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GEACH ON ATOMICITY AND SINGULAR PROPOSITIONS

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In a recent article we find this assessment of Frege's contribution to the theory of atomic propositions.

> When in his *Begriffsschrift*, Frege introduced quantifiers, he was thus compelled to make the subject-predicate form the unique form of his prime sentences. Frege is generally credited with having introduced, to get full quantification theory, the generalized subject-predicate form, that is one in which there is more than one subject. This is true. But, it is also true that he was making more than a generalization of something already existing. He was putting the subject-predicate form squarely at the basis of logic in its generalized (more than one subject) and in its simple (just one subject) version.[1]

I too share this estimate of the importance of Frege's innovations for the monadic case. Before Frege, there was no effective procedure for determining the logical analysis of expressions involving multiple generality. The Scholastics had introduced the doctrine of Supposito to help determine the truth conditions of such expressions, but the doctrine was so clumsy that it became apparent that it would not serve well. Furthermore, Frege's solution to the problem of multiple generality was not at all within the Scholastic tradition. In fact his analysis suggested a more radical departure from what, until then, had been regarded as the basic form of the proposition than just a different analysis of multiple generality might suggest. It required the 'atomicity' of the singular propositional form *Fa*, *Rab*, etc.

The purpose of this paper is twofold. First, I will assess Frege's discovery of the atomic proposition, since it seems to me that only when we see what the new analysis of the underlying propositional form of Frege's primitive propositions means for inference power can we fully understand Frege's achievement properly. Secondly, I will scrutinize an argument deployed against the traditional doctrine of singular propositions that represents a line of reasoning accepted by the vast majority of contemporary philosophers for a Fregean-like analysis of the primitive propositions of quantification theory. This is of interest for the following reason. No

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one will deny that by changing the logical form of the standard proposition entering into deductive reasoning, Frege revolutionized logical theory. What we must be on the lookout for, however, are certain sophistic arguments for a Fregean point of view that in the end only blur the real significance of his discovery. One such argument is given by Peter Geach in *Reference and Generality*.[2]

1 The logicians prior to Frege had relied mainly upon grammatical features of the languages in which they conducted their logical investigations, exploiting these features in the structure they took to underlie the syntax of their logic. The logical form of their basic proposition, the categorical proposition is as follows: a syntactically complex subject, i.e. 'some/every *S*,' which consists of a sign of quantity followed by a term joined to a complex predicate, i.e. 'is/is not *P*,' which consists of a positive or negative copula (a sign of quality) followed by a term. The general form of this proposition is represented as follows:

some/every *S* is/is not *P*

A term could occur in a categorical proposition in either the subject or the predicate position. For instance the term 'dog' occurs in the subject position in "Some dog is in my office" and in predicate position in "Some animal is a dog." But the syntactic fact that a complex subject contains not only a term but also a sign of quantity (either 'some' or 'every'), and a predicate contains, besides a term, a sign of quality (either 'is' or 'is not'), signals a difference of semantic role between the complex subject and predicate. The semantic role of the complex subject is to refer; the predicates' role, on the other hand, is to characterize what the subject is referring to. In the two propositions mentioned before the term 'dog' occurs in the first as part of the complex subject 'some dog' and in the second as part of the complex predicate 'is a dog.' The complex subject 'some dog' in the first proposition refers to (but does not stand for) some of the dogs denoted by the term 'dog' while the predicate 'is a dog' in the second proposition is an expression that asserts something about the complex subject 'some animal'; such that, the complex subject refers to what the assertion has been made about. And so it will be important for the sequel to realize now that even though terms in categorical propositions are interchangeable in their *A*, *E*, *I*, and *O* forms, it is not this that is essential to understanding the semantic character of traditional logic but rather the difference in function between the complex subject and complex predicate, i.e., the referential function of the complex subject and the characterizing function of the predicate.

For our purposes we are mainly interested in the function of complex subjects since they carry the burden of reference. For traditional logic complex subjects do not refer by way of identifying or naming an object and so are what might be called weakly referential. Thus 'some man' in "Some man is immortal" is weakly referring, and so the question of which man or men the complex subject refers to need not be answered by identifying a

man. For a speaker who asserts that "some man is immortal" may not have any specific man in mind. It is enough that some man or other be immortal for the proposition to be true. But, as we know, it was just because categorical propositions were thus weakly referential that the doctrine of Supposito was introduced and the eventual dissolution of traditional logic ensued.

