraries, of a statement.

3 *The Square* I have said that the following square is a *general* square of opposition. If this is so, then all the logical relations among the four categoricals which are represented on the traditional square must hold for any four general statements consisting of an affirmation, its logical contrary, and the negations (denials) of each.



This is a genuine square of opposition if and only if all the following hold.

- (1) 'S is P ' and 'S is not- P ' cannot both be true.
- (2) 'S is P ' and 'S is not P ' are contradictories.
- (3) 'S is not- P ' and 'S is not not- P ' are contradictories.
- (4) 'S is not not- P ' and 'S is not P ' cannot both be false.
- (5) 'S is P ' implies 'S is not not- P '.
- (6) 'S is not- P ' implies 'S is not P '.

All the logical relations among the four categoricals and, indeed, among our four can be guaranteed by just two simple Aristotelian rules.

The Law of Contradiction: *An affirmation and its negation cannot both be true.*

The Law of Excluded Middle: *Either an affirmation or its negation must be true.*²⁰

The Law of Incompatibility (An affirmation and its logical contrary cannot both be true) is derivable from the Law of Contradiction. "Now since it is impossible that contradictories should be at the same time true of the same thing, obviously contraries also cannot belong at the same time to the same thing."²¹ In other words, given the impossibility of both 'S is P' and 'S is not P' being true, the impossibility of both 'S is P' and 'S is not-P' being true follows. Now this is so only because the logical contrary of a statement implies its negation ('S is not-P' implies 'S is not P'). Thus we establish (6).

If 'S is P' is true, its negation 'S is not P' is false by the Law of Contradiction. If 'S is not P' is false, then by (6) 'S is not-P' is false. If 'S is not-P' is false, then by the Law of Excluded Middle 'S is not not-P' must be true. Thus we establish (5). By the Law of Incompatibility 'S is P' and 'S is not-P' cannot both be true. Thus (1). From (1) and the Law of Excluded Middle it follows that 'S is not not-P' and 'S is not P' cannot both be false. Thus (4). Finally, both laws taken together immediately give us (2) and (3).

Notice that two statements are contraries (logical or nonlogical) if and only if they cannot both be true. The question of whether they can both be false is open. Two statements are subcontraries if and only if their negations are contraries. Some logicians have worried about how necessary statements could appear on the square of opposition because they took the possibility of both of two statements being false as a necessary condition for their contrariety.²² If this were so then a necessarily true statement could not have a contrary (it and its contrary could not both be false), and thus could not appear on a square of opposition. However, there is no sound reason for believing that the impossibility of two statements both being true is not the only necessary condition for their contrariety.²³ To repeat, the

20. This law should not be confused with the Law of Bivalence: A statement must be either true or false. Note that when Aristotle's term and predicate denials are abandoned in favor of propositional negation we get 'S is not-P' and 'S is not P' both equivalent to 'not (S is P)', so that statements like '2 is red' and '2 is not-red', since they are both false, would break the Law of Excluded Middle.

21. *Metaphysics*, 1011b15-17.

22. See D. H. Sanford, "Contraries and subcontraries," *Noûs*, vol. 2 (1968), pp. 95-96.

23. Obviously this means that contradictories are contraries (as well as subcontraries) and that any two necessarily false statements will be contrary. I see no logical danger in this. See my "Knowledge, negation and incompatibility," *Journal of Philosophy*, vol. 66 (1969), especially pp. 584-585.

question of whether both an affirmation and its logical contrary are false is open. We will return to this later.

4 Existence We have seen that our square is general at least in that it "works" for both necessary and contingent statements. But there are at least two other ways in which modern logicians have attempted to restrict the square. The first is the requirement that all subject-predicate statements must have logically singular subjects. I will say little about this restriction here since Sommers has given in "On a Fregean dogma" (see footnote 7) a clear and totally adequate defense of the view that such a restriction is unwarranted. Suffice it to say that Aristotle never envisaged such a restriction and that the compulsion to feel so restricted only falls upon the quantificationalist who thinks of subject-predicate statements as disguised quantified conditionals, conjunctions, etc.

