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AN ALTERNATIVE TO BRIAN SKYRMS' APPROACH TO THE LIAR

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1 In a recent paper[1] Brian Skyrms has formulated an approach to the Liar paradox for a trivalent language which has the following characteristics:

a) substitution of identicals is valid in general only in the weak sense that it never leads directly from truth to falsehood;[2]
b) "The result of concatenating 'is true' with a quotation-mark name of a sentence is itself either true or false";[3]
c) the language has a "mildly global" truth predicate.[4]

The semantics of Skyrms' language proceeds by first assigning to each sentence of the language a "level" with respect to each model. On the basis of this level-assignment, the atomic sentences are assigned one of three truth values in that model. Molecules are then evaluated by a supervaluation.[5]

Skyrms points out that his model theory is a "conservative one; it makes many more sentences neuter in a model than need be."[6] He also observes that in some cases this conservatism is a defect. He says:[7]

> On the other hand, consider the case in which "$a = Q(\sim Ta)$" and "$b = Q(\sim Ta)$" are both true. My model theory makes not only "Ta" but "Tb" neuter in this case. But here there is no reason why we cannot take "Tb" at face value without risk, in which case it is false. In such cases, it seems to me that the conservatism of the model theory is unfounded.

The intuition expressed in this passage may be put like this. Where T is a truth predicate and a and b are two distinct names with the same denotation, then we shall say that $\ulcorner Ta \urcorner$ and $\ulcorner Tb \urcorner$ are different truth ascriptions *of the same sort*.[8] Skyrms' observation, then, is that truth ascriptions of the same sort need not always have the same truth-value. In some cases, there is no reason not to make the one bivalent (in the example, false) and the other neuter.

Skyrms' approach already incorporates this feature to some extent. Where a is a quotation-functor name, for instance, $\ulcorner Ta \urcorner$ will always be

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bivalent, even when a different truth ascription $\ulcorner Tb \urcorner$ of the same sort is neuter. And in that case $\ulcorner Ta \urcorner$ will in fact be false. On the other hand, where a is a name other than a quotation-functor name, then if $\ulcorner Tb \urcorner$ is neuter, $\ulcorner Ta \urcorner$ will be neuter too. Skyrms' semantics provides a structural criterion, in the case of a neuter sentence A, for distinguishing those truth ascriptions about A which are neuter from those which are bivalent.

Now Skyrms' intuition in the above passage, one with which I agree, is that this dividing line is perhaps not where it ought to be. In section **2**, I shall outline an alternative to Skyrms' approach. I shall set out a language $\mathcal{L}$ incorporating what are perhaps plausible criteria for determining when a truth ascription about a neuter sentence is itself neuter and when it is bivalent. On these criteria, the $\ulcorner Tb \urcorner$ of Skyrms' example will turn out false, while $\ulcorner Ta \urcorner$ will be neuter. Nevertheless, features b) and c), above, of Skyrms' approach fail for $\mathcal{L}$. In section **3** the semantics of $\mathcal{L}$ is revised slightly to incorporate these advantages of Skyrms' system. The revised semantics may be viewed as an alternative, less "conservative" semantics for Skyrms' own system.

2 *The Language* $\mathcal{L}$[9]

Vocabulary

1) Individual constants $c_1, c_2, \ldots$.
2) Predicates of all finite degrees $P_1^1, P_2^1, \ldots, P_1^2, P_2^2, \ldots$ (P_i^n is the i-th predicate of degree n). We single out P_1^1 as a truth predicate and denote it by 'T'.
3) Logical constants $\sim$, &, $\vee$, $\supset$, $\equiv$, (,), Q (quotation functor), $=$. (We also use '$=$' as a metalinguistic identity sign.)

Expressions and Names An *expression* of $\mathcal{L}$ is a finite sequence of items from the vocabulary of $\mathcal{L}$. Where $e_1, \ldots, e_n$ are items from the vocabulary of $\mathcal{L}$, we write $\langle e_1, \ldots, e_n \rangle$ simply as $\ulcorner e_1 \ldots e_n \urcorner$. Then if E is an expression of $\mathcal{L}$, we call $\ulcorner Q(E) \urcorner$ a *name* of $\mathcal{L}$. The names of $\mathcal{L}$ will consist of the individual constants of $\mathcal{L}$ together with the quotations of expressions of $\mathcal{L}$. The names of $\mathcal{L}$ are $a_1, a_2, \ldots$.

