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LEIBNIZ'S SYLLOGISTICO-PROPOSITIONAL CALCULUS

HECTOR-NERI CASTAÑEDA

In his research in logical theory after 1680 Gottfried Wilhelm Leibniz worked intensively at devising a formal calculus that would be interpreted as a general logic. He was convinced that the classical types of categorical propositions were the fundamental ones, so that the central core of his general calculus had to consist of the theory of the syllogism. He conceived the rest of the logical relationships between propositions as developments of syllogistics. Thus, the calculus he was aiming at was *not* to be merely a calculus that could have a double interpretation: as a syllogistic calculus and, alternatively, as a calculus of other types of implication. His general calculus was to have the principles of syllogistics *and* the other logical principles together at once. His calculus would be powerful enough to handle mixed inferences involving classical syllogisms and other reasonings.

Undoubtedly because of his algebraic researches and his view on the nature of truth, Leibniz took it for granted that a fully satisfactory logical calculus must be an equational one. Thus, his first efforts at treating the theory of syllogism were directed toward the construction of an equational syllogistic calculus. (And he anticipated some equations of Boolean algebra.) His *General Inquiries about the Analysis of Concepts and Truths* (1686), referred to here as [1],[1] contains a series of reflections on the syllogistic equations. He tries several approaches, and discovers some valid equations. But two things are of special interest to us now: (i) Leibniz formulates there principles that allow the generalization of syllogistics to propositional logic; (ii) Leibniz moves toward a higher level of abstraction. In his [2], *The Primary Bases of a Logical Calculus* (August 1, 1690), he takes up the topic again, and this time he begins with general logical principles and discusses applications to syllogistics. He distills the general system in [3], *The Bases of a Logical Calculus* (August 2, 1690).

My purpose here is to examine critically the system slowly developed by Leibniz in those three papers. I want to determine how close he came to formulate, or conceive of, a calculus that is at once adequate for both syllogistic and propositional logic. Naturally, the precise way of measuring the degree of accomplishment of his efforts consists of the completion of

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his system. The contrast between the finished whole and the missing part illuminates the furnished fragment. Of course, this involves a distortion here and a distortion there. But reconstructive or rational history of ideas is founded on the postulate that the sole means of assessing philosophical ideas is a series of properly delineated distortions that make an adumbrated pattern explicit or complete. Briefly, in [1] Leibniz furnishes formation rules for his general logical calculus and makes several attempts at formulating axioms and rules of inference. In [2] he makes another inconclusive attempt at formulating axioms and rules of inference. His set of axioms and rules in [3] is almost completely adequate for the propositional calculus. It is lamentable that he did not publish an article summarizing the main results in these three works. Symbolic logic and Boolean algebra would have been developed earlier and faster. The system of logic that Leibniz almost formulated fully is worth enjoying for its own sake.

I LEIBNIZ'S STRUCTURAL PRINCIPLES

1 *The official principles* First I want to set up Leibniz's official principles about the composition of propositions in his own words. The statements are for the most part taken from [1], where they are scattered; but together they make a better impression.

At least since 1679 Leibniz was firmly persuaded that all non-syllogistic inferences are embellishments of the syllogism, since

> (1) the categorical proposition is the basis of the rest, and modal, hypothetical, disjunctive, and all other propositions presuppose it. ([4], par. 2.)

Leibniz, following the Aristotelian tradition, conceives of all propositions as being of a subject-predicate form, so that

> (2) A proposition is that which states what term is or is not contained in another. ([1], par. 195.)

He also includes as propositions structures of terms that attribute *coincidence* or identity to such terms. But Leibniz himself formulates containment of a term in another as a special case of coincidence. Thus, where '≡' replaces Leibniz's own sign '∞' for coincidence:

> (3) Generally, "A is B" is the same as "A ≡ AB"; for it is clear from this that B is contained in A. ([1], par. 83; see also pars. 16, 36, 54, 88, 189, 199, *et al.*)

Hence, Leibniz justifiably characterizes a proposition more simply, thus:

> (4) A proposition is [of one of the forms] "A coincides with B," "A does not coincide with B." (A and B can signify terms, or other propositions.) ([1], par. 4.)

The important generalizing parenthetical remark after his characterization of propositions repeats a note attached to a previous passage, just two paragraphs before Leibniz begins numbering the sections of [1]. It is, of course, based on the principle that

> (5) Any proposition can be conceived as a term ([1], par. 109; see also pars. 75, 138, 189, 197, 198, and par. 55, where Leibniz declares: "I count the whole syllogism as proposition also.")

This principle allows Leibniz to transfer the syllogistic machinery to other propositions:

> (6) Whatever is said of a term which contains a term can be said of a proposition from which another proposition follows. ([1], par. 189.)

In par. 198 he says: "that a proposition follows from a proposition is simply that a consequent is contained in an antecedent." But there is an ambiguity here. The word 'follows' may be taken to mean implication, rather than taken to represent the connective of conditionalization. On the former interpretation the principle would be a special case of the Deduction Theorem, i.e., a special case of the principle of conditional proof. In the latter case the passage says that a conditional of the form "If p, then q" is, in the light of (3) above, to be analyzed as the proposition "$p \equiv pq$." Undoubtedly both are true. But the second interpretation is the stronger one. There is additional evidence in C 423:

> (7) Une proposition catégorique est vrai quand le prédicat est contenu dans le sujet; une proposition hypothetique est vrai quand le conséquent est contenu dans l'antécédent.

Leibniz himself analyzes "If L then M" as "$L \equiv LM$" (C 408).

One of the fundamental principles in Leibniz's philosophy is this: "In every proposition the (concept of the) predicate is included in the (concept of the) subject." From this principle he derives the important metaphysical conclusion about each monad including a representation of the whole universe, which conclusion is itself the basis for his preestablished harmony. Thus, there cannot be true negative propositions, i.e., true propositions in which negation has the largest possible scope. Hence, for profound philosophical reasons Leibniz is committed to the view that negation cannot be, in the end, a genuine propositional connective, but only a term operator. This view is reinforced by his experience as an algebraist. The elementary algebra he knew is primarily a theory of equations. Thus, the general logical theory he was searching for had to be fundamentally an equational theory.

Although Leibniz discusses negative propositions, i.e., propositions with negation as their primary connective, without apparent discomfort, he is very much concerned with their reduction to non-negative propositions. He is most desirous to formulate a strict equational calculus in which all propositions are about the coincidence of terms (or propositions). Thus, he argues that:

> (8) "Not every" and "not some" may not properly occur in propositions; for they only negate the proposition affected by the sign "every" or "some" and do not make a new sign "not-every" or "not-some." ([1], par. 185.)

Leibniz claims somewhat inexactly, because of (3) and (4) above, that "Every proposition which is commonly used in speech comes to this, that it is said what term contains what." ([1], par. 184.) At any rate, since Leibniz knows how to eliminate negative propositions he can afford the luxury to speak of them. One method of elimination (discussed in [1], par. 185) consists of predicating falsehood of a term, rather than of negating truth of it. He discusses a second method:

> (9) If we use "AB exists" and "A not-B exists" for particular propositions, and if for universal propositions we use "A contains B" or "A contains not-B," we shall be able to dispense with negative propositions. (Marginal note at the end of [1], par. 198; C 398, P 87nl.)