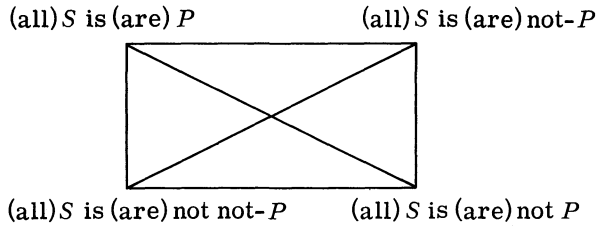
The second unwarranted restriction is that statements whose logical subjects fail to exist cannot be represented on the square.²⁴ Consider an empty subject term. It may be individual (e.g., 'the man now residing on the moon'), or universal but not universally characterized (e.g., 'men now residing on the moon'), or universal and universally characterized (e.g., 'all men now residing on the moon'). The contemporary logician would render '*a* is *F*' as '*Fa*' if '*a*' is individual, and as ' $(x)(Mx \supset Fx)$ ' otherwise. Now 'All men now residing on the moon are *F*' will be true whenever there are no men residing on the moon (i.e., when the subject term is empty). For, '*x* is a man residing on the moon' will be false for all values of *x*, i.e., no value of *x* renders '*Mx*' true. Since '*Mx*' is false for all values of *x*, ' $Mx \supset Fx$ ' is true for all values of *x*. But this holds also for the quantificational version of the contrary ($(x)(Mx \supset \neg Fx)$), making such statements true in both their universal forms (viz. the affirmation and its logical contrary)! Consequently, logicians like Quine not only do not use the square of opposition for analyzing the logical features of such statements, but go on to develop unnecessary cautions about nonexistent subjects and avoid the empty domain altogether.²⁵

If, like Aristotle, we view statements with universal subjects simply as subject-predicate in form, nothing forces us to make statements like 'All *S* are *P*' and 'All *S* are not-*P*' both true. The easiest thing to do is consider them both false and their negations both true. We can do the same for statements with individual subjects.²⁶ Then we have the following completely general square of opposition.

24. See, for example, Sanford, *op. cit.*

25. For an extended discussion of this see my "Sommers on empty domains and existence," *Notre Dame Journal of Formal Logic*, vol. XIII (1972), pp. 350-358.

26. In *Categories*, 13b14-35, Aristotle says that when Socrates does not exist 'Socrates is ill' and its contrary are both false but that the negations of each are true.



By recognizing propositional negation in place of predicate denial and predicate term denial the modern logician, when confronted with a statement having an individual subject recognizes only 'a is F' and 'not (a is F)'. Their inability to discern four genuinely different forms for singular statements prohibits them from displaying such statements on a square of opposition.

Now when a subject is empty (e.g., 'the present king of France', 'the round square', 'the man now residing on the moon') we can truly deny but not affirm predicates of it. Thus, if 'a' is empty, 'a is F' and 'a is not-F' are false, while 'a is not not-F' and 'a is not F' are true. Strawson recognized that when 'a' is empty 'a is F' and 'a is not-F' must both fail to be true. But because he took 'a is not-F' to be 'not (a is F)' the Law of Excluded Middle forced him to conclude that they could not both be false either.²⁷ Russell, on the other hand, recognized our point, briefly at least, when he allowed that for one sense of 'is not bald' (obviously the sense in which 'not' is a predicate term operator rather than a propositional operator), 'The present king of France is bald' and 'The present king of France is not bald' are both false.²⁸

We saw earlier that the question of whether an affirmation and its logical contrary were both false (the rules governing the square only prohibit them from both being true) was an open question. A statement which is false in both these forms can still be displayed on the square as long as it obeys all the rules governing the square. Statements with empty subjects are false in both their affirmative and logical contrary forms. Let us call any statement false in both these forms (the A and E forms) a *vacuous* statement.