Formation Rules

1) $\ulcorner a_i = a_j \urcorner$ is an (atomic) sentence of $\mathcal{L}$.
2) $\ulcorner P_i^n a_1 \ldots a_n \urcorner$ is an (atomic) sentence of $\mathcal{L}$.
3) If A and B are sentences of $\mathcal{L}$, so are $\ulcorner \sim A \urcorner$, $\ulcorner A \,\&\, B \urcorner$, $\ulcorner A \vee B \urcorner$, $\ulcorner A \supset B \urcorner$, $\ulcorner A \equiv B \urcorner$.

Models A *model* for $\mathcal{L}$ is a couple $\mathfrak{M} = \langle D, f \rangle$, where D is a set including the set of all expressions of $\mathcal{L}$, and f is a function such that

1) for each i, $f(a_i) \in D$;
2) if E is an expression of $\mathcal{L}$, $f(\ulcorner Q(E) \urcorner) = E$;
3) for each i and n, if $P_i^n \neq T$, then $f(P_i^n) \subseteq D^n$.

The basis of our approach is a certain relation between atomic sentences. Intuitively, this relation is founded on (but not identical with) the relation of one sentence's being "semantically about" a (perhaps different) sentence. More specifically, it is based on the notion of being "semantically about" a sentence via a truth predicate. We say that A is "about" B in this sense just in case A is of the form $\ulcorner Ta_i \urcorner$ and a_i denotes B.

The relation we shall define is not itself the relation we have just described. For our relation holds only between atomic sentences, and it is obvious that truth ascriptions can be about a sentence that is not atomic. Nor is it the subrelation of that "aboutness" relation consisting of those ordered pairs both of whose elements are atomic. With these caveats, the reader will readily see what is going on. For a model $\mathfrak{M} = \langle D, f \rangle$ we define the relation $R_{\mathfrak{M}}$ as follows:

$R_{\mathfrak{M}}(A, B) =_{def}$ for some name a_i, $A = \ulcorner Ta_i \urcorner$ and $f(a_i)$ has B as an atomic constituent.

Then I suggest that the following principles codify sound intuitions:

1) An atomic sentence is neuter in $\mathfrak{M}$ only if it is $R_{\mathfrak{M}}$-ungrounded.[10]
2) Not all $R_{\mathfrak{M}}$-ungrounded sentences are neuter in $\mathfrak{M}$. For then we could never have two truth ascriptions of the same sort, one of which is neuter in $\mathfrak{M}$ and the other bivalent in $\mathfrak{M}$. If $\ulcorner Ta_i \urcorner$ and $\ulcorner Ta_j \urcorner$ are of the same sort, and $\ulcorner Ta_i \urcorner$ is $R_{\mathfrak{M}}$-ungrounded, then so is $\ulcorner Ta_j \urcorner$. Thus, we must provide some way to distinguish bivalent $R_{\mathfrak{M}}$-ungrounded sentences from neuter ones.
3) Not all bivalent $R_{\mathfrak{M}}$-ungrounded sentences are false in $\mathfrak{M}$. Not all the cases are like that in Skyrms' passage quoted in section 1. For let $f(a_2) = f(a_1) = \ulcorner \sim Ta_1 \urcorner$. Then, as in that quotation, we want $\ulcorner Ta_1 \urcorner$ (and $\ulcorner \sim Ta_1 \urcorner$) to be neuter in $\mathfrak{M}$, and $\ulcorner Ta_2 \urcorner$ to be false in $\mathfrak{M}$. But then $\ulcorner \sim Ta_2 \urcorner$ will be true in $\mathfrak{M}$. And if $f(a_3) = \ulcorner \sim Ta_2 \urcorner$, then it seems we should want $\ulcorner Ta_3 \urcorner$ to be true in $\mathfrak{M}$, even though it is $R_{\mathfrak{M}}$-ungrounded.