Considerations such as these are why I wish to agree with those who find that Frege's main contribution to logic was the insistence that the basic propositions of logical theory were singular propositions devoid of all syncategormatic elements. Frege's primitive proposition, in fact, consisted of only a logically simple name and predicate. This is, of course, in contrast to the traditional analysis of the general propositional form of the categorical proposition, which not only contained syncategormatic elements in its structure but, as we shall see later on, subsummed singular as well as general propositions under this one propositional form. Furthermore, unlike the traditional analysis, the distinction between subject and predicate was no longer tied to the grammatical forms of natural language but became analyzed much along the lines of a mathematical formula consisting of a functional expression (the 'concept'), an argument for the function (the 'logical subject'), and a value for the function (the 'truth-value' of the proposition). An important consequence of this shift, especially for our purposes, is that a Fregean-like analysis of atomic propositions requires a powerful referential apparatus. That is, an expression functioning as a logical subject Frege took to *name* or *stand for* the object uniquely denoted by the expression while an expression functioning as a logical predicate referred to a concept that *characterized* the object so named or designated. The difference between an expression functioning as a logical subject and an expression functioning as a logical predicate was crucial for Frege. For it was the role an expression played in a language that determined its use as either part of the logical subject or the logical predicate. Thus, if an expression, grammatically simple or complex, functions to *name* or *stand for* an object it is a logical subject, if not it is simply part of the predicate. There are no elements in common.

Since a Fregean-like system is strongly referential, the criteria to determine whether an expression is functioning as a logical subject or not is fairly straightforward; some of the tests that are employed are (a) existential generalization, (b) whether there is a definite answer to the question, To whom/To what are you referring?, and (c) establishing whether an expression carries with it a criterion of identity associated with its use.[3] Only when such tests are satisfactorily satisfied is an expression functioning as a name. Furthermore, because an atomic proposition has no syncategormatic elements in its structure and even though names themselves may be phonemically or typographically complex a logical subject has no parts that determine its sense, i.e., it is logically simple, and, because it *stands for* an object, it is, for Frege, semantically complete.

A predicate expression, on the other hand, functions to characterize the object referred to by the subject expression in such a way that if the

predicate *applies to* or correctly *characterizes* the object the proposition is true, but if it fails to apply or mis-characterizes the object the proposition is false. However, predicates are not logically simple or complete in the way names are since they may consist of parts that determine their sense. Even so, predicates *stand for* concepts and in this sense, they are complete. Also, since Frege finds no use for the copula except as a grammatical marker of predicate expressions, predicates may be thought to be syntactically simple! But then names and predicates are distinct and can have no syntactic parts in common.

So a name can never *stand for* a concept nor can a concept *stand for* an object, although, what may superficially look like a name can occur in a predicate and what resembles a predicate might occur in a subject expression. But then these expressions do not have their usual naming of characterizing function. As Frege says,

> . . . To this distinction among the symbols there naturally corresponds an analogous distinction in the realm of references: to a proper name corresponds an object, and to the predicate corresponds what I will call a concept. This is not meant to be a definition. For the decomposition into saturated and unsaturated parts must be regarded as a primitive feature of logical structure, which must simply be recognized and accepted but which cannot be reduced to something more primitive.[4]

Thus, the distinction between name and predicate is absolute and primitive from a Fregean point of view. Atomic propositions are formed from these primitives by syntactic rules that govern their formation. For example, a predicate like 'is a man' *stands for* a concept that either *characterizes* or *mis-characterizes* a certain object, say John, just in case it is true or false that the object denoted by the logical proper name 'John' is a man. So, if we name the concept 'is a man' by the predicate constant M and we use the letter a as a name of John then the logical form of the proposition "John is a man" becomes Ma. This propositional form is the general logical form of an atomic proposition from a Fregean point of view.

We have only to look to contemporary logical theory to see how far reaching the consequences of Frege's re-interpretation of the basic propositions have been for inference power. Today we not only have a much clearer notion of logical entailment but we also, by utilizing the tools Frege has given us, have been able to better understand the nature of mathematics, of truth in a formal language, and ontic commitment. These are some of the philosophic problems that have virtually been solved by the introduction of Frege's primitive propositions. At least it can easily be seen that the solutions we now have on hand could only be stated in terms of Frege's analysis of propositions into atomic parts logically primitive and devoid of logical signs.