5 Vacuity In "The ordinary language tree" Sommers says, "The reason one would rule out a sentence like 'K is tall and not tall' is not because a category mistake was committed. It is because of other rules than those of sense. In fact, if (T, not T) were a category mistake it would make no sense to call it an inconsistent or self-contradictory sentence. A sentence which is a category mistake cannot get to be contradictory."²⁹ The

27. "On referring," *Mind*, vol. 59 (1950), pp. 320-344.

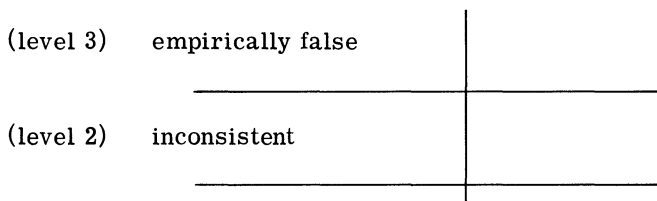
28. "Descriptions," *Classics of Analytic Philosophy*, ed. by R. R. Ammerman, McGraw-Hill, New York (1965), especially p. 23.

29. *Mind*, vol. 68 (1959), p. 181.

suggestion here seems to be that a sentence is ruled out as categorically incorrect at a lower level (earlier) than when it is ruled out as inconsistent. Questions of logical consistency do not get asked about sentences ruled out at the category correctness level. This suggestion is more fully expounded in Sommers' discussion of "levels of rectitude" in his later "Types and ontology":

A linguistic sequence may be correct or incorrect in different ways. I shall consider three such ways by way of illustrating the general character of clarification. A sequence may be grammatical or ungrammatical, it may be category correct or category mistaken, it may be consistent or inconsistent. We may call these ways of being correct or incorrect "levels of rectitude." The reason for calling them levels is that a sequence which is incorrect in one way must be correct in other ways and the ways it must be correct are therefore "lower" than, because presupposed by, the way it is incorrect. Also, an incorrect sequence is neither correct nor incorrect with respect to other ways, and these ways are "higher" since they presuppose the rectitude of the sequence. For example, an ungrammatical sentence is not a sentence at all; it cannot therefore make a category mistake. Thus, the incorrectness we call a category mistake presupposes the grammaticalness of the sentence. Again, a category mistake is neither consistent nor inconsistent. If I say "his anger was triangular and not triangular" I have not contradicted myself; I have said nothing and retracted nothing. An inconsistent sentence is neither true nor false empirically. Thus, inconsistency as a way of being incorrect presupposes both the grammaticalness and the category correctness of the sequence. Again, empirical falsity presupposes that the sequence is grammatical, category correct, and consistent. In short, any sequence which is incorrect at one level of rectitude must be correct at all lower levels and is neither correct nor incorrect at any higher level.³⁰

Little more needs to be said by way of clarifying what Sommers means by "levels of rectitude". The point to be emphasized, however, is that a sequence incorrect at some level must be correct at all lower levels and is *neither correct nor incorrect* at any higher level. We might construct this 'divided line' to illustrate Sommers' notion.



30. *Philosophical Review*, vol. 72 (1963), p. 384. The literature on category mistake grammaticalness and levels of language rules in general is fairly extensive. See, for example, the appropriate references in T. Drange, *Type Crossings* (Mouton, The Hague, 1966) and J. A. Fodor and J. J. Katz, *The Structure of Language* (Prentice-Hall, Englewood Cliffs, New Jersey, 1964); also see P. Ziff, "About ungrammaticalness," *Mind*, vol. 73 (1964), pp. 204-214, and A. Pap, "Logical nonsense," *Philosophy and Phenomenological Research*, vol. 9 (1948), pp. 269-283.

| | | |
|-----------|-------------------|--|
| (level 1) | category mistaken | |
| (level 0) | ungrammatical | |

Formally, if a sequence is incorrect at level n , it is correct at every $m < n$ and neither correct nor incorrect at any $m > n$.