Pictorially we can say that a sentence A is $R_{\mathfrak{M}}$-ungrounded just in case we can start from A and move along an $R_{\mathfrak{M}}$-path without stopping. This is possible in two ways: either at some point the $R_{\mathfrak{M}}$-path doubles back on itself, so that a "cycle" is formed, or else the $R_{\mathfrak{M}}$-path stretches on endlessly, as it were in a straight line, without turning back on itself at any point.

In the latter case I think we should want to say that A is neuter in $\mathfrak{M}$. This leaves the first, "cyclic" case in which to locate the bivalent $R_{\mathfrak{M}}$-ungrounded truth ascriptions. But we certainly do not want *every* sentence along an $R_{\mathfrak{M}}$-path that doubles back on itself to be bivalent, for they include such cases as $f(a_1) = \ulcorner \sim Ta_1 \urcorner$. Here $\ulcorner Ta_1 \urcorner$ is involved in a cycle at the very first step, and it is pretty generally agreed that $\ulcorner Ta_1 \urcorner$ ought to be neuter in this case.

If we look back once again to Skyrms' remark quoted in section 1 (using 'a_1' for his 'a' and 'a_2' for his 'b'), we notice that $\ulcorner Ta_1 \urcorner$ and $\ulcorner Ta_2 \urcorner$ lie on the same $R_{\mathfrak{M}}$-path that doubles back on itself. But there is an important difference. $\ulcorner Ta_1 \urcorner$ is itself a member of the cycle, for $R_{\mathfrak{M}}(\ulcorner Ta_1 \urcorner, \ulcorner Ta_1 \urcorner)$. $\ulcorner Ta_2 \urcorner$ is *not* a member of any cycle, but only *leads into* one. This suggests that if A is $R_{\mathfrak{M}}$-ungrounded, it is neuter only if either it is itself a member of an $R_{\mathfrak{M}}$-cycle, or else it heads an infinite (non-cyclic) $R_{\mathfrak{M}}$-path. On the third possibility, that it *leads into* an $R_{\mathfrak{M}}$-cycle of which it is not a member, it is bivalent.

We can formulate these notions more precisely thus:[11]

a) $R_{\mathfrak{M}}/{}^*R_{\mathfrak{M}}(A,A)$. That is, A is a member of an $R_{\mathfrak{M}}$-cycle.
b) $\sim R_{\mathfrak{M}}/{}^*R_{\mathfrak{M}}(A,A)$, but $R_{\mathfrak{M}}/{}^*R_{\mathfrak{M}}(A,B)$ and $R_{\mathfrak{M}}/{}^*R_{\mathfrak{M}}(B,B)$ for some B. That is, A leads into an $R_{\mathfrak{M}}$-cycle of which it is not a member.
c) Where $A = A_1$, for all $i,j \geqslant 1$ there is an A_{i+1} such that $R_{\mathfrak{M}}(A_i, A_{i+1})$ and $A_i = A_j$ just in case $i = j$. That is, A heads an infinite (non-cyclic) $R_{\mathfrak{M}}$-path.

Then an atomic sentence A is neuter in $\mathfrak{M}$ just in case either condition a) or condition c) obtains. It is bivalent otherwise.

We know the condition under which an $R_{\mathfrak{M}}$-ungrounded sentence is bivalent. But how do we determine when it is true and when it is false? I suggest that our procedure here should be the same as for any other bivalent atomic sentence (i.e., for atomic sentences that are not $R_{\mathfrak{M}}$-ungrounded), and should be a standard one. Since if A is $R_{\mathfrak{M}}$-ungrounded, it is of the form $\ulcorner Ta_i \urcorner$, this means that A is true in $\mathfrak{M}$ just in case $f(a_i)$ is true in $\mathfrak{M}$, and is false in $\mathfrak{M}$ otherwise.

Molecular sentences are evaluated on the basis of the values of their atomic constituents by a supervaluation. Our semantics agrees with Skyrms' on this point.[12] We have then:

For atomic sentences which are not $R_{\mathfrak{M}}$-ungrounded:

1) $\ulcorner a_i = a_j \urcorner$ is true in $\mathfrak{M}$ just in case $f(a_i) = f(a_j)$, and is false in $\mathfrak{M}$ otherwise.
2) If $P_i^n \neq T$, then $\ulcorner P_i^n a_1 \ldots a_n \urcorner$ is true in $\mathfrak{M}$ just in case $\langle f(a_1), \ldots, f(a_n) \rangle \in f(P_i^n)$, and is false in $\mathfrak{M}$ otherwise.
3) $\ulcorner Ta_i \urcorner$ is true in $\mathfrak{M}$ just in case $f(a_i)$ is true in $\mathfrak{M}$, and is false in $\mathfrak{M}$ otherwise.

For $R_{\mathfrak{M}}$-ungrounded atomic sentences $A = \ulcorner Ta_i \urcorner$:

1) A is true (false) in $\mathfrak{M}$ just in case

i) $\sim R_{\mathfrak{M}}/{}^*R_{\mathfrak{M}}(A,A)$, and
ii) for some B, $R_{\mathfrak{M}}/{}^*R_{\mathfrak{M}}(A,B)$ and $R_{\mathfrak{M}}/{}^*R_{\mathfrak{M}}(B,B)$ and
iii) $f(a_i)$ is true (not true) in $\mathfrak{M}$.

2) A is neuter in $\mathfrak{M}$ just in case it is neither true nor false in $\mathfrak{M}$.

For molecular sentences: A *classical valuation for a molecular sentence A* with respect to $\mathfrak{M}$ is an assignment v of truth values to the atomic constituents B of A such that

1) If B is true (false) in $\mathfrak{M}$, v assigns 'true' ('false') to all occurrences of B in A.
2) If B is neuter in $\mathfrak{M}$, v assigns either 'true' to all occurrences of B in A, or 'false' to all such occurrences.

Then

1) A molecular sentence A is true (false) in $\mathfrak{M}$ just in case A is classically true (false)—i.e., by a classical bivalent truth table analysis—under all classical valuations for A with respect to $\mathfrak{M}$.
2) A molecular sentence is neuter in $\mathfrak{M}$ just in case it is neither true nor false in $\mathfrak{M}$.

Now while all this is plausible, it leads to certain rather non-standard results. In particular, $\ulcorner Ta_i \urcorner$ is not always true when $f(a_i)$ is true. The Tarski-biconditionals (and the corresponding inferences[13]) do not always hold. For consider the following case:

$$f(a_1) = \ulcorner a_2 = a_2 \vee Ta_3 \urcorner$$
$$f(a_3) = \ulcorner Ta_1 \urcorner.$$

Then $R_{\mathfrak{M}}(\ulcorner Ta_1 \urcorner, \ulcorner Ta_3 \urcorner)$ and $R_{\mathfrak{M}}(\ulcorner Ta_3 \urcorner, \ulcorner Ta_1 \urcorner)$. So $\ulcorner Ta_1 \urcorner$ and $\ulcorner Ta_3 \urcorner$ are both neuter in $\mathfrak{M}$. And yet $\ulcorner a_2 = a_2 \vee Ta_3 \urcorner$, which is just $f(a_1)$, is evaluated true in $\mathfrak{M}$ by a supervaluation.

A somewhat similar situation obtains on Skyrms' semantics. The conditional from $f(a_i)$ to $\ulcorner Ta_i \urcorner$ is not always true. For instance, where $f(a_i)$ is neuter, and a_i is a quotation-functor name, $\ulcorner Ta_i \urcorner$ is false, and the conditional is neuter by a supervaluation. Nor does the corresponding inference hold generally. But it does hold for Skyrms where a_i is a quotation-functor name. Since our semantics does not make such names inherently "safe," even this restricted inference may fail.[14]

Moreover, on our semantics the conditional from $\ulcorner Ta_i \urcorner$ to $f(a_i)$ is not generally true. *Neither* half of the Tarski-biconditionals holds universally. Let $f(a_1) = \ulcorner TQ(Ta_1) \urcorner$. Then $R_{\mathfrak{M}}(\ulcorner TQ(Ta_1) \urcorner, \ulcorner Ta_1 \urcorner)$ and $R_{\mathfrak{M}}(\ulcorner Ta_1 \urcorner, \ulcorner TQ(Ta_1) \urcorner)$. So $\ulcorner TQ(Ta_1) \urcorner$ and $\ulcorner Ta_1 \urcorner$ are both neuter in $\mathfrak{M}$. The conditional from $\ulcorner TQ(Ta_1) \urcorner$ to $\ulcorner Ta_1 \urcorner$ (which is just $f(\ulcorner Q(Ta_1) \urcorner)$) goes from a neuter sentence to a neuter sentence. It itself is neuter by a supervaluation. Such conditionals do hold for Skyrms where a_i is a quotation-functor name, and indeed this fact is recorded in his fourth axiom schema.[15] On the other hand, the *inference* from $\ulcorner Ta_i \urcorner$ to $f(a_i)$ is truth preserving in our semantics. For $\ulcorner Ta_i \urcorner$ is true in $\mathfrak{M}$ only if it either leads into an $R_{\mathfrak{M}}$-cycle of which it is not a member, or else is not $R_{\mathfrak{M}}$-ungrounded at all. In either case, $\ulcorner Ta_i \urcorner$ is true in $\mathfrak{M}$ just in case $f(a_i)$ is true in $\mathfrak{M}$.

In short, our semantics embodies only a restricted portion of the correspondence theory of truth. Nevertheless, the correspondence theory can be salvaged at least to this extent, that the Tarski-biconditionals are never *false*. For if the biconditional for $f(a_i)$ were false by a supervaluation, either $\ulcorner Ta_i \urcorner$ would be true and $f(a_i)$ false or *vice versa*. In either case, both would be bivalent. But then $\ulcorner Ta_i \urcorner$ would be true just in case $f(a_i)$ is true, and false just in case $f(a_i)$ is false. This contradicts the assumption. Similarly, the Tarski-inferences from $f(a_i)$ to $\ulcorner Ta_i \urcorner$ are valid in the weak sense that they never lead from truth to falsehood. Moreover, the opposite inferences, as we have seen, are valid in the strong sense that they are truth-preserving.

These considerations are similar to some made by Hans Herzberger. He says:[16]

> Perhaps the general principles of semantic theory are simply not true. . . . They are at any rate inherently secure.[17] On the strict transcendence view they would be not determinate and thereby not true.
>
> A case in point is the equivalence-principle itself. On the projected account this principle is inherently secure, as are each of the Tarski biconditionals that can be derived as instances of it.

Herzberger's approach is nevertheless not the same as ours. For one, he evaluates molecular sentences by Bochvar's "internal" matrices,[18] while we employ supervaluations. "Bochvar's internal matrices then yield the satisfactory result that indeterminate sentences uniformly have indeterminate Tarski biconditionals, whereas determinate sentences uniformly have true Tarski biconditionals."[19] No such clean division obtains on our approach. For where $f(a_1) = \ulcorner Ta_1 \urcorner$, $\ulcorner Ta_1 \equiv Ta_1 \urcorner$ is a Tarski biconditional. It is *true* in $\mathfrak{M}$ by a supervaluation, even though $\ulcorner Ta_1 \urcorner$ is neuter in $\mathfrak{M}$.

Skyrms sets out the following axiom schemata and rules:[20]

Axiom Schemata

S1) Schemata adequate to generate just the tautologies of the classical propositional calculus.
S2) $TQ(A) \supset TQ(TQ(A))$.
S3) $\sim TQ(A) \supset TQ(\sim TQ(A))$.
S4) $TQ(A) \supset A$.
S5) $a_i = a_i$.
S6) $(TQ(a_i = a_j)\ \&\ TQ(A)) \supset \sim TQ(\sim A^*)$ where A^* is the result of substituting a_j for some or all occurrences of a_i in A and provided that a_i does not occur within the scope of a Q-functor in A.
S7) $a_i = a_j \supset TQ(a_i = a_j)$.
S8) $\sim(a_i = a_j) \supset TQ(\sim(a_i = a_j))$.
S9) $(TQ(A)\ \&\ TQ(A \supset B)) \supset TQ(B)$.

Rules:

R1) From A and $\ulcorner A \supset B \urcorner$, infer B.
R2) From A, infer $TQ(A)$.

S2 and S3 embody Skyrms' principle that truth ascriptions made via Q-names are always bivalent. S4 and R2 contain his version of the correspondence theory. S6 is a weakened schema for substitution. S7 and S8 formulate the bivalence of all identity statements. For our purposes, the interesting ones are S2-S4, S6, S9, and R2. Counterexamples to S2-S4 can be constructed in our semantics by assuming that $f(a_i) = \ulcorner TQ(Ta_i) \urcorner$, and then replacing '$A$' in the schemata by '$\ulcorner Ta_i \urcorner$'. A counterexample to S6 can be constructed as follows:

$$(TQ(a_1 = Q(TQ(Ta_1)))\ \&\ TQ(Ta_1)) \supset \sim TQ(\sim TQ(TQ(Ta_1))).$$

If the first conjunct of the antecedent is true in $\mathfrak{M}$ (and there are models in which it is), then the whole is neuter in $\mathfrak{M}$. For a counterexample to S9, let $f(a_1) = \ulcorner TQ(a_1 = a_1 \vee Ta_1)\urcorner$. Then

$$(TQ(a_1 = a_1) \;\&\; TQ(a_1 = a_1 \supset (a_1 = a_1 \vee Ta_1))) \supset TQ(a_1 = a_1 \vee Ta_1)$$

is neuter in $\mathfrak{M}$. A counterexample to R2 may be constructed thus: For all $i, j \geqslant 1$, let $f(a_i) = \ulcorner Ta_{i+1}\urcorner$, where $a_i = a_j$ only if $i = j$. Then the inference from $\ulcorner a_1 = a_1 \vee Ta_1\urcorner$ to $\ulcorner TQ(a_1 = a_1 \vee Ta_1)\urcorner$ leads from a truth to a neuter sentence. The reader may work through these cases for himself.

It remains to be seen what happens to substitution of identicals on our approach. First we show that substitution is not always truth-preserving. Let $f(a_2) = f(a_1) = \ulcorner{\sim}Ta_1\urcorner$. Then $R_{\mathfrak{M}}(\ulcorner Ta_1\urcorner, \ulcorner Ta_1\urcorner)$, and so $\ulcorner Ta_1\urcorner$ is neuter in $\mathfrak{M}$. So also is $\ulcorner{\sim}Ta_1\urcorner$. But $\ulcorner Ta_2\urcorner$ is false in $\mathfrak{M}$, since it leads into an $R_{\mathfrak{M}}$-cycle of which it is not a member, and since $f(a_2)$ is not true in $\mathfrak{M}$. Hence $\ulcorner{\sim}Ta_2\urcorner$ is true in $\mathfrak{M}$. So the move from $\ulcorner{\sim}Ta_2\urcorner$ to $\ulcorner{\sim}Ta_1\urcorner$ by substitution does not preserve truth. The same example suffices to show that substitution does not preserve non-falsehood. The move from $\ulcorner Ta_1\urcorner$ to $\ulcorner Ta_2\urcorner$ leads from a neuter sentence to a falsehood.

For Skyrms, as we saw at the beginning of section 1, substitution is valid in general only in the weak sense that it never leads directly from a truth to a falsehood. This is also so on our approach. We first argue the case where A is atomic. Suppose $f(a_i) = f(a_j)$, and A has a different truth-value than $A^{a_i}/\!\!/a_j$ (where $A^{a_i}/\!\!/a_j$ is the result of replacing zero or more occurrences of a_j—not within the scope of a quotation-functor—in A by occurrences of a_i). Then A and $A^{a_i}/\!\!/a_j$ are both $R_{\mathfrak{M}}$-ungrounded, $A = \ulcorner Ta_j\urcorner$ and $A^{a_i}/\!\!/a_j = \ulcorner Ta_i\urcorner$. The only possibility is that one is a member of an $R_{\mathfrak{M}}$-cycle, and the other leads into an $R_{\mathfrak{M}}$-cycle of which it is not a member. On any of the other possibilities, $\ulcorner Ta_j\urcorner$ and $\ulcorner Ta_i\urcorner$ will have the same truth-value, contrary to the hypothesis. But if one is a member of an $R_{\mathfrak{M}}$-cycle, it is neuter in $\mathfrak{M}$, and so the substitution cannot lead from a truth to a falsehood. The same argument of course shows also that the substitution cannot lead from a falsehood to a truth.

On the basis of this result, we can argue the general case. Let $\ulcorner a_i = a_j\urcorner$ and A be true in $\mathfrak{M}$. Suppose that $A^{a_i}/\!\!/a_j$ is not true in $\mathfrak{M}$. Then one or more atomic constituents of $A^{a_i}/\!\!/a_j$ must be of the form $\ulcorner Ta_i\urcorner$ and have a value in $\mathfrak{M}$ different than that which $\ulcorner Ta_j\urcorner$ has in $\mathfrak{M}$. Hence, by the above result, either $\ulcorner Ta_i\urcorner$ or $\ulcorner Ta_j\urcorner$ must be neuter in $\mathfrak{M}$. But then there must be a classical valuation v for A with respect to $\mathfrak{M}$, exactly like some classical valuation v' for $A^{a_i}/\!\!/a_j$ with respect to $\mathfrak{M}$, except that where v assigns a value to $\ulcorner Ta_j\urcorner$, v' assigns the *same* value to $\ulcorner Ta_i\urcorner$. Since A is true in $\mathfrak{M}$, it is classically true under all classical valuations for A with respect to $\mathfrak{M}$, and in particular under v. Thus $A^{a_i}/\!\!/a_j$ is classically true under v'. Hence, $A^{a_i}/\!\!/a_j$ cannot be false in $\mathfrak{M}$. This argument can obviously be revised to handle simultaneous substitution as well.

Finally, $\mathcal{L}$ has no class of "safe" names available for truth ascriptions that are guaranteed bivalent.[21] Nor does it have even a mildly global truth predicate. With respect to the latter claim, for all $i, j \geqslant 2$, let $f(c_i) = \ulcorner Tc_{i+1}\urcorner$. Then each $\ulcorner Tc_i\urcorner$ heads an infinite (non-cyclic) $R_{\mathfrak{M}}$-path, and is neuter in $\mathfrak{M}$. Now if a_1 is a name of some $f(c_i)$ (i.e., if $f(a_1) = f(c_i)$ for some $i \geqslant 2$), then $\ulcorner Ta_1 \urcorner$ also heads an infinite (non-cyclic) $R_{\mathfrak{M}}$-path. (In particular, it "enters" the path headed by $\ulcorner Tc_2 \urcorner$ at the i-th place.) So $\ulcorner Ta_1 \urcorner$ will also be neuter. There is no name of c_i by which to make a bivalent truth ascription. Thus the language does not have a mildly global truth predicate. The same example also justifies the claim that it has no class of "safe" names.

3 In view of these facts, it would seem that our semantics has little to recommend it over Skyrms' except that it incorporates the intuition expressed in the quotation in section **1**. Now it seems certainly a desirable feature to have at least a mildly global truth predicate. And in the application of such a truth predicate it would be desirable to have a class of "safe" names. Skyrms' quotation-functor names are obvious candidates. Let us see then how we can revise our semantics to incorporate the advantages of Skyrms' approach, without sacrificing the advantage claimed above for our own.

In effect what we want is a guarantee that truth ascriptions made via quotation-functor names are always bivalent. This guarantee is very simply obtained by a slight change in the definition of $R_{\mathfrak{M}}$. In $\mathcal{L}$ we had

$R_{\mathfrak{M}}(A, B) =_{def}$ for some i, $A = \ulcorner Ta_i \urcorner$ and $f(a_i)$ has B as an atomic constituent.

Recall that the *names* $a_1, a_2, \ldots$ of $\mathcal{L}$ comprise the *quotation-functor names* of $\mathcal{L}$ (of the form $\ulcorner Q(E) \urcorner$) and the *individual constants* $c_1, c_2, \ldots$ of $\mathcal{L}$. We now revise the above definition of $R_{\mathfrak{M}}$ by replacing 'a_i' by 'c_i' at both occurrences. Call this revised language $\mathcal{L}'$. In $\mathcal{L}'$ the only truth ascriptions bearing $R_{\mathfrak{M}}$ to a sentence are truth ascriptions made via individual constants. Truth ascriptions made via quotation-functor names bear $R_{\mathfrak{M}}$ to nothing at all. *A fortiori* no such truth ascription can be $R_{\mathfrak{M}}$-ungrounded. Hence by the truth-rules for atomic sentences that are not $R_{\mathfrak{M}}$-ungrounded, truth ascriptions made via quotation-functor names are always bivalent.