Moreover, another important feature of the Fregean point of view that has come down to us from the work of Tarski, Quine and others is one that calls for our close attention. Namely the notion of a standard interpretation for quantification theory. In a standard interpretation for a language of this sort once a well defined domain of objects U is specified over which the

variables are said to range, the interpretation function assigns objects from U to the individuals. This is usually done by associating with those constants names or descriptions of objects. The predicate constants are assigned properties which in an extensional language may be identified with sets the members of which all have that property. This is done by associating with that predicate constant, a predicate name, or description. Sentences are assigned propositions which are usually identified with one of two truth-values. The truth of an atomic sentence 'Pa' is true if and only if the interpretation function assigns a to an individual, P to a predicate and $a \in P$, otherwise false. However, when we consider general propositions this is not the case. Here we are confronted with the logical constants—the universal and the existential quantifiers. And they are not, in general, reducible to conjunctions or disjunctions of atomic propositions. In this theory then, there is a radical difference between the logical forms of singular and general propositions. But this by no means must be the case. As an alternative we might, taking a clue from Leibniz, treat a singular proposition like "Socrates is wise" as a special case of the general form of the categorical proposition. Leibniz's view can be found in a passage from "A Paper on 'Some Logical Difficulties.'"

> "How is it that opposition is valid in the case of singular propositions-. . . since elsewhere a universal affirmative and a particular negative are opposed. Should we say that a singular proposition is equivalent to a particular and to a universal proposition? Yes, we should. So also when it is objected that a singular proposition is equivalent to a particular proposition, since the conclusion in the third figure must be particular, and can nevertheless be singular; e.g., 'Every writer is a man, Some writer is the Apostle Peter, therefore the Apostle Peter is a man.' I reply that here also the conclusion is really particular and it is as if we had drawn the conclusion 'Some Apostle Peter is a man.' For 'Some Apostle Peter' and 'Every Apostle Peter' coincide, since the term is singular.[5]

So, if we take Leibniz seriously then an analysis of singular propositions might be as follows: "Socrates is wise" "Some Socrates is wise and Every Socrates is wise".

2 So far only Frege's view has been fully stated and its importance noted and discussed. We have also assessed why Frege felt it necessary to propose such a drastic shift in perspective as well as seeing that it was Frege's major achievement to have changed the very notion of the primary sentences of logic. Also, inspired by Leibniz, we have introduced an alternative to the Fregean atomicity of singular propositions based on a more traditional account of propositions. It is now time to review the argument given by Geach in favor of the subject-predicate distinction in the atomic proposition. We should also understand that Geach's argument is implicitly meant to count against the Leibnizian line on singular propositions.

The basis for Geach's argument is an asymmetry between subject and predicate noted as long ago as Aristotle when he said that substances unlike qualities have no contraries. In the formal mode: an expression A has a

contrary if and only if its complement -A, is well-formed. Thus predicates have contraries but logical subjects do not. This makes for a fundamental asymmetry of subject and predicate. Geach puts the point this way:

> When a proposition is negated, the negation may be taken as going with the predicate in a way in which it cannot be taken to go with the subject. For predicables always occur in contradictory pairs; and by attaching such a pair to a common subject we get a contradictory pair of propositions. But we never have a pair of names so related that by attaching the same predicates to both we always get a pair of contradictory proposions.[6]

Geach believes this can be proved. I will quote his argument in full.

> The conjunction of a pair of predicables when attached to a name 'x' signifies the same as the conjunction of the propositions that we get by attaching each predicate separately to 'x', this is precisely what conjunction means when applied to predicables rather than propositions. Now suppose we had a pair of names 'x' and 'y' such that by attaching the same predicate to both we always got a pair of contradictory propositions. Thus we have, for example
>
> "$(P \,\&\, Q)x$" is contradictory to "$(P \,\&\, Q)y$"
>
> in view of what the conjunction of predicables has to mean
>
> "$Px \,\&\, Qx$" is contradictory to "$Py \,\&\, Qy$"
>
> But by our supposition "Px" is contradictory to "Py" and "Qx" is contradictory to "Qy". We may thus infer:
>
> "$Px \,\&\, Qx$" is contradictory to "not (Px) & not (Qx)"
>
> and from this it is easily proved by way of the truth-functional tautology
>
> $$(\neg(p \,\&\, q) \equiv (\neg p \,\&\, \neg q)) \equiv (p \equiv q)$$
>
> that for this name 'x' arbitrary predications "Px" and "Qx", assuming they can be significantly formed into one predication must always have the same truth-value—which is absurd."[7]