Rules that govern sequences at level 0, rules for distinguishing sequences which are sentences from those which are not, are *grammatical* rules. Rules which govern sequences at level 1, rules for distinguishing sentences which are category mistakes from those which are not, are *sense* rules (Sommers has formalized such rules in "Types and ontology"). Rules which govern sequences at level 2, rules for distinguishing inconsistent sentences from those which are consistent, are *logical* rules. In a sense, grammatical, sense, and logical rules are all *linguistic* rules.³¹ If there are rules governing sequences at level 3, rules for distinguishing between empirical truth and falsity, they are not linguistic. The few obvious candidates for rules at this level are the laws of physical science (e.g., 'Nothing is faster than light' which rules as empirically false a sentence like 'Jack drove his dog sled faster than the speed of light', or 'Mules are sterile' used to rule as false 'This is the off-spring of two mules').

Category mistakes (e.g., '2 is red', 'This stone is blind', 'The moon is invalid') are vacuous³²—they are false in both their A and E forms.

Given the rules governing the square of opposition we can say now that the square "works" for any statement which is such that

(1) It is false in either its A or E form,

and

(2) It is true in either its I or O form.

The square is restricted in no way except by (1) and (2). Two kinds of statements satisfy (1): (a) those which are false in their A or E form but not false in both, and (b) those which are false in both their A and E forms. Statements belonging to (a) are nonvacuous; those belonging to (b) are vacuous. Nonvacuous statements clearly satisfy (2) as well as (1) and so can be displayed on the square.

Vacuous statements are of two kinds: (b_i) those which are true in both their I and O forms (viz. those with empty subjects), and (b_{ii}) those which

31. These have all been frequently referred to as grammatical rules and thus Sommers' levels of rectitude have been conceived as degrees of grammaticalness. See, for example, N. Chomsky, "Degrees of grammaticalness," in Fodor and Katz, *op. cit.*

32. I have distinguished category mistakenness and vacuousity in "Vacuousity," *Mind*, vol. 81 (1972), pp. 273-275.

are false in both their I and O forms. In "Types and ontology" we find that "when a sentence . . . is significant i.e. category correct it remains significant under all the normal logical operations such as conversion, negation, contraposition and so forth. And, similarly, if the sentence is category nonsense then all such transformations are also nonsensical."³³ This seems quite unobjectionable and indicates that category mistakes remain category mistaken in all their logical forms.³⁴ From this fact and the fact that category mistakes, by virtue of their vacuousity are false in both their A and E forms, it follows that category mistakes satisfy (1) but not (2).

Conditions (1) and (2) hold, then, for any statement (whether necessary or contingent, individual or universal, vacuous or nonvacuous) which is not category mistaken. The square of opposition "works" for *all* category correct statements. This confirms what we have seen in Sommers' theory of "levels of rectitude". The question of logical correctness or incorrectness does not get asked of statements which are category mistaken. Such statements are neither logically correct nor logically incorrect. Their failure is more fundamental. The square of opposition is a way of displaying *logical* features of statements. It is not at all surprising, therefore, that it is restricted to just those statements about which logical questions can be asked.

In summary, the usual analysis of categoricals has not only ignored the distinction between propositional negation and Aristotle's two modes of denial, but has even failed to see that such statements are indeed subject-predicate in form. Moreover, the frequent insistence upon the fulfillment of requirements concerning such things as existence and universality has blinded us to the genuine restriction on the square of opposition which I have tried to indicate here.

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33. "Types and ontology," p. 333.

34. The notion that statements which mean "nothing at all" (Loeb) or which have no "significance" (Ross) break the Laws of Contradiction and Excluded Middle and are thus false in all four forms can be seen in *On Interpretations*, 18a24-26.