This elimination of negative propositions is surprising in its lack of uniformity. Not only does Leibniz leave existence out of the analysis of the universal propositions, but he does not treat here existence (or truth) as a genuine term. Leibniz, having analyzed "Some A's are B's" as 'AB exists," does not go on to interpret this as "AB contains existence," which by principle (3) above would be symbolized as "$AB \equiv AB$ (Existence)." He takes this step in [8] (G 213, P 117), but he leaves the concept *existence* or *entity* somewhat isolated.

Leibniz comes, in [8], to the verge of distinguishing a universal proposition with existential import from one without existential import. In fact he fails to make the distinction and claims that the subject term is an entity in full stare of his own symbolism that exhibits the distinction quite openly. (See G ii 213ff or P 117ff.) He now proposes the following analysis of the form of the classical categorical propositions:

A:	All S's are P's	E:	No S is P
	S not-$P \equiv S$ not-P not-Entity		$SP \equiv SP$ not-Entity
I:	Some S's are P's	O:	Some S's are not P's
	$SP \equiv SP$ Entity		S not-$P \equiv S$ not-P Entity

Clearly the A- and O-propositions are contradictory of each other just as the I- and the E-propositions are each other contradictories. But it is not at all clear that the A-proposition implies its corresponding I-proposition. Undoubtedly, the A-proposition as symbolized above and the proposition that S's exist do imply together that $SP \equiv SP$ Entity. But it is not clear that "S not-$P \equiv S$ not-P not-Entity" implies "$S \equiv S$ Entity," that is, that S's exist. We will say more about this in Part II, section 2.

We must note that the above chart is important also because it contains a third way in which negation as a propositional connective can be eliminated in favor of negation as a term operator. Leibniz is fully aware of this and proclaims with satisfaction:

> (10) So we have reduced all categorical propositions of logic to a calculus of equivalences [i.e., of coincidence]. ([10], G vii 214, P 118.)

2 *Unofficial principles* The preceding principles (1)-(10), with the exception of (7), are so to speak anatomic principles. They deal with the form of propositions. In the development of a calculus they pertain to the formation rules which determine the well-formed formulas of the calculus. Now I

pass to discuss other formation rules that Leibniz did not state, but are everywhere present in his discussion. They are extracted from his total practice in calculus designing in the papers under discussion:

> (11) Molecular terms are formed by negating a term and by conjoining two terms. That is, if A and B are terms, not-A and not-B are terms, and so are AB and BA.
>
> (12) The copula which forms propositions out of terms is *coincidence*, and no proposition has more than one copula.

Principle (11) needs no commentary. Leibniz has often been chastized for his neglect of disjunction. (See, for instance, P lxi.)

The first part of (12) merely repeats part of (3) and (4) above. The second part is my generalization from Leibniz's uniform practice. We noted how in [1] Leibniz insists on treating propositions as terms and that his term variables have both propositions and proper terms as substituends. Yet in the essays we are discussing only in [1], (25) and (61), does he discuss a formula with iterated coincidence sign. In all three essays under examination here Leibniz comes to junctures at which he should iterate his coincidence sign, but he refrains from doing it and speaks, in Latin, metalinguistically, of coincidence or equivalence or sameness. We shall see some examples later on. The explanation seems to be the fact, too obvious for him to mention in notes not intended for publication, that algebraic equations do not contain equations as parts. Of course, it is open to the reader of those texts to apply the Leibnizian principles (1)-(10) and introduce, in spite of Leibniz, iterated copulae. Thus, we must distinguish two Leibnizian logical calculi: (i) the straightforward calculus with iterated copula, and (ii) the non-iterative calculus.

The non-iterative calculus creates an unavoidable problem. Propositions can be treated as terms; propositions have a copula, but terms do not; hence, *in order to treat a proposition as a term there must be a transformation scheme* that yields terms out of propositions. Of course, there is no hint in the three essays under discussion that Leibniz was fully aware of this problem. Actually, the solution to the problem is not far off in the light of principle (11). Because of (11) the problem is that of transforming a formula of the form "$A \equiv B$" (in our notation) into an expression made up of the terms in A or B, and negation and conjunction. I think that, although Leibniz did not have á fully satisfactory conception of conjunction, or a useful heuristic model for it, he would have solved this problem if he had considered it in detail. He was close enough and he would have interpreted conjunction as overlapping, so that a term like AB can be interpreted as the overlapping of the concepts A and B. Thus, the coincidence of the concepts (or terms) A and B is identical with the overlapping of the complement of the overlapping of A and B and the complement of the overlapping of not-A and not-B. For clearly, A and B will fail to coincide if either A overlaps not-B or not-A overlaps B. All of this was within the reach of Leibniz.

It is perplexing to see Leibniz not pursuing his ideas further in ways that seem obvious to us (anachronistically, of course). If he had taken the iteration step he would have fulfilled his claim to treat propositions as terms. The non-iterative calculus is also interesting, and if he had dwelled upon his assumption (12) he would also have made some important discoveries. It seems to me that the non-iterative calculus is the calculus that Leibniz adumbrated and the one he was in the process of building up. Besides his steadfast practice of refusing iteration there is his general practice of algebraic equations and his idea of producing a mathematics of reason, or an algebra of logic. Even his later calculi ([5] and [6]) are uniform in treating negation and addition and subtraction as the only term-forming operators, and identity or coincidence as the only proposition-forming operator.

The same inspiration from algebra is the one we already saw operating before in Leibniz's desire both to eliminate negation as a proposition-forming operator, and to restrict it to forming terms.

II LEIBNIZ'S AXIOMS AND INFERENCE RULES

1 *The official axioms and rules* Leibniz's investigations in the three essays under examination involve several attempts at axiomatizing his system of logic. The most mature fruit appears in [3]. This is a two-page piece containing axioms and principles with some deductions, but with no discussion of applications or rules of formation. This is the relevant text for us at this juncture. Again, we must consider the principles he introduces explicitly and officially, so to speak, as well as those which he introduces in his practice, i.e., in his demonstrations of theorems.

Leibniz puts forward the principles below each of which, in Leibniz's own words, is numbered with the number Leibniz himself gives either to it or to the paragraph where he discusses it:

(1) "$A \equiv B$" is the same as "$\overline{A \equiv B}$ is a *true* proposition."
(2) "Not-($A = B$)" is the same as "$\overline{A \equiv B}$ is a *false* proposition."
(3) $A \equiv AA$.
(4) $AB \equiv BA$.
(5) $A \equiv B$ means that one can be substituted for the other, B for A or A for B, i.e., they are equivalent.
(6) 'Not' immediately repeated destroys itself.
(7) Hence, $A \equiv$ not-not-A.
(8) Moreover, "$A \equiv B$" and "not-not-($A \equiv B$)" are equivalent.
(9) That in which there is $\overline{A \text{ not-}A}$ is a *non-entity* or a *false term*; e.g., if $C \equiv AB$ not-B, C would be a non-entity.
(17) Not-$B \equiv$ not-B not-(AB), i.e., not-B contains not-(AB), or not-B is not-(AB).
(20) "Not-[not-(AB) $\equiv$ not-(AB) not-B]" and "Not-(AB) $\equiv$ not-(AB) not-A" are equivalent.

Principles (1) and (2) are presumably meta-linguistic, or second-order, principles showing how propositions can be terms. Leibniz puts a bar over "$A \equiv B$," which is not exactly the equivalent of our quotation marks. The overbar seems, rather, to be mainly a scope indicator as well as a signal that what is under it is a term. Thus, we may construe (1) and

(2) as not really having the coincidence sign at all, except for the implicit one contained in the 'is' of 'is a true [false] proposition'. Yet Leibniz refuses to go on to symbolize his analyses in (1) in the form (P): "$t(A \equiv B) \equiv t(A \equiv B)$ Truth," where t maps propositions into terms, and is, thus, in part like Leibniz's overbar. In any case (1) is the rule that (P) can be inferred from "$A \equiv B$" and that "$A \equiv B$" can be inferred from (P).

Principle (2) is the schema, discussed in [1], par. 185, for the elimination of propositional negation. If so it is *not* an inferential schema like (1). Principles (1) and (2) together determine the bivalence of propositions. They do not apply to proper terms. Clearly, if a term AB is true, its negation not-(AB) may also be true, for the former truth amounts to the claim that AB's exist, which is compatible with there being A not-B's as well as not-AB's and also not-A not-B's. This shows that Leibniz erred in not distinguishing carefully the falsity of a term, which we saw above he treats as *non-entity* (or *no-existence*), from the falsity of a proposition. This is perplexing because one would expect his interest in separating propositions from terms to have provided the basis for an insight into the distinction between no-existence and falsity.

Leibniz did not seem to have realized that the bivalence of propositions requires that they, or their corresponding concepts or terms, be treated as singular concepts. Thus, the conditional proposition *If P then Q* is analyzed as *$t(P)$ is in $t(Q)$*, where the subject $t(P)$ is like an individual, or an individual concept, like the Apostle Peter or the concept *the Apostle Peter*. Hence, *$t(P)$ is in $t(Q)$* is like *The Apostle Peter is friendly.* Metaphysically this is fine, for it may be argued that just as the concept *the Apostle Peter* is, if instantiated at all, instantiated by just one object, so, if a proposition is true (or a propositional concept is instantiated), it will be made true by (or be instantiated by) just one fact.

The bivalence of propositions is precisely what corresponds to Leibniz's view that singular propositions are at once universal and particular:

> Should we say that a singular proposition is equivalent to a particular and to a universal proposition? Yes, we should. . . . For "Some Apostle Peter" and "Every Apostle Peter" coincide [and coincide with "Peter"], since the term ["Peter"] is singular. ([8], very first paragraph; G vii 211 and P 115.)

Unfortunately, Leibniz apparently never put together the special principles required by singular propositions with the general principles of his logical system.

2 *Unofficial principles* In paragraph (13) of [3] Leibniz attempts to prove "not-[$AB \equiv C$ not-(EB)]," which is perhaps a lapsus for "not-[$AB \equiv CE$ not-B]." In his proof he uses:

1. Indirect proof, i.e., *reductio ad absurdum,*

and

2. The associativity of conjunction, i.e., $A(BC) = (AB)C$.

Thus we should consider his total system as having the latter axiom and either an axiom or a rule for indirect proof. Here is another place where Leibniz has to be chastized for not having been careful in formulating his logical system. The case of association is tantalizing. Apparently he never realized the need for a principle allowing it, even though he uses it widely, especially in his proof that $2 + 2 = 4$ is analytic, and in his late [7], IV, vii 10 (G v 394), he even indicates its use by means of braces. This is evidently like the non-iteration of the copula because so obviously equations do not contain equations: similarly the associativity of conjunction is so obvious that it can easily escape notice.

On the other hand, Leibniz did come to list a principle of indirect proof under the name of inference by regress—i.e., regress to the premises so as to posit one as false. This principle is stated in [9] (C 412 and P 107) in the process of applying it to the derivation of the moods of the second and third figures of the syllogism. In [10] (G vii 208, P 112), the principle of regress is the fifth general principle listed on its own right. (This suggests that [10] is later than [9].)

The development of his awareness of the primitivity of *reductio ad absurdum* that Leibniz underwent beyond 1686 compares with the development of his awareness of the primitivity for his calculus of the commutativity of conjunction. This property of conjunction is assumed in his several attempts at axiomatics in [1], but it is listed separately as an axiom only in [2] and [3].

In Part I, section 1, above we put forward a chart pieced together from Leibniz's discussion in [8], where he eliminates negation as a propositional connective in terms of the negation of the term *Entity* or *existence*. The main point is this:

E: No S is P	(a) $SP \equiv (SP)$ not-*Entity*
I: Some S's are P	(b) $SP \equiv (SP)$ *Entity*

Leibniz has already in [3] given two other analyses of the E-proposition:

(c) $S \equiv S$ not-P
(d) not-($SP \equiv (SP)$ *Entity*)

The crucial thing is that Leibniz so understands the concept of *Entity* (*ens*) that (a), (c), and (d) are equivalent to one another. Leibniz is here groping for an account of the existential quantifier in monadic quantification.

The equivalence between (a) and (c) can be illuminated by considering the contraposition of (a), a principle which Leibniz derived in [3] (19) from [3] (17). The contraposition of (a), after applying double negation, is "*Entity* $\equiv$ *Entity* not-(SP)." Now, "not-(SP)," although just a term, is undoubtedly equivalent to (c), "$S \equiv S$ not-P." Presumably this holds on the ground that what *Entity* implies is true. Hence, (a) and (c) are equivalent. Thus, the term *Entity* seems well-behaved.

The equivalence between (a) ("$SP \equiv (SP)$ not-*Entity*") and (d) ("not-($SP \equiv SP$ *Entity*)") is both troublesome and peculiar. It cannot be part of Leibniz's system if propositional negation is eliminated. The equivalence would have to be, on the one hand, an external procedure introducing negative propositions as terms into the system, and, on the other, an equivalence reflected by the propositional terms representing (a) and (d). Clearly, one important case is the very fruitful representation used in the preceding paragraph to show that (a) is equivalent to (c). Using it again we find:

(a) $SP \equiv (SP)$ not-*Entity*	(a′) not-((SP) *Entity*)
(d) not-($SP \equiv SP$ *Entity*)	(d′) (SP) not-*Entity*

We should have the equivalence between (a) and (d) reflected by the equivalence between (a′) and (d′):

(ϵ) not-((SP) *Entity*) $\equiv$ (SP) not-*Entity*

But (ϵ) leads to serious trouble:

1.	not-(SP) $\equiv$ not-(SP)	[3] (3) and (5)
2.	not-(SP) $\equiv$ not-((SP) not-*Entity*)	(a), 1; [3] (5)
3.	not-(SP) $\equiv$ not-not-((SP) *Entity*)	(ϵ), 2; [3] (5)
4.	not-(SP) $\equiv$ (SP) *Entity*	[3] (7), 3; [3] (5)
5.	not-(SP) *Entity* $\equiv$ ((SP) *Entity*) *Entity*	4; [3] (5)
6.	*Entity* $\equiv$ *Entity* not-(SP)	(a), Contraposition: [3] (19)
7.	*Entity* $\equiv$ not-(SP) *Entity*	6; [3] (4) and (5)
8.	*Entity* $\equiv$ ((SP) *Entity*) *Entity*	7, 5; [3] (5)
9.	((SP) *Entity*) *Entity* $\equiv$ (SP) (*Entity*) *Entity*	Association
10.	*Entity* $\equiv$ (SP) *Entity*	9; [3] (3) and (5)
11.	not-(SP) $\equiv$ not-(SP) not-*Entity*	10; [3] (19)

Now, step 10 says that everything is SP, while its contrapositive 11 says that the complement of SP does not exist. Each contradicts (a), which says that SP does not exist. Thus, the E-proposition implies a contradiction. By similar reasoning, the A-proposition also implies a contradiction.

Leibniz's *Entity* analysis of the classical categorical propositions disturbs the relationships between the universal and the existential quantifiers. Consider the proposition "Everything is A." On Leibniz's analysis it is "not-A $\equiv$ not-A not-*Entity*." By his very own principle of contraposition this is equivalent to "*Entity* $\equiv$ *Entity* A," i.e., *Entity* or existence implies A. On the other hand, "A's exist" or "Something is A" is "$A \equiv A$ *Entity*," i.e., A implies *Entity*. Thus, the above universal proposition is the converse of its own existential proposition and does not imply it without further principles.

Leibniz did not see the difficulties with his treatment of existence, or *Entity*, as a term. The most obvious difficulty lies in the transitivity of coincidence. On his approach, "A's exist" is analyzed as "A implies existence." Hence, since AB implies A, regardless of the nature of A and B, if A's exist, AB also implies existence, and, hence, AB's exist. This difficulty can initially be avoided by keeping *Entity*, or existence, with a fixed scope, i.e., by refusing *Entity* the property of associativity. This would be the beginning of the treatment of existence, *not* as itself a term, but rather as a term-forming operator. Thus, Leibniz could have granted that "$A \equiv A$ *Entity*" implies "$AB \equiv (A$ *Entity*$)\ B$," without the latter implying "$AB \equiv (AB)$ *Entity*" or "$B \equiv B$ *Entity*." He could then have allowed the term "(A *Entity*) (not-A *Entity*)" to be consistent: it does not have the term "*Entity* not-*Entity*." The non-associativity of *Entity* would also stop the above formal argument at step 9. But Leibniz was never really aware of his assuming and using the associativity of term composition. Hence, he was not in a position, when he wrote [8], to think of existence, or *Entity*, as a term-forming operator.

A difficulty that cannot be solved by merely excepting the term *Entity* from associativity or other axioms is this. "AB's exist" implies "A's exist," yet "AB implies *Entity*" does not in general, without special principles, imply "A implies *Entity*." We need a principle like: from "$AB \equiv (AB)$ *Entity*" infer "$A \equiv (A)$ *Entity*." Incidentally, the converse approach, to treat "A's exist" as "Existence implies A," has the converse malady. While "A's exist and B's exist" does not imply "AB's exist," "Existence implies A and Existence implies B" does imply "Existence implies AB."[2]

Apparently, then, Leibniz did not reach a theoretical understanding of the existential quantifier. This, however, does not ruin his treatment of Aristotelian syllogistics. On the Aristotelian assumption that the atomic terms of his calculus all have instances (or as Leibniz puts it: "In all of them is tacitly assumed that the ingredient term is an entity" ([8], G vii 214, P 119), his equational calculus can handle the classical syllogisms.[3]

III THE LEIBNIZIAN CALCULI L.C AND L.Ci

In the light of our discussion in section 3, Part II, we conclude that Leibniz did not have an adequate incipient grasp of the monadic predicate calculus. Given his paramount concern with the syllogism one would have expected that the monadic functional calculus would have been the natural fruit of his attempts at generalizing the logical theory of his time. Somewhat ironically, his generalizing efforts came very close to yielding an adequate formulation of the propositional calculus. This is what we establish in this section.

We will formulate first the non-iterative logical calculus L.C, which incorporates Leibniz's own axioms as nearly in his own versions as possible. Besides the streamlining of Leibniz's formulations, L.C essentially adds, to what Leibniz propounded or assumed, a scheme for transforming propositions into terms. L.C contains a complete version of the propositional calculus.

By the introduction of a transformation scheme as an axiom we produce the iterative calculus L.Ci.

1 *The calculus* L.C

A. *Primitive signs* The primitive terms of L.C are, (1) a denumerable supply of primitive proper terms; (2) the term connectors; $\sim$ (read as 'not'), '&' (read as 'and' or 'but'), and 'ϵ' (read as 'exist(s)'); (3) the propositional copula '≡'; (4) parentheses '(' and ')' as scope indicators. From now on we shall use the signs exhibited above autonomously. The primitive terms are supposed to denote concepts or properties, just as Leibniz wanted. Existence is, thus, treated as a term-forming operator.

B. *Formation rules* These rules define the terms and the well-formed formulas or sentences of L.C.

1. The *terms* of L.C are each of the sequences A of primitive signs of L.C that has one of the following forms: (1) A is a primitive term; (2) A is $(\sim B)$, where B is a term; (3) A is $(B \& C)$, where B and C are terms; (4) A is (ϵB), where B is a term.
2. The *well-formed formulas* (wffs) of L.C are all and only those sequences of primitive terms of L.C that are of the form: $A \equiv B$, where A and B are both terms of L.C.

Convention We shall delete the sign '&' as well as the outermost parentheses of a term.

C. *Proposition-term transformation scheme* Leibniz's idea that propositions can be treated as terms requires a mechanism that transforms the former into the latter and vice versa. A simple mechanism that suffices here as a primitive is this:

TS. A wff of L.C of the form $A \equiv B$ corresponds to the term $\sim(A \ \& \sim B) \ \& \sim(B \ \& \sim A)$.

Convention on Variables In order to give more generality to our formulation of L.C we adopt the convention of using square brackets around a wff or the name of a wff of L.C in order to indicate the term corresponding to the formula, regardless of the transformation scheme being adopted. Thus, while "$A \equiv B$" is a wff where A and B are terms, "$[A \equiv B]$" is the term corresponding to it. We shall also use the capital letters 'P', 'Q', 'R', and 'S' with subscripts if necessary as meta-linguistic variables ranging over propositional terms. We shall use the corresponding lower case letters as meta-linguistic variables ranging over propositions. In a given formula or set of formulas, 'P', 'Q', 'R', and 'S' will each represent the term corresponding, respectively, to the proposition represented by 'p', 'q', 'r', and 's'. We shall continue to use 'A', 'B', and 'C' as general meta-linguistic variables ranging over all types of terms.

D. *Axioms of* L.C The axioms of L.C are wffs of L.C that have at least one of the following forms:

General Axioms

[3] (13u-)	A1. $(AB)C \equiv A(BC)$.
[3] (4)	A2. $AB \equiv BA$.
[3] (3)	A3. $A \equiv AA$.
[3] (17)	A4. $\sim B \equiv \sim B \sim (AB)$.
[3] (7)	A5. $A \equiv \sim\sim A$.

Existence Axioms: We shall not provide them. The ones that Leibniz adumbrated for his term *Entity* are not useful for term-operator ϵ.

Special Term Axioms: Since **L.C** is not general enough to include the logic of singular propositions, it does not include the needed axioms for the treatment of individual concepts.

The expressions 'A1' through 'A5' are our labels for the axioms. The numeral within parentheses following '[3]' before each axiom label is the number of the paragraph of [3], quoted above in Part II, section 2, where Leibniz formulates the axiom in question. The letter 'u' means that axiom A1 is unofficial, and the sign '-' means that some distortion is involved in our listing it, namely, the distortion of making explicit what is implicit or unofficial in Leibniz.

E. *Primitive rules of inference* We assume that the sequences of symbols mentioned below are all wffs of **L.C**. The symbol '$p_1, \ldots, p_n \vdash q$' means that the wff q of **L.C** can be derived from the wffs $p_1, \ldots, p_n$ by the rules of **L.C**. '$G(B)$' represents any wff of **L.C** containing one or more occurrences of the term B. We include as a wff $G(B)$ the wff p itself. '$G(B/A)$' represents any wff of **L.C** resulting from a wff $G(B)$ by the replacement of one or more occurrences of B with occurrences of A. If $G(P)$ is p, then $G(P/Q)$ is q.

General Rules

[3] (5-) R*. $A \equiv B,\ G(B) \vdash G(B/A)$.
[3] (13u-) R1. If $p_1, \ldots, p_n,\ P \equiv Q \vdash A \sim A \equiv \sim(B \sim B)$,
$p_1, \ldots, p_n \vdash P \equiv \sim Q$.
[3] (1, 2, 9-) R2. $p \vdash \sim P \equiv A(B \sim B)$.
[3] (1, 2, 9-) R3. $\sim P \equiv A(B \sim B) \vdash p$.

The small distortions R* makes to Leibniz's paragraph (5) of [3] are: (i) R* is weaker than Leibniz's own rule of substitution of coincidents or equivalents by not including the replacement of the lefthand term A; (ii) R* introduces more explicitly the special case $P \equiv Q,\ p \vdash q$.

R.1 involves the distortion of reaching to [10] and then of using bivalence to introduce the negation of the assumption. This fits his plan of not using negation as a propositional connective.

R2 and R3 involve the distortion of Leibniz's statement of them consisting of taking a negated proposition rather than an unnegated one to equate with a contradiction.

F. *Some theorems and derived rules of* **L.C** We adopt the standard definitions of 'theorem' and 'derivation with premises'. Here we list some simple theorems and derived rules that will for the most part be referred to in the next sections. We do not give proofs, but indicate the major steps in the proofs. Many applications of R* will go unnoted.

T1. $\vdash A \equiv A$.

Proof: A3: $A \equiv AA,\ A \equiv AA$; by R*: $A \equiv A$.

Definitions

Def. T. '$\underset{\approx}{\mathrm{T}}$' is short for '$[P \equiv P]$'.
Def. t. '$\underline{\mathrm{t}}$' is short for '$P \equiv P$'.
Def. F. '$\underset{\approx}{\mathrm{F}}$' is short for '$\sim[P \equiv P]$'.

T2. $\vdash \underline{\mathrm{t}}$.
T3. $\vdash \underset{\approx}{\mathrm{T}} \equiv \sim(P \sim P)$.

Proof: By TS: $\underset{\approx}{\mathrm{T}} \equiv \sim(P \sim P) \sim (P \sim P)$; A3, by R*.

T4. $\underset{\approx}{\mathrm{F}} \equiv \sim \underset{\approx}{\mathrm{T}}$.

Proof: By TS, Def. F and A3.

DR1. $A \equiv B \vdash B \equiv A$.

Proof: By A2 and R*.

DR2. $A \equiv B,\ G(A) \vdash G(A/B)$.
DR3. $A \equiv B \vdash \sim A \equiv \sim B$.

Proof: By T1: $\sim B \equiv \sim B$; hypothesis; R*.

T5. $\vdash \underset{\approx}{\mathrm{F}} \equiv A\, \underset{\approx}{\mathrm{F}}$.

Proof: From T2 by R2 and T4 by R*.

T6. $\vdash \underset{\approx}{\mathrm{F}} \equiv B \sim B$.

Proof: From T2 and R2 taking A to be B, then by A1 and A2.

DR4. $A \equiv AB \vdash \sim B \equiv \sim B \sim (A)$.

Proof: (Given by Leibniz in [3] (19).) By A4: $\sim B \equiv \sim B \sim (AB)$; hypothesis; replacement by R*.

T7. $\vdash A \equiv A\, \underset{\approx}{\mathrm{T}}$.

Proof: From T5 by DR4, T4, and A5.

DR5. $AB \equiv \underset{\approx}{\mathrm{T}} \vdash A \equiv \underset{\approx}{\mathrm{T}}$.

Proof:

1. $AB \equiv \underset{\approx}{\mathrm{T}}$	Hypothesis
2. $\vdash A \equiv A\, \underset{\approx}{\mathrm{T}}$	T7
3. $A \equiv A(AB)$	1, 2; R*
4. $A \equiv (AA)B$	3; A1; R*
5. $A \equiv AB$	4; A3; R*
6. $A \equiv \underset{\approx}{\mathrm{T}}$.	1, 6; DR2

DR6. $p \vdash \sim P \equiv \underset{\approx}{\mathrm{F}}$.

Proof: From R2 by A1 and A3 taking C and A as P.

DR7. $p \vdash P \equiv \underset{\approx}{\mathrm{T}}$.

Proof: From DR6, DR3, T4, and A5.

DR8. $A \equiv B,\ C \equiv D \vdash AC \equiv BD$.

Proof: From $AC \equiv AC$, T1, DR2, and hypothesis.

DR9. $A \equiv B,\ C \equiv B \vdash AC \equiv B$.

Proof: From DR8 and A3.

DR10. (*Modus Ponens*) $P \equiv PQ,\ P \equiv \underset{\approx}{\mathrm{T}} \vdash Q \equiv \underset{\approx}{\mathrm{T}}$.

Proof:

1.	$P \equiv PQ$	Hypothesis
2.	$P \equiv \underset{\approx}{\mathrm{T}}$	Hypothesis
3.	$\vdash Q \equiv Q\underset{\approx}{\mathrm{T}}$	T7
4.	$Q \equiv QP$	2, 3; R*
5.	$Q \equiv P$	1, A2, 4; R*
6.	$Q \equiv \underset{\approx}{\mathrm{T}}$	2; 5; DR2

DR11. $P \equiv PQ,\ P \equiv \underset{\approx}{\mathrm{T}} \vdash q$.

Proof: From DR10, T2, and R*.

DR12. *If* $\Gamma,\ P \equiv \sim Q \vdash \underset{\approx}{\mathrm{T}} \equiv \underset{\approx}{\mathrm{F}}$, *then* $\Gamma \vdash P \equiv Q$.

Proof: From R1, T6, T3, T4.

T8. $\vdash [P \equiv \underset{\approx}{\mathrm{T}}] \equiv P$.

Proof:

1.	$\underset{\approx}{\mathrm{T}}P \equiv P$	T7, DR1, A2
2.	$\sim \underset{\approx}{\mathrm{F}}P \equiv P$	2, T4, DR3, A5
3.	$\underset{\approx}{\mathrm{F}} \equiv P\underset{\approx}{\mathrm{F}}$	T5
4.	$\sim (P\underset{\approx}{\mathrm{F}})\, P \equiv P$	2, 3; DR2
5.	$\sim (P\underset{\approx}{\mathrm{F}}) \sim \sim P \equiv P$	4, A5; R*
6.	$\sim (P \sim \underset{\approx}{\mathrm{T}}) \sim (\sim P\underset{\approx}{\mathrm{T}}) \equiv P$	5; T4; T7; R*
7.	$\sim (P \sim \underset{\approx}{\mathrm{T}}) \sim (\underset{\approx}{\mathrm{T}} \sim P) \equiv P$	6; A2; R*
8.	$[P \equiv \underset{\approx}{\mathrm{T}}] \equiv P$	7; TS; R*

T9. $\vdash [P \equiv \underset{\approx}{\mathrm{F}}] \equiv \sim P$.

Proof: Similar to the preceding one.

T10. $\vdash P \sim (PQ) \equiv (P \sim Q)$.

Proof:

1.	$\sim (P \sim (PQ)) \equiv (P \sim Q)$	Assumption for R1
2.	$\sim P \equiv \sim P \sim (\sim (PQ)P)$	A4
3.	$\sim P \equiv \sim P \sim (P \sim (PQ))$	2, A2; R*
4.	$\sim P \equiv \sim P\, (P(P \sim Q))$	1, 3; DR2
5.	$\sim P \equiv \underset{\approx}{\mathrm{F}}$	4, A1, T6, T5, R*
6.	$P \equiv \underset{\approx}{\mathrm{T}}$	5; DR3, A5, T4
7.	$\sim (\underset{\approx}{\mathrm{T}} \sim (\underset{\approx}{\mathrm{T}}\, Q)) \equiv \underset{\approx}{\mathrm{T}} \sim Q$	6, 1; DR2
8.	$\sim \sim Q \equiv \sim Q$	7, T7, A2; R*
9.	$\sim \sim Q \equiv \sim Q \sim \sim Q$	8, T1, A2; DR9

10. $\sim Q \equiv \sim Q \sim\sim Q$	8, T1; DR9
11. $\sim\sim Q \equiv \sim(\sim Q \sim\sim Q)$	10; DR3
12. $\sim Q \sim\sim Q \equiv \sim(\sim Q \sim\sim Q)$	9, 11; DR2
13. $\sim(P \sim(PQ)) \equiv \sim(P \sim Q)$	1-12; R1
14. $P \sim(PQ) \equiv P \sim Q$	13; DR3, A5

DTS. *A wff of* **L.C** *of the form* $P \equiv PQ$ *corresponds to the term* $\sim(P \sim Q)$, i.e., $\vdash[P \equiv PQ] \equiv \sim(P \sim Q)$.

Proof: By TS, $[P \equiv PQ]$ is: $\sim(P \sim(PQ)) \sim ((PQ) \sim P)$. Now:

1. $\sim(P \sim(PQ)) \sim ((PQ) \sim P) \equiv \sim(P \sim(PQ)) \sim ((P \sim P)Q)$	T1, A1, A2; R*
2. $\sim(P \sim(PQ)) \sim (PQ \sim P) \equiv \sim(P \sim(PQ))$	T6, T5, T4, A5, T7
3. $\sim(P \sim(PQ)) \equiv \sim(P \sim Q)$	T10, DR2, A5
4. $[P \equiv PQ] \equiv \sim(P \sim Q)$	

DR13. $[P \equiv \underset{\approx}{\underline{F}}] \equiv \underset{\approx}{\underline{F}} \vdash \underline{p}$.

DR14. $\sim(\sim(P \sim Q) \sim (Q \sim P)) \equiv \underset{\approx}{\underline{T}} \vdash \sim(PQ) \sim (\sim P \sim Q) \equiv \underset{\approx}{\underline{T}}$.

Proof:

1. $(\sim(PQ) \sim (\sim P \sim Q)) \equiv \underset{\approx}{\underline{F}}$	Assumption for R1
2. $\sim(P \sim Q) \sim (Q \sim P) \equiv \underset{\approx}{\underline{F}}$	hypothesis, DR3, A5
3. $P \equiv P \sim(Q \sim P)$	A4, A5
4. $Q \equiv Q \sim(P \sim Q)$	A4, A5
5. $PQ \equiv PQ(\sim(P \sim Q) \sim (Q \sim P))$	3, 4, DR8, A1, A2
6. $PQ \equiv \underset{\approx}{\underline{F}}$	5, 2, T6, T5, DR2, R*
7. $\sim P \equiv \sim P \sim(\sim QP)$	A4
8. $\sim Q \equiv \sim Q \sim(\sim PQ)$	A4
9. $\sim P \sim Q \equiv \sim P \sim Q(\sim(P \sim Q) \sim (Q \sim P))$	7, 8; DR8, A1, A2
10. $\sim P \sim Q \equiv \underset{\approx}{\underline{F}}$	9, 2; T6, T5, DR2, R*
11. $\sim(\underset{\approx}{\underline{F}}) \sim (\underset{\approx}{\underline{F}}) \equiv \underset{\approx}{\underline{F}}$	10, 1; DR2
12. $\underset{\approx}{\underline{T}} \equiv \underset{\approx}{\underline{F}}$	11; T4, DR2, A3
13. $\sim(PQ) \sim (\sim P \sim Q) \equiv \underset{\approx}{\underline{T}}$	R1; T4, etc.

DR15. $\sim[P \equiv Q] \equiv \underset{\approx}{\underline{T}} \vdash [P \equiv \sim Q] \equiv \underset{\approx}{\underline{T}}$.

G. *The deduction theorem in* **L.C** *If* $\Gamma, p \vdash q$, *then* $\Gamma \vdash P \equiv PQ$.

Proof: Let $s_1, \ldots, s_m$ be a proof of q from Γ and p. Then each S_i is (i) an axiom, (ii) a member of Γ, (iii) p, or (iv) a consequence of previous s's by R*, or (v) by R1, or (vi) by R2, or (vii) by R3. We can construct a proof from Γ to $P \equiv PS_i$ by inserting steps of the form and making further derivations as indicated below.

Cases (i) and (ii) We have s_i as premise. Therefore:

DR16. $s_i \vdash P \equiv PS_i$.

1. s_i	
2. $\sim S_i \equiv \underset{\approx}{\underline{F}}$	1; DR6
3. $P \sim S_i \equiv P\underset{\approx}{\underline{F}}$	2, T1; DR8
4. $\vdash \underset{\approx}{\underline{F}} \equiv P\underset{\approx}{\underline{F}}$	T5

5. $P \sim S_i \equiv \underset{\approx}{F}$	4, 3; R*
6. $\sim(P \sim S_i) \equiv \underset{\approx}{T}$	5, DR3; Def. T
7. $\vdash \underline{t}$	T1, Def. T
8. $P \equiv PS_i$	6, 7; R* and DTS

Case (iii) $\vdash P \equiv PP$. A3

For cases (iv)-(vii) we assume that we have a proof up to s_{i-1}.

Case (iv) We have three subcases: (a) s_h is p, (b) s_j is p, (c) both are p.

s_h: $A \equiv B$
s_j: $G(B)$
s_i: $G(B/A)$

To derive $P \equiv P[G(B/A)]$ we apply R1 proceeding as follows for (a) - (c):

1. $P \equiv P[A \equiv B]$	A3, or DR16
2. $P \equiv P[G(B)]$	A3, or DR16
3. $\Gamma, \sim P \equiv P[G(B/A)]$	Assumption for R1
4. $\sim P \sim P \equiv \sim P(P[G(B/A)])$	3; T1, DR8
5. $\sim P \equiv \underset{\approx}{F}$	4; A3, A1, T6, T5
6. $P \equiv \underset{\approx}{T}$	5; DR3, A5
7. $[A \equiv B] \equiv \underset{\approx}{T}$	1, 7; DR10
8. $A \equiv B$	7, T2; R*
9. $G(B)$	2, 7; DR10; T2, R*
10. $G(B/A)$	8, 9; R*
11. $[G(B/A)] \equiv \underset{\approx}{T}$	10, DR7
12. $P[G(B/A)] \equiv \underset{\approx}{T}$	6, 11; DR9
13. $\sim P \equiv \underset{\approx}{T}$	3, 12; R*
14. $P \sim P \equiv \underset{\approx}{T}$	6, 13; DR9

Hence, $\Gamma \vdash \sim P \equiv \sim(P[G(B/A)])$ 3-14; R1
$\Gamma \vdash P \equiv P[G(B/A)]$ DR3, A5; R*

Case (v) We have in the proof of s_i: $\ldots, S_i' \equiv S_i'', \ldots, A \sim A \equiv \sim(B \sim B)$, $S_i' \equiv \sim S_i''$. We assume $\sim P \equiv P[S_i' \equiv \sim S_i'']$ in order to derive, by R1, $P \equiv P[S_i' \equiv \sim S_i'']$. We derive thus: $\sim P \equiv \sim PP[S_i' \equiv \sim S_i'']$; $\sim P \equiv \underset{\approx}{F}$; $P \equiv \underset{\approx}{T}$; $\sim[S_i' \equiv \sim S_i''] \equiv \underset{\approx}{T}$; $S_i' \equiv S_i'', \ldots, A \sim A \equiv \sim(B \sim B)$. Hence, by R1, $\sim P \equiv \sim P[S_i' \equiv \sim S_i'']$; then, $P \equiv P[S_i' \equiv \sim S_i'']$.

Case (vi) We have in the proof of s_i: $\ldots s_j, \ldots, \sim S_j \equiv A(B \sim B)$. By DR16 or A3 the proof being constructed has: $P \equiv PS_j$. We want to derive $P \equiv P[\sim S_j \equiv A(B \sim B)]$. We assume $\sim P \equiv P[\sim S_j \equiv A(B \sim B)]$ in order to derive a contradiction. As in the preceding case we derive $P \equiv \underset{\approx}{T}$ and $[\sim S_j \equiv A(B \sim B)] \equiv \underset{\approx}{F}$. Then by DR10 we derive $S_j \equiv \underset{\approx}{T}$, and hence by R*, s_j. Then as in the original proof we derive $\sim S_j \equiv A(B \sim B)$, and, then by DR7 we derive $[\sim S_j \equiv A(B \sim B)] \equiv \underset{\approx}{T}$. Then we apply DR1 and DR4.

Case (vii) We establish this case by deriving a contradiction from the assumption $\sim P \equiv PS_i$, introducing also $P \equiv P[\sim S_i \equiv A(B \sim B)]$ by DR16 or by A3.

H. *Completeness of* **L.C** In order to prove the propositional completeness of **L.C** we prove first the following:

Lemma *Let Q be a term of* **L.C** *made up of occurrences of $\sim$, &, and terms $P_1, \ldots, P_n$. Consider an ordinary two-valued truth-table for $P_1, \ldots, P_n$, each row of the table assigning $\underset{\approx}{T}$ or $\underset{\approx}{F}$ to each P_1. Interpret $\sim$ as negation and & as conjunction. We represent each row r of the table by the sequence $(p_1)', \ldots, (p_n)'$, where each $(p_1)'$ is $P_i \equiv \underset{\approx}{T}$ if r assigns $\underset{\approx}{T}$ to P_i, and $P_i \equiv \underset{\approx}{F}$ if r assigns $\underset{\approx}{F}$ to P_i. Similarly for $(q)'$. Let row r_j assign $\underset{\approx}{T}$ to q. Then: $(P_1)', \ldots, (P_n)' \vdash (q)'$.*

Proof: Case 1 Q has only one occurrence of a symbol. Then $(q)'$ is one of the premises $(p_i)'$, and the lemma holds.

Case 2 We assume the lemma true for up to $m - 1$ occurrences of symbols. Then: (i) Q is of the form $\sim R$, or (ii) Q is of the form RS, where R and S have less than m occurrences of symbols.

Subcase (i) R has $m - 1$ occurrences of symbols. Hence,

$$(p_1)', \ldots, (p_n)' \vdash (r)'.$$

By A5 and DR3 the lemma holds.

Subcase (ii) If r_j assigns $\underset{\approx}{T}$ to Q, it assigns $\underset{\approx}{T}$ to R and to S. Hence, $(p_1)', \ldots, (p_n)' \vdash (r)', (s)'$. Now, by DR9, $R \equiv \underset{\approx}{T}$, $S \equiv \underset{\approx}{T} \vdash RS \equiv \underset{\approx}{T}$. Hence, $(p_1)', \ldots, (p_n)' \vdash (q)'$. If r_j assigns $\underset{\approx}{F}$ to Q, then r_j assigns: (a) $\underset{\approx}{T}$ to R and $\underset{\approx}{F}$ to S, or (b) $\underset{\approx}{F}$ to R and $\underset{\approx}{T}$ to S, or (c) $\underset{\approx}{F}$ to R and $\underset{\approx}{F}$ to S. In subcase (a), $(p_1)', \ldots, (p_n)' \vdash R \equiv \underset{\approx}{T}, S \equiv \underset{\approx}{F}$. Hence, $(p_1)', \ldots, (p_n)' \vdash RS \equiv \underset{\approx}{F}$, i.e., $Q \equiv \underset{\approx}{F}$, by DR8 and T5.

Subcase (b) is like (a).

In subcase (c), $(p_1)', \ldots, (p_n)' \vdash R \equiv \underset{\approx}{F}$. Hence, $(p_1)', \ldots, (p_n)' \vdash RS \equiv \underset{\approx}{F}$, $Q \equiv \underset{\approx}{F}$, by DR9. Thus, the lemma holds in all cases.

Metatheorem *If Q is a term of* **L.C**, *as in the preceding lemma, and every row of the truth-table for $P_1, \ldots, P_n$ assigns $\underset{\approx}{T}$ to Q, then $\vdash Q \equiv \underset{\approx}{T}$.*

Proof: By Lemma, $(p)', \ldots, \vdash Q \equiv \underset{\approx}{T}$ for every row. Hence, $(p)', \ldots, (p_{n-1})'$, $P_n \equiv \underset{\approx}{T} \vdash Q \equiv \underset{\approx}{T}$ and $(p)', \ldots (p_{n-1})', P_n \equiv \underset{\approx}{F} \vdash Q \equiv \underset{\approx}{T}$. By the deduction theorem in section G, from $(p_1)', \ldots, (p_{n-1})'$ we can derive:

1.	$[P_n \equiv \underset{\approx}{T}] \equiv [P_n \equiv \underset{\approx}{T}]\,[Q \equiv \underset{\approx}{T}]$	
2.	$[P_n \equiv \underset{\approx}{F}] \equiv [P_n \equiv \underset{\approx}{F}]\,[Q \equiv \underset{\approx}{T}]$	
3.	$\sim[Q \equiv \underset{\approx}{T}] \equiv \sim[Q \equiv \underset{\approx}{T}] \sim [P_n \equiv \underset{\approx}{T}]$	1; DR4
4.	$\sim[Q \equiv \underset{\approx}{T}] \equiv \sim[Q \equiv \underset{\approx}{T}] \sim [P_n \equiv \underset{\approx}{F}]$	2; DR4
5.	$\sim[Q \equiv \underset{\approx}{T}] \equiv \sim[Q \equiv \underset{\approx}{T}]\,(\sim[P_n \equiv \underset{\approx}{T}] \sim [P_n \equiv \underset{\approx}{F}])$	3, 4; A2, DR9
6.	$[P_n \equiv \underset{\approx}{T}] \equiv P_n$	T8
7.	$[P_n \equiv \underset{\approx}{F}] \equiv \sim P_n$	T9
8.	$\sim[Q \equiv \underset{\approx}{T}] \equiv \sim[Q \equiv \underset{\approx}{T}]\,(\sim P_n \sim\sim P_n)$	6, 7, 8; R*
9.	$\sim[Q \equiv \underset{\approx}{T}] \equiv \underset{\approx}{F}$	8, T5; R*

10. $[Q \equiv \underset{\approx}{T}] \equiv \underset{\approx}{T}$ 9; DR3, Def. T

11. $Q \equiv \underset{\approx}{T}$ 10; T1, R*

By n - 1 similar derivations, we arrive at $\vdash Q \equiv \underset{\approx}{T}$.

2 *The iterative calculus* **L.C***i* As we noted in Part I, section 2, Leibniz spoke of equivalence of propositions as if he were ready to consider a logical system that allows the iteration of the copula in well-formed formulas. Such a calculus will have to include an axiom relating conjunction and the copula in its new additional role as a term-connector. An adequate axiom that serves well is the derived transformation scheme DTS, which was proved for **L.C** with the help of theorem T10. Appropriate modifications of the rules of term-formation, and adjunction of DTS as an axiom yield the Leibnizian logical system **L.C***i*. This is adequate for propositional logic.

We will not consider variations of **L.C** which are nicer in that they have only R* as the rule of primitive detachment. With appropriate axiomatic versions of rules R2 and R3, axiom A5 of double negation may be immediately proven. We will not investigate other changes of **L.C**. After all our primary interest is the historical one of attaining a detailed insight into Leibniz's logical works in [1]-[3].

III CONCLUSION

We have examined Leibniz's logical system as he developed it through [1]-[3], with some glimpses of related later thoughts. Our formulation of **L.C** uses the principles that Leibniz formulated as well as those which he assumed in discussing it. His idea of a general calculus for syllogisms and propositional logic works rather well on the assumption that all atomic terms have existential import (i.e., are not vacuous). His brief effort at working the logic of existence was a strepitous failure. Yet the system **L.C** we have constructed upon Leibniz's work, containing really only few simple distortions, is adequate for propositional logic.

I do not claim that Leibniz had a clear idea of **L.C** and that he was deliberately constructing it. I claim that Leibniz had in his logical reflections seen the principles involved in **L.C** with different degrees of awareness—perhaps the only exception is the term-formation schemes TS and DTS. He had insights into the distinctions among rules of formation, axioms, and rules of inference. He had a good idea of all the rules and axioms, as indicated at the proper place.

It is really a pity that Leibniz did not publish a paper containing the main results of [1]-[3], even without his clearer conception of indirect proof in [10]. That paper would undoubtedly have attracted the attention of Euler, if of nobody else, and logic would have developed faster.

NOTES

1. See the index to references at the end of this essay. Our citations from [1]-[4] are taken from Parkinson's translations with minor modifications.

2. In [1] (144) ff, whose translation by Parkinson is not reliable, Leibniz treats existence as a copula. This idea leads to his developing, in [8], an equational calculus that can be interpreted either as having no existential import or as lacking it. See next footnote.

3. There is a verbal issue between Couturat and Parkinson on this point. Couturat interprets the Leibniz quotation just given as showing that for Leibniz all (categorical) propositions have existential import. Parkinson argues (LR 21f) that Leibniz holds, instead, that neither universal nor particular propositions have existential import. Parkinson bases his argument on the fact that in [8] Leibniz is engaged in a protracted discussion of the validity of the inference "Every laugher is a man, therefore, some man is a laugher." Leibniz notes that if there are no men, the inference appears invalid. He holds, on the other hand, that the inference is for the most part valid, even though there are two domains of interpretation for the quantifiers, (i) possible entities, (ii) actually existing entities. He claims that if we hold fast to the same domain in both the premise and the conclusion, the inference is valid, but, of course, there is invalidity if we take (i) for the premise and (ii) for the conclusion. Hence, Parkinson is right in claiming that Leibniz does not take either the universal premise or the particular conclusion as having existential import in the sense that the domain of quantification has to be a set of actually existing objects. Yet formal logicians tend to speak of existential import in a generalized sense, namely, as the assumption that the atomic terms being dealt with are not empty. To this extent Couturat is correct. The interesting things in Leibniz's discussion are: (a) his conception of different domains of quantification, (b) his consideration of existence as a logical term determining the logical form of the categorical propositions, and (c) his inability to conceive of a logical system without vacuous terms, i.e., without existential import in the generalized sense. Of course (c) is connected with his not conceiving the peculiar features of *existence* as a term-forming operator, rather than as a mere term, and this in its turn depends in part on his never really seeing the role of associativity.

REFERENCES

I. Major Sources:

C: Couturat, L., ed., *Opuscules et fragments inédits de Leibniz*, Presses Universitaires de France, Paris (1903).

G: Gerhardt, C. I., ed., *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, Weidmann, Berlin (1875-1890), in seven volumes.

P: Parkinson, G. H. R., ed. and trans., *Leibniz's Logical Papers*, Clarendon Press, Oxford (1966).

II: Individual works by Leibniz referred to in the present essay:

[1] *General Inquiries about the Analysis of Concepts and Truths* (1686) in C 356-399, and in P 47-89.

[2] *The Primary Bases of a Logical Calculus* (August 1, 1690), in C 235-237 and in P 90-92.

[3] *The Bases of a Logical Calculus* (August 2, 1960), in C 421-423, and in P 93-94.

[4] *Elements of a Calculus* (April, 1679), in C 49-57 and in P 17-24.

[5] *A Study in the Plus-Minus Calculus* (After 1690), in G vii 228-235 and in P 122-130.

[6] *A Study in the Calculus of Real Addition* (after 1690), in G vii 236-247 and in P 131-144.

[7] *New Essays Concerning Human Understanding*, translated and edited by A. G. Langley, The Open Court Publishing Company, Chicago (1916), all of G v.

[8] *Some Logical Difficulties* (after 1690), in G vii 211-217 and in P 115-121.

[9] *Of the Mathematical Determination of Syllogistic Forms* (after 1690?), in C 410-416 and in P 105-111.

[10] *Logical Definitions* (undoubtedly after 1690), in G vii 208-210 and in P 112-114.

III. Other Items:

LR G. H. R. Parkinson, *Logic and Reality in Leibniz's Metaphysics*, The Clarendon Press, Oxford (1965).

Indiana University
Bloomington, Indiana