Geach concludes, on the basis of this argument, that "No names come in contradictory pairs; therefore no name is a predicable."[8]

But this in itself is not an argument against the traditional logical analysis of singular propositions. It begs the question entirely. For if Leibniz is right, Geach's assumption that negation effects no change in the subject of "Socrates is wise" is simply mistaken. "Socrates is wise" from a Leibnizian point of view, is equivalent to the conjunction of "Some Socrates is wise" and "Every Socrates is wise." The principle of tolerance that allows a singular term such as "Socrates" to have both particular and universal quantity can account just as well for the logical facts as the positive parts of Geach's arguments for the primacy of singular predication.

On a Leibnizian view the denial of "Socrates is wise" is "(Every) Socrates is not wise" which is equivalent to "(Some) Socrates is not wise." In the latter, it will be noted there is a shift in the sign of quantity. But since "Every Socrates is not wise" and "Some Socrates is not wise" are

equivalent the shift is unmarked and is idle. That it is idle should by no means dissuade us from accepting the traditional account of singular propositions since it might equally account for the fact that logicians like Geach can overlook its existence. Commenting on Geach's argument against the traditional analysis Sommers observes

> The closest thing to an argument that I have seen is in *R & G* where Geach contrasts singular propositions with those of form 'some/every *X* is/isn't *Y*' saying that singular propositions can be contradicted by merely changing the 'quality' of the predicate while the latter cannot be contradicted without also changing the quantity of the subject. But this argument, if it be one, against the traditional view that all predication, singular as well as general, have complex logical subjects simply begs the question. If 'Ed is ill' has a syntactically complex subject, its explicit form may be "Every Ed is ill' and its contradictory will then be 'Some Ed isn't ill.' The change of quantity is there but it is unmarked in the case of singular propositions. The same point vitiates Geach's thesis that 'by attaching a pair of contradictory predicates to a common subject we get a pair of contradictory propositions.' Note the double dogmatic assumption that the logical subjects must be singular and that the subject of a contradictory pair of singular propositions is the same in both.[9]

That is Geach's argument assumes, first of all, the name-predicate distinction, the distinction it is attempting to justify. Secondly, it assumes that atomic proposition differ in logical form from categorical propositions, since the terms in standard categorical propositions are interchangeable; there being no syntactic distinction between term occurring in subject and those occurring in predicate position. Geach says,

> A term, as conceived in Aristotelian logic, is supposed capable of being a subject in one proposition and a predicate in another; since only names, not predicables, can be logical subjects, this notion of terms has no application whatsoever. This initial confusion has led to a multitude; *pessima in principiis corruptio*.[10]

We have already observed where Frege's subject and predicate are material distinct in the traditional doctrine, subjects and predicates differ in containing different *logical* signs. Let us call the traditional difference 'logical asymmetry.' The fact that '*a* is *P*' the subject and predicate consist of syntactically different parts is not an argument for a Fregean point of view, which treats this as a primitive difference not further analyzable, since we have not yet ruled out the logical asymmetry that exists between the complex subject and the complex predicate of a categorical proposition, and we have not shown that '*a* is *P*' is not of this form. The traditional asymmetry is today not much discussed and I will explain it further.

We recall the traditional doctrine concerning the different semantic roles of the complex subject and predicate in a categorical proposition: the complex subjects' ('some/every *S*') role is to refer; the role of the predicate ('is/is not *P*'), on the other hand, is to characterize what the subject refers to. This semantic difference is itself signalized by a syntactic difference in logical signs, i.e. a sign of quantity in the subject and a sign

of quality in the predicate. Now we must examine the logical roles of the distinct syncategormatic elements.

