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# A FURTHER EXAMINATION OF SACCHERI'S USE OF THE "CONSEQUENTIA MIRABILIS"

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Gerolamo Saccheri (1667-1733) is perhaps best known as a geometer because of his *Euclides ab omni naevo vindicatus*. However, he is also deserving of attention as a logician because of his *Logica Demonstrativa*. The most interesting part of this book is to be found in Chapter 11, where Saccheri sets out to prove a series of theorems of syllogistic by means of a method which has come to be known as the "consequentia mirabilis" or the law of Clavius.[1] Saccheri himself notes that this method is not original with him. What is original and unique is the deliberate, systematic use of this method to prove a number of theorems.

Saccheri's procedure can best be understood by examining an example from the proof of Proposition 1: "In prima figura minor non potest esse negativa."[2] He must first assume its contradictory, i.e., some syllogism having a negative minor premise does conclude in the first figure, and then from this derive the proposition to be proved.

Saccheri begins by observing that if any syllogism having a negative minor premise concludes in the first figure it will be of the form AE or EE. Considering AE first, he constructs the following syllogism: "Omnis syllogismus habens majorem universalem, & minorem affirmativam, concludit in prima figura: atqui nullus syllogismus AE habet majorem universalem, & minorem affirmativam ergo omnis, vel aliquis syllogismus AE non concludit in prima figura."[3]

We can represent this symbolically as:

(I) $\bigwedge_s .\ s \in \mathrm{MU} \wedge s \in \mathrm{MinA} \wedge s \in \mathrm{1st} \rightarrow s \in \mathrm{Concl}.$
(II) $\bigwedge_s .\ s \in \mathrm{AE} \rightarrow \neg(s \in \mathrm{MU} \wedge s \in \mathrm{MinA}).$
(III) $\bigwedge_s .\ s \in \mathrm{AE} \wedge s \in \mathrm{1st} \rightarrow \neg s \in \mathrm{Concl}.$
(IV) $\bigvee_s .\ s \in \mathrm{AE} \wedge s \in \mathrm{1st} \wedge \neg s \in \mathrm{Concl}.$

where '$s$' ranges over syllogistic premises, 'MU' stands for 'has a universal major premise', 'MinA' for 'has an affirmative minor premise', '1st' for 'belongs to the first figure', and 'Concl' for 'yields a conclusion'.

Now (I) has already been proven by Saccheri, and (II) is evident. But

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(I), (II), and (III) form an AE-E syllogism in the first figure; and (I), (II), and (IV) form an AE-O syllogism in the first figure. Thus the assumption that syllogisms of the form AE conclude in the first figure leads, via a syllogism of the form AE in the first figure, to the conclusion that no syllogism of the form AE concludes in the first figure.[4] In short, the assumption leads to its own denial.

Saccheri must also show that the same holds true for AE-A and AE-I as well as for the syllogisms of the form EE. By doing this, he can conclude that no syllogism having a negative minor premise concludes in the first figure.

It is interesting to note here that the assumption that syllogisms of the form AE conclude in the first figure was not used as a premise but rather as the rule of inference—that is, the form of the syllogism—according to which its negation was derived. This use of the assumption is common to all of Saccheri's proofs in this chapter.[5] And it is this method of employing the "consequentia mirabilis", even more than its systematic use, which makes Saccheri's work so interesting.

However, as Ignacio Angelelli points out,[6] there are several instances in which the conclusion which Saccheri derives through the use of this method is not the denial of his assumption but is simply a contradiction. And because the assumption leads to a contradiction, he concludes that the assumption must be false. But this is to use the method of indirect proof rather than the "consequentia mirabilis". Saccheri himself notes in the scholium to Chapter 11 that these cases seem to be instances of the "via negativa" rather than the "via nobilior" which he had intended to employ. Angelelli has shown that these cases, at least for Proposition 2, can be seen as instances of the "consequentia mirabilis" through the development of the suggestions provided by Saccheri in the scholium. In what follows I will attempt to do the same for the other cases: Proposition 3, first figure (i) IA-A and IA-I, (ii) OA-E and OA-O, (iii) IA-E and IA-O, (iv) OA-A and OA-I; Proposition 5, fourth figure (v) AO-I, (vi) OA-A and OA-I; Proposition 6, (vii) third figure II-I, (viii) fourth figure II-I.

Cases (i)-(vi) can be handled using the same form of argument as was used by Angelelli. Let us schematize the crucial steps of his approach as follows: (Throughout the paper, numbers on the right will indicate the number of the corresponding formula in Angelelli's paper.)

$$\bigwedge_s .\ s \epsilon \mathrm{A} \wedge s \epsilon \mathrm{N} \rightarrow s \epsilon \mathrm{Concl.} \qquad [5]$$

$$\bigwedge_s .\ s \epsilon \mathrm{A} \rightarrow s \epsilon \mathrm{B}. \qquad [6]$$

$$\bigvee_s .\ s \epsilon \mathrm{A} \wedge s \epsilon \mathrm{B}. \qquad [7]$$

$$\bigwedge_s .\ s \epsilon \mathrm{B} \rightarrow \neg s \epsilon \mathrm{A}. \qquad [15]$$

$$\bigwedge_s .\ s \epsilon \mathrm{A} \rightarrow \neg s \epsilon \mathrm{A}. \qquad [16]$$

$$\bigwedge_s .\ \neg s \epsilon \mathrm{A}. \qquad [17]$$

$$\bigwedge_s .\ s \epsilon \mathrm{N} \wedge s \epsilon \mathrm{Concl} \rightarrow \neg s \epsilon \mathrm{A}. \qquad [14]$$

The steps above allow us directly to derive [14], the denial of [5], from [6]. To do the same for [7], we show that it implies [6] by establishing that 'B' is invariant with respect to similarity of form, i.e.,

$$\bigwedge_s . \bigwedge_t . s \in \mathrm{B} \wedge s, t \in \mathrm{Sim} \rightarrow t \in \mathrm{B}$$

where '$s$' and '$t$' range over syllogistic premises and 'Sim' means that both $s$ and $t$ belong to the same combination AA, AI, etc.

In the case considered by Angelelli, 'N' is '2nd', 'A' is 'AA', and 'B' is 'DQ'. For cases (i)-(vi) the following substitutions are required:

| Case | N | A | B |
|---|---|---|---|
| (i) | 1st | IA | MU |
| (ii) | 1st | OA | ¬MP |
| (iii) | 1st | IA | ¬MP |
| (iv) | 1st | OA | MU |
| (v) | 4th | AO | AA |
| (vi) | 4th | OA | MU |

where 'MP' stands for 'has a particular major premise'. Let us examine the cases in detail to see how this works.

Cases (i)-(iv) arise in the proof of Proposition 3: "In prima & secunda figura major debet esse universalis."[7] The syllogism for case (i)—"Aliquis syllogismus concludens in prima figura habet majorem universalem; sed omnis syllogismus IA concludit in prima figura; ergo omnis vel aliquis syllogismus IA habet majorem universalem."[8]—can be symbolized as follows:

(1) $\bigvee_s . s \in \mathrm{Concl} \wedge s \in \mathrm{1st} \wedge s \in \mathrm{MU}.$

(2) $\bigwedge_s . s \in \mathrm{IA} \wedge s \in \mathrm{1st} \rightarrow s \in \mathrm{Concl}.$ [5]

(3) $\bigwedge_s . s \in \mathrm{IA} \rightarrow s \in \mathrm{MU}.$ [6]

(4) $\bigvee_s . s \in \mathrm{IA} \wedge s \in \mathrm{MU}.$ [7]

Now (1), (2), and (3) form an IA-A syllogism of the first figure, and (1), (2), and (4) form an IA-I syllogism of the first figure. The two conclusions, (3) and (4), involve a contradiction, and they are not the denial of the assumption. In order to obtain this directly from them, we follow Angelelli's procedure and observe that it is evident that

(5) $\bigwedge_s . s \in \mathrm{MU} \rightarrow \neg s \in \mathrm{IA}.$ [15]

and this with (3) yields

(6) $\bigwedge_s . s \in \mathrm{IA} \rightarrow \neg s \in \mathrm{IA}.$ [16]

and hence by the "consequentıa mirabilis"

(7) $\bigwedge_s . \neg s \in \mathrm{IA}.$ [17]

And thus

(8) $\bigwedge_s . s \in \mathrm{1st} \wedge s \in \mathrm{Concl} \rightarrow \neg s \in \mathrm{IA}.$ [14]

which is the denial of (2), which is the assumption. We then show that (4) implies (3) by establishing that 'MU' is invariant with respect to form, i.e.,

(9) $\bigwedge_s . \bigwedge_t . s \in \mathrm{MU} \wedge s, t \in \mathrm{Sim} \rightarrow t \in \mathrm{MU}.$

The syllogism for case (ii)—"Aliquis syllogismus concludens in prima figura non habet majorem particularem; sed omnis syllogismus OA concludit in prima figura: ergo omnis vel aliquis syllogismus OA non habet majorem particularem."[9]—can be symbolized as:

(10) $\bigvee_s .\ s \in \mathrm{Concl} \wedge s \in \mathrm{1st} \wedge \neg s \in \mathrm{MP}.$
(11) $\bigwedge_s .\ s \in \mathrm{OA} \wedge s \in \mathrm{1st} \rightarrow s \in \mathrm{Concl}.$ [5]
(12) $\bigwedge_s .\ s \in \mathrm{OA} \rightarrow \neg s \in \mathrm{MP}.$ [6]
(13) $\bigvee_s .\ s \in \mathrm{OA} \wedge \neg s \in \mathrm{MP}.$ [7]

Here (10), (11), and (12) form an OA-E syllogism of the first figure, and (10), (11), and (13) form an OA-O syllogism of the first figure. Now it is evident that

(14) $\bigwedge_s .\ \neg s \in \mathrm{MP} \rightarrow \neg s \in \mathrm{OA}.$ [15]

and this with (12) yields

(15) $\bigwedge_s .\ s \in \mathrm{OA} \rightarrow \neg s \in \mathrm{OA}.$ [16]

and hence by the "consequentia mirabilis"

(16) $\bigwedge_s .\ \neg s \in \mathrm{OA}.$ [17]

Thus

(17) $\bigwedge_s .\ s \in \mathrm{1st} \wedge s \in \mathrm{Concl} \rightarrow \neg s \in \mathrm{OA}.$ [14]

which is the denial of (11). We then show that (13) implies (12) by establishing that '$\neg$MP' is invariant with respect to form, i.e.,

(18) $\bigwedge_s .\ \bigwedge_t .\ \neg s \in \mathrm{MP} \wedge s, t \in \mathrm{Sim} \rightarrow \neg t \in \mathrm{MP}.$

The syllogism for case (iii)—"Aliquis syllogismus concludens in prima figura habet majorem particularem; sed omnis syllogismus IA concludit in prima Figura: ergo omnis vel aliquis syllogismus IA non habet majorem particularem."[10]—can be symbolized as:

(19) $\bigvee_s .\ s \in \mathrm{1st} \wedge s \in \mathrm{Concl} \wedge s \in \mathrm{MP}.$
(20) $\bigwedge_s .\ s \in \mathrm{IA} \wedge s \in \mathrm{1st} \rightarrow s \in \mathrm{Concl}.$ [5]
(21) $\bigwedge_s .\ s \in \mathrm{IA} \rightarrow \neg s \in \mathrm{MP}.$ [6]
(22) $\bigvee_s .\ s \in \mathrm{IA} \wedge \neg s \in \mathrm{MP}.$ [7]

Now (19), (20), and (21) form an IA-E syllogism of the first figure, and (19), (20), and (22) form an IA-O syllogism of the first figure. It is evident that

(23) $\bigwedge_s .\ \neg s \in \mathrm{MP} \rightarrow \neg s \in \mathrm{IA}.$ [15]

and this with (21) yields

(24) $\bigwedge_s .\ s \in \mathrm{IA} \rightarrow \neg s \in \mathrm{IA}.$ [16]

and hence by the "consequentia mirabilis"

(25) $\bigwedge_s .\ \neg s \in \mathrm{IA}.$ [17]

Thus we have

(26) $\bigwedge_s.\ s \in \text{1st} \wedge s \in \text{Concl} \rightarrow \neg s \in \text{IA}.$ [14]

which is the denial of (20). We then show that (22) implies (21) by means of (18).

The syllogism for case (iv)—"Aliquis syllogismus concludens in prima figura non habet majorem universalem; sed omnis syllogismus OA concludit in prima Figura: ergo omnis vel aliquis syllogismus OA habet majorem universalem."[11]—can be symbolized as:

(27) $\bigvee_s.\ s \in \text{1st} \wedge s \in \text{Concl} \wedge \neg s \in \text{MU}.$

(28) $\bigwedge_s.\ s \in \text{OA} \wedge s \in \text{1st} \rightarrow s \in \text{Concl}.$ [5]

(29) $\bigwedge_s.\ s \in \text{OA} \rightarrow s \in \text{MU}.$ [6]

(30) $\bigvee_s.\ s \in \text{OA} \wedge s \in \text{MU}.$ [7]

Now (27), (28), and (29) form an OA-A syllogism of the first figure, and (27), (28), and (30) form an OA-I syllogism of the first figure. It is evident that

(31) $\bigwedge_s.\ s \in \text{MU} \rightarrow \neg s \in \text{OA}.$ [15]

and this with (29) yields

(32) $\bigwedge_s.\ s \in \text{OA} \rightarrow \neg s \in \text{OA}.$ [16]

and hence by the "consequentia mirabilis"

(33) $\bigwedge_s.\ \neg s \in \text{OA}.$ [17]

And thus

(34) $\bigwedge_s.\ s \in \text{1st} \wedge s \in \text{Concl} \rightarrow \neg s \in \text{OA}.$ [14]

which is the denial of (28). We then show that (30) implies (29) by means of (9).

Case (v) arises in connection with the proof of Proposition 5: "In quarta figura neutra praemissa potest esse particularis negativa."[12] The syllogism in question—"Omnis syllogismus AO concludit in quarta figura: sed aliquis syllogismus concludens in quarta figura non est syllogismus AA; ergo aliquis syllogismus AA est syllogismus AO."[13]—can be symbolized:

(35) $\bigwedge_s.\ s \in \text{AO} \wedge s \in \text{4th} \rightarrow s \in \text{Concl}.$ [5]

(36) $\bigvee_s.\ s \in \text{4th} \wedge s \in \text{Concl} \wedge \neg s \in \text{AA}.$

(37) $\bigvee_s.\ s \in \text{AA} \wedge s \in \text{AO}.$ [7]

Here (35), (36), and (37) form an AO-I syllogism of the fourth figure. We must first show that (37) implies

(38) $\bigwedge_s.\ s \in \text{AO} \rightarrow s \in \text{AA}.$ [6]

and this is done by establishing that 'AA' is invariant with respect to form, i.e.,

(39) $\bigwedge_s.\ \bigwedge_t.\ s \in \text{AA} \wedge s, t \in \text{Sim} \rightarrow t \in \text{AA}.$

It is evident that

(40) $\bigwedge_s .\ s \epsilon \mathrm{AA} \rightarrow \neg s \epsilon \mathrm{AO}.$ [15]

and this with (38) yields

(41) $\bigwedge_s .\ s \epsilon \mathrm{AO} \rightarrow \neg s \epsilon \mathrm{AO}.$ [16]

and hence by the "consequentia mirabilis"

(42) $\bigwedge_s .\ \neg s \epsilon \mathrm{AO}.$ [17]

And thus

(43) $\bigwedge_s .\ s \epsilon \mathrm{4th} \wedge s \epsilon \mathrm{Concl} \rightarrow \neg s \epsilon \mathrm{AO}.$ [14]

which is the denial of (35).

Case (vi) is also part of Proposition 5, and the syllogism—"Aliquis syllogismus OA non concludit in secunda figura: sed omnis syllogismus concludens in secunda figura habet majorem universalem; ergo omnis, vel aliquis syllogismus habens majorem universalem, est syllogismus OA."[14]—can be symbolized as:

(44) $\bigvee_s .\ s \epsilon \mathrm{OA} \wedge s \epsilon \mathrm{2nd} \wedge \neg s \epsilon \mathrm{Concl}.$
(45) $\bigwedge_s .\ s \epsilon \mathrm{2nd} \wedge s \epsilon \mathrm{Concl} \rightarrow s \epsilon \mathrm{MU}.$
(46) $\bigwedge_s .\ s \epsilon \mathrm{MU} \rightarrow s \epsilon \mathrm{OA}.$
(47) $\bigvee_s .\ s \epsilon \mathrm{MU} \wedge s \epsilon \mathrm{OA}.$ [7]

Here (44), (45), and (46) form an OA-A syllogism of the fourth figure, and (44), (45), and (47) form an OA-I syllogism of the fourth figure. We first show that (47) implies

(48) $\bigwedge_s .\ s \epsilon \mathrm{OA} \rightarrow s \epsilon \mathrm{MU}.$ [6]

by means of (9). Now it is evident that

(49) $\bigwedge_s .\ s \epsilon \mathrm{MU} \rightarrow \neg s \epsilon \mathrm{OA}.$ [15]

And this with (48) yields

(50) $\bigwedge_s .\ s \epsilon \mathrm{OA} \rightarrow \neg s \epsilon \mathrm{OA}.$ [16]

and by the "consequentia mirabilis"

(51) $\bigwedge_s .\ \neg s \epsilon \mathrm{OA}.$ [17]

And thus

(52) $\bigwedge_s .\ s \epsilon \mathrm{4th} \wedge s \epsilon \mathrm{Concl} \rightarrow \neg s \epsilon \mathrm{OA}.$ [14]

Thus we have derived the denial of the assumption directly from (47). We must also show that it can be derived from (46). But this is trivial, since (46) implies (47).

Cases (vii) and (viii) concern Proposition 6: "Ex puris negativis, aut particularibus nihil sequitur."[15] Case (vii)—"Aliquis syllogismus concludens in tertia Figura habet unam ex praemissis universalem; sed aliquis syllogismus concludens in tertia Figura habet utramque praemissam particularem: ergo aliquis syllogismus habens utramque praemissam particularem habet unam ex praemissis universalem.''[16]—can be symbolized:

(53) $\bigvee_s .\ s \in \text{3rd} \wedge s \in \text{Concl} \wedge s \in \text{Univ}.$
(54) $\bigvee_s .\ s \in \text{3rd} \wedge s \in \text{Concl} \wedge s \in \text{BP}.$
(55) $\bigvee_s .\ s \in \text{BP} \wedge s \in \text{Univ}.$

and case (viii)—''Aliquis syllogismus habens unam ex praemissis universalem concludit in quarta Figura; sed aliquis syllogismus concludens in quarta Figura habet utramque praemissam particularem: ergo aliquis syllogismus habens utramque praemissam particularem habet unam ex praemissis universalem.''[17]—can be symbolized:

(56) $\bigvee_s .\ s \in \text{Univ} \wedge s \in \text{4th} \wedge s \in \text{Concl}.$
(57) $\bigvee_s .\ s \in \text{4th} \wedge s \in \text{Concl} \wedge s \in \text{BP}.$
(58) $\bigvee_s .\ s \in \text{BP} \wedge s \in \text{Univ}.$

where 'Univ' stands for 'has a universal premise' and 'BP' stands for 'has both premises particular'. Here (53), (54), and (55) form an II-I syllogism of the third figure, and (56), (57), and (58) form an II-I syllogism of the fourth figure.

In neither case do I see how the method used above could be applied to produce the desired result, nor have I been able to discover any alternative method which will accomplish this. It is easy, however, to produce an alternative syllogism of the required form which yields the desired conclusion, but this is to abandon the argument as presented by Saccheri. It seems, then, that these two cases are exceptions to Saccheri's claim.

There is one additional case which must also be considered. This arises in Proposition 1, in the consideration of the cases AE-A and AE-I. Saccheri constructs the following syllogism: ''Omnis syllogismus EA concludit negative in prima figura: sed nullus syllogismus AE est syllogismus EA; ergo omnis vel aliquis syllogismus AE concludit negative in prima figura.''[18] This can be symbolized as:

(59) $\bigwedge_s .\ s \in \text{EA} \wedge s \in \text{1st} \rightarrow s \in \text{ConclN}.$
(60) $\bigwedge_s .\ s \in \text{AE} \rightarrow \neg s \in \text{EA}.$
(61) $\bigwedge_s .\ s \in \text{AE} \wedge s \in \text{1st} \rightarrow s \in \text{ConclN}.$
(62) $\bigvee_s .\ s \in \text{AE} \wedge s \in \text{1st} \wedge s \in \text{ConclN}.$

where 'ConclN' stands for 'yields a negative conclusion'.

Now (59), (60), and (61) form an AE-A syllogism of the first figure, and (59), (60), and (62) form an AE-I syllogism of the first figure. But the conclusions (61) and (62) are not the denial of the assumption—what we would like are the conclusions, (III) and (IV) above, which were reached for the cases AE-E and AE-O—nor are they contradictory. Rather, Saccheri points out, they are false as was shown by the consideration of AE-E and AE-O.

Since the premises are true, (59) having already been proven and (60) being evident, we have derived a false conclusion from true premises and so the forms AE-A and AE-I must not conclude in the first figure. However, this is not a direct derivation of the denial of the assumption but only the construction of a counterexample.

This difficulty is easily resolved by considering the cases AE-E and AE-O. We began there by assuming that syllogisms of the form AE in the first figure do yield negative conclusions—that is, we assume that (61) and (62) are true. And from this we directly derived the conclusion that syllogisms of the form AE in the first figure do not conclude. Thus the steps used for the cases AE-E and AE-O are exactly the steps needed to directly derive the denial of the assumption from (61) and (62).

Consequently we see that in all but two cases Saccheri is correct in his claim that, for those cases in which the conclusion of the syllogism is not the denial of the assumption, it is possible directly to derive that denial from the conclusion. And those two cases are so similar, differing only in respect to the figure, that they can be seen as two instances of the same case. Still, even a single exception is sufficient to render his claim false. But this does not make his procedure any less interesting nor does it lose any of its historical importance.

## NOTES

1. Saccheri expresses his procedure as: "Sumam contradictorium propositionum demonstrandarum, ex eoque ostensive, ac directe propositum eliciam." (I will assume the contradictory of the proposition to be proved, and from it I will derive the proposition in an ostensive and direct manner.) *Logica demonstrativa, theologicis, philosophicis et mathematicis disciplinis accomodata*; auctore R. P. Hieronymo Saccherio . . . Augustae Ubiorum, sumtu Henrici Noethen, 1735 (original in Universitätsbibliothek, Münster), p. 80. The name "consequentia mirabilis" is usually given to the formula $\neg p \rightarrow p . \rightarrow p$.

2. *Ibid.*, p. 81. In the first figure, the minor premise cannot be negative.

3. *Ibid.*, p. 81. Every syllogism having a universal major premise and an affirmative minor premise concludes in the first figure. But no syllogism of the form AE has a universal major premise and an affirmative minor premise. Therefore every or some syllogism of the form AE does not conclude in the first figure.

4. (IV) does not immediately give the result that no syllogism of the form AE concludes, only that some syllogism of the form AE does not conclude. But Saccheri obtains the stronger conclusion by application of the lemma: "Si quispiam syllogismus taliter constructus, non recte concludit, nullus alius similiter constructus, ratione formae concludet." (If any syllogism of a certain form does not yield a conclusion, no other syllogism of the same form will yield a conclusion.) *Logica*, p. 80-81.

5. This is not to say that Saccheri never uses the assumption as a premise. In fact he does so in six instances: the case considered by Angelelli (Prop. 2, second figure AA-A and AA-I) as well as my cases (i) - (v). Also, in cases (vii) and (viii), the premises (54) and (57) follow from the assumption.

6. Ignacio Angelelli, "On Saccheri's use of the *consequentia mirabilis*" Proceedings of the Second Leibniz Congress, Hanover, July 1972, forthcoming.

7. *Logica*, p. 84. In the first and second figures the major premise must be universal.

8. *Ibid.*, p. 84. Some syllogism which concludes in the first figure has a universal major premise. But every syllogism of the form IA concludes in the first figure. Thus every or some syllogism of the form IA has a universal major premise.

9. *Ibid.*, p. 84. Some syllogism which concludes in the first figure does not have a particular major premise. But every syllogism of the form OA concludes in the first figure. Therefore every or some syllogism of the form OA does not have a particular major premise.

10. *Ibid.*, p. 84. Some syllogism which concludes in the first figure has a particular major premise. But every syllogism of the form IA concludes in the first figure. Thus every or some syllogism of the form IA does not have a particular major premise.

11. *Ibid.*, p. 84-85. Some syllogism which concludes in the first figure does not have a universal major premise. But every syllogism of the form OA concludes in the first figure. Therefore every or some syllogism of the form OA has a universal major premise.

12. *Ibid.*, p. 87. In the fourth figure neither premise can be both particular and negative.

13. *Ibid.*, p. 87. Every syllogism of the form AO concludes in the fourth figure. But some syllogism which concludes in the fourth figure is not of the form AA. Thus some syllogism of the form AA is a syllogism of the form AO.

14. *Ibid.*, p. 87. Some syllogism of the form OA does not conclude in the second figure. But every syllogism which concludes in the second figure has a universal major premise. Thus every or some syllogism which has a universal major premise is a syllogism of the form OA.

15. *Ibid.*, p. 87. If both premises are negative, or if both are particular, there will be no conclusion.

16. *Ibid.*, p. 88. Some syllogism which concludes in the third figure has a premise which is universal. But some syllogism which concludes in the third figure has two particular premises. Therefore some syllogism which has two particular premises has a premise which is universal.

17. *Ibid.*, p. 88. Some syllogism which has a universal premise concludes in the fourth figure. But some syllogism which concludes in the fourth figure has two particular premises. Therefore some syllogism which has two particular premises has a premise which is universal.

18. *Ibid.*, p. 82. Every syllogism of the form EA yields a negative conclusion in the first figure. But no syllogism of the form AE is a syllogism of the form EA. Thus every or some syllogism of the form AE yields a negative conclusion in the first figure.

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