Skyrms' S2-S4, S6, S9, and R2 all failed in $\mathcal{L}$. Each of them holds, however, in $\mathcal{L}'$. For all but S6, this follows directly from the fact that truth ascriptions made via quotation-functor names cannot be $R_{\mathfrak{M}}$-ungrounded in $\mathcal{L}'$. Moreover, substitution of identicals is valid in $\mathcal{L}'$ as in $\mathcal{L}$, in the weak sense that it cannot lead directly from a truth to a falsehood. The proof for $\mathcal{L}$ in section **2** carries over to $\mathcal{L}'$. Hence it follows also that axioms under S6 are all valid in $\mathcal{L}'$. In short, the revised semantics of $\mathcal{L}'$ satisfies Skyrms' axioms and rules.

What we have then is an alternative semantics to Skyrms' own, a semantics that retains the advantages of his approach and which satisfies the axiomatization he provides, but which is less objectionally "conservative" than his in the sense described at the beginning of this paper.

NOTES

1. See [4].

2. *Cf.* [4], p. 155.

3. *Cf.* [4], p. 155.

4. "Let us say that a truth predicate is *strongly* global if and only if every sentence of $\mathcal{L}$ has a name and it is provable in $\mathcal{L}$ that for *every* name of every sentence of $\mathcal{L}$ the resultant truth ascription statement is true or false; and that it is *mildly* global if and only if for every sentence there is *at least one* name such that [the] resultant truth ascription statement is provably either true or false." See [4], p. 160.

5. [4], pp. 158f.

6. [4], p. 160.

7. *Cf.* [4], p. 160. 'T' is a truth predicate and 'Q' a quotation-functor.

8. This usage of the term 'sort' has nothing directly to do with the notion of a "sortal."

9. The syntax of our language $\mathcal{L}$ can be made to coincide with that of Skyrms' language, *cf.* [4], p. 157, if we restrict our supply of predicates to a truth predicate only. Skyrms' identity *predicate* is classed among the *logical constants* of our language. Note also that in Skyrms' models ([4], p. 158) only quotations of *sentences* have denotations, not quotations of *expressions* generally. These differences between the two languages are rather inessential. In the following pages, when we compare our language with Skyrms', it will be under the tacit assumption that these minor differences have been eliminated, either by restricting our own language or by extending his in obvious ways.

10. See [1]. Note also that for a language with the syntax and models of Skyrms' language, an atomic sentence is $R_{\mathfrak{M}}$-ungrounded just in case it is of level ω. See Skyrms, [4], p. 159, for the assignment of levels.

11. I am using Quine's notion and notation for ancestrals, according to which every element bears the ancestral of every relation to itself. The "proper" ancestral, which is used here, eliminates these trivial cases. See [3], p. 221.

12. [4], p. 159.

13. See [4], p. 153.

14. The inference from $f(a_i)$ to $\ulcorner Ta_i \urcorner$ is not in general even non-falsehood preserving. Where $f(a_2) = f(a_1) = \ulcorner Ta_i \urcorner$, $f(a_2)$ is neuter in $\mathfrak{M}$, but $\ulcorner Ta_2 \urcorner$ is false.

15. [4], p. 157.

16. See [2], p. 36.

17. "Let us say that a sentence is *secure* in case 'things are as the sentence signifies', and *contrasecure* otherwise. If furthermore the requisite presuppositions are satisfied, the sentence will be true in the former case and false in the latter. Security then, which is my term for 'correspondence with reality', is a constituent of truth—but not the whole of it." [2], p. 31.

18. [2], p. 32.

19. [2], p. 36.

20. See [4], p. 157, with the notation modified.

21. Except, of course, for the uninteresting class of quotation-functor names of identity sentences and quotation-functor names of expressions that are not sentences at all.

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