One key to understanding the asymmetry of subject and predicate in traditional logic is to examine the way negatives like 'not' and 'non' may be distributed into a proposition and how they may affect the syncategormatic elements. The operation of negation, first of all, acts on the entire proposition, affecting both subject and predicate and effecting a change in the sign of quantity of the subject and the sign of quality in the predicate. Thus, "Not: every S is P" is equivalent to "Some S is not P". The terms are as yet not negatively inflected; all that has so far changed are the syncategormatic terms 'every' to 'some' and 'is' to 'is not'. This fact, that negation operates on the predicate to change its quality and on the subject to change its quantity, is thus one logical manifestation of the difference between subject and predicate.

Still one may argue that no real asymmetry has been shown. After all is not "Some S is P" equivalent to "Some P is S". This might suggest to some philosophers that 'some' behaves like 'is' so that even though in the subject there is, in some cases, a change of quantity and in the predicate a change of quality this may not amount to anything but a trivial asymmetry. This, however, is not the case. A more pronounced manifestation of the logical difference between subject and predicate is displayed when we apply the traditional operation of obversion to categorical propositions. Now, "Some S is not P" is equivalent by obversion to "Some S is non-P" where the syncategormatic term 'non' attaches directly to the term in predicate position. That is, obversion permits a change in the quality of the predicate term, i.e. 'is not P' changes to 'is non-P' where 'is not' changes to 'is' and 'P' to 'non-P'. But obversion is only permitted in the predicate; we cannot abvert the subject phrase and derive from, for example, "Some S is P" an equivalent proposition by negating the term in subject position and changing the syncategormatic element 'some' to 'every.' Generally this may be put as follows: For predicates we have

(a) is P = is not non-P
(b) is not P = is non-P.

If there were subject obversion we would also have

(c) some S = every non-S
(d) every S = some non-S.

But subject obversion is not permitted. One example should suffice to show that while predicate obversion is legitimate subject obversion is not. From

(1) "Some book is not red"

we may, by predicate obversion, infer

(2) "Some book is non-red".

This is simply an instance of (b) above. That (1) and (2) are equivalent is obvious, and that we may infer that obversion in the predicate holds

generally can, I think, be granted. But if we attempt to obvert the subject of (1) by our proposed rule (c) we would get

(3) "Every non-book is red"

which is obviously not equivalent to (1). In fact it is impossible to infer (3) from one by any of the permitted rules of immediate inference. Of course, similar results obtain for propositions in the *A*, *E*, and *I* forms.

Now, we have seen that when we negate a categorical proposition we only change the sign of quantity and the sign of quality. Furthermore we have seen that while we may obvert the predicate it is never permissible, if we are to preserve inference, to change the quality of the term in subject position. Thus, Aristotles' dictum—that substances have no contraries while qualities do—holds for the traditional account of propositions as well as in contemporary logical theory. So, Geach's argument for the primacy of singular propositions devoid of logical signs fails, as an argument for a Fregean interpretation of singular propositions against the traditional theory presented here. And it should because after all the asymmetry between subject and predicate does not depend upon whether the proposition is singular or not, nor whether the primitive proposition is devoid of syncategormatic elements or not, but rather upon the referring role of the subject as opposed to the characterizing role of the predicate (be they complex or simple). And since the different roles of subject and predicate are analogous in the two theories, albeit each utilizing different notations and each having their own metatheoretical considerations, Geach's argument had to fail.[11]

The one question that remains is whether there is still a reason to accept a Fregean point of view over the traditional viewpoint. I think that there is. It should be accepted simply because, for now at least, it has yielded the only acceptable semantics that we have. I would say that atomicity is justified by semantics, nothing more, nothing less.

FOOTNOTES

1. Jean van Heijenoort, "Subject and predicate in Western logic," *Philosophy East & West*, vol. 24 (1974), p. 258.

2. Geach's argument, of course, is not the only argument for the atomicity of singular propositions that blurs Frege's achievements. Both Strawson and Dummett tend to do so as well. The reason Geach's argument has been chosen is that to date it is the most clearly formulated argument of its type.

3. See P. T. Geach, *Reference and Generality*, Cornell University Press, Ithaca (1962), and M. Dummett, *Frege: Philosophy of Language*, Harper & Row, New York (1972).

4. G. Frege, "Uber die Grundlagen der Geometrie, II," *Jahresbericht der deutschen Mathematiