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A NOTE ON PEIRCE ON BOOLE'S ALGEBRA OF LOGIC

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Peirce was quite naturally led to the development of innovative improvements in logic through critical investigation of the writings of other logicians, e.g., the traditional system of syllogistic, Boole's algebra, Hamilton's system, and Mill's logic. In the following I will consider his early reaction to Boole's algebra of logic. In his 1865 lecture series, Peirce indicated great respect for Boole's accomplishments but also pointed out many failings in Boole's system. By 1865, twelve years before the first publication of Schroeder, Peirce had read and had begun to work on revisions of the algebra of logic developed by Boole.

One of Boole's most significant achievements, in Peirce's view, was his contribution of an effective symbolic notation.[1] Indeed, Boole's was not the first attempt at a symbolic notation for logic, but Peirce maintains, it is more adequate in fulfilling the aims desired from using such a notation. Ordinary language, with its ambiguities and richness, is inadequate for the investigation of logical form. The symbols most effective for the science of logic should have the power of diagramming significant linguistic forms and of aiding in the analysis of the laws of the necessary relations between such forms. Boole's symbolization, according to Peirce, is the first significant approach towards fulfilling these objectives. Thus Peirce attempts to convince his 1865 lecture audience that Boole's approach is of great value and well worth studying; still, he explains, there is much that is wrong with the system.

The notation is not adequate, for example, to express all types of propositions. In particular, Peirce, in his 1865 lecture series, expressed dissatisfaction with:

(1) Boole's symbolization of particular propositions. Boole used the symbol 'v' to indicate the indefinite class; for example, he symbolized the particular proposition 'Some X's are Y's' as '$v . x = v . y$'. This method of symbolizing particular propositions, Peirce argued, is not adequate to indicate the existential presupposition of particular propositions.

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(2) Boole's symbolization of universal and conditional propositions using '=' as the connective. Boole, that is, symbolized 'All men are animals' as '$m = ma$', where 'm' denotes the class of men and 'a' denotes the class of animals. Conditional propositions are symbolized in the following way by Boole. Let 'A' stand for 'There is an east wind'; let 'B' stand for 'The barometer would rise'. Boole further introduces 'a', which stands for 'that portion of time for which the proposition 'A' is true' and 'b', which stands for 'that portion of time for which the proposition 'B' is true'. Boole symbolizes the conditional proposition 'If there is an east wind, then the barometer would rise' as '$a = ab$' (i.e., '$a = a \cap b$). According to Peirce, Boole's symbolization was not adequate to express the logical relation between the subject and predicate of a universal proposition or that between the antecedent and consequent of a conditional proposition.

(3) Boole's mathematical symbols. Peirce recognized the need to clearly distinguish between mathematical and logical symbols.

In Peirce's 1865 lecture series, then, we find the motivating concerns that led to many of his later important developments. As a result of his dissatisfaction with Boole's algebra, in March of 1867, Peirce published a revision of the Boolean algebra (3.1ff). In this paper he distinguished logical from mathematical operations, presenting distinct logical symbols. He distinguished inclusive disjunction, as a logical notion, from exclusive disjunction used by Boole; this allowed Peirce to introduce the law of duality. He introduced the law '$a \vee a = a$' (not in Boole's algebra because of his use of exclusive disjunction) and he revised Boole's law '$a^2 = a$' as '$a . a = a$', thus presenting the laws of tautology. He also introduced and proved the law: '$(a . b) \vee c = (a \vee c) . (b \vee c)$'. In a paper of September 1867 (3.20ff), Peirce dropped logical subtraction and logical division from his system of logic for their use leads to uninterpretable expressions; he also introduced the notion of material identity, distinguishing it from mathematical equality.

Because of his dissatisfaction with the use of the identity theory of the copula in the work of Boole and also in the writings of Hamilton, De Morgan and others, in 1870 Peirce introduced the symbol '$-\!\!<$' to indicate class inclusion and implication (3.47). Thus by 1870 Peirce presented a complete set of operations for what is commonly called Boolean algebra, clearly distinguishing, in his system, between the logical notions of class inclusion and identity. Using this new symbol, in Peirce's view, he was able to present a more adequate symbolization of universal and conditional propositions than Boole could in his system.

Peirce found the Boolean system inadequate to treat mathematical propositions without introducing relations. In 1870, Peirce presented his first published study on the logic of relations (3.45ff). He introduced the notion of the relative product and examined properties of relations, e.g., symmetry, transitivity and reflexivity. In this paper of 1870 Peirce also attempted to symbolize the existential quantifier in such a way as to indicate existence ("case of the existence of--") to revise Boole's method of symbolizing particular propositions. Peirce's dissatisfaction with Boole's manner of symbolizing particular propositions led eventually to his independent discovery of quantifiers and indices.[2]

Schroeder, in letters and published writings, expresses indebtedness to Peirce for many of his own developments in logic. It is not Boole's algebra but fundamentally the Boolean system as revised by Peirce that is what is currently referred to as Boolean algebra or the Boole-Schroeder algebra of logic.

NOTES

1. This is discussed in Ms. 344, Lecture 6, 1865 Lecture Series.
2. *Cf.* 3.328ff. For the discovery of quantifiers, Peirce acknowledges a debt to his student, O. H. Mitchell; Peirce himself though introduced indices.

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