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A QUESTION ABOUT INCOMPLETENESS

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At the conclusion of his discussion of the incompleteness of the nineteen rules of inference, Irving M. Copi presents a valid argument form which cannot be proved valid by the nineteen rules alone. This argument, $A \supset B \therefore A \supset (A \cdot B)$, known as "absorption," is easily proved valid by using the method of conditional proof. Thus the rule of conditional proof is a "genuine addition" to the proof apparatus. "Not only does it permit the construction of shorter proofs of validity for arguments which could be proved valid by appealing to the original list of nineteen Rules of Inference alone, but it permits us to establish the validity of valid arguments whose validity could *not* be proved by reference to the original list alone."[1]

I do not doubt that *absorption* cannot be proved using just the nineteen rules, but I find it interesting that a proof is possible which does not employ the rule of conditional proof. Instead, the principle of Excluded Middle is introduced as an additional premise:

1. $A \supset B$	$\therefore A \supset (A \cdot B)$
2. $\sim A \vee B$	1, Impl.
3. $A \vee \sim A$	additional premise (Excluded Middle)
4. $\sim A \vee A$	3, Com.
5. $(\sim A \vee A) \cdot (\sim A \vee B)$	4, 2, Conj.
6. $\sim A \vee (A \cdot B)$	5, Dist.
7. $A \supset (A \cdot B)$	6, Impl.

In his discussion of arguments involving relations, Copi sanctions the introduction of additional or enthematic premises in cases where the premise is clearly or obviously true. In the case of the argument, "Tom has the same weight as Dick. Dick has the same weight as Harry. Therefore, Tom has the same weight as Harry," it is necessary to add the

1. Irving M. Copi, *Symbolic Logic*, 4th edition, The Macmillan Co., New York (1973), p. 53.

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premise that *having the same weight as* is transitive in order to provide a formal proof of validity. "In most discussions," Copi remarks, "a large body of propositions can be presumed to be common knowledge. The majority of speakers and writers save themselves trouble by not repeating well-known and perhaps trivially true propositions that their hearers or readers can perfectly well be expected to supply themselves."[2] Surely the principle of Excluded Middle qualifies as a "trivially true proposition" that everyone can be expected to accept as true. Hence, there can be no logical objection to introducing it as an additional premise in order to construct a formal proof.

Independently of any discussion of enthemenes in class, I have found that students will sometimes construct proofs employing the principle of Excluded Middle as an additional premise. In most of these cases it is easy to construct an alternative proof which employs only the original nineteen rules. But it is difficult to come up with any logically compelling reasons for not using the rule of Excluded Middle which can be a useful tool in shortening many proofs.

A further justification for this use of the principle of Excluded Middle is that it, along with the principles of Identity and Non-Contradiction, is fundamental to logic. As Copi puts it, "The three Laws of Thought can be regarded as the basic principles governing the construction of truth tables."[3] As basic principles they are somehow enthematic premises of all valid arguments. Actually they can be routinely derived from either formulation of rule nineteen, Tautology.[4]

Hence the Laws of Thought are *included* in the nineteen rules, in at least the sense of "included" that they may be logically derived from the rules. It seems reasonable then to maintain that the alternative proof of absorption which employs the law of Excluded Middle is a proof that utilizes the nineteen rules alone. If this conclusion is correct, then there must be something wrong with the incompleteness proof that Copi presents.[5] According to that proof absorption cannot be proved with the nineteen rules alone. But if the arguments offered here are good, it can.

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2. *Ibid.*, p. 133.

3. Irving M. Copi, *Introduction to Logic*, 4th edition, The Macmillan Co., New York (1972), p. 286.

4.

1.	$P \equiv (P \cdot P)$	$/ \therefore P \vee \sim P$
2.	$[P \supset (P \cdot P)] \cdot [(P \cdot P) \supset P]$	1, Equiv.
3.	$P \supset (P \cdot P)$	2, Simp.
4.	$P \supset P$	3, Taut. (Identity)
5.	$\sim P \vee P$	4, Impl.
6.	$P \vee \sim P$	5, Com. (Excluded Middle)
[7.	$\sim(P \cdot \sim P)$	5, De M (Non-Contradiction)]

5. *Symbolic Logic*, pp. 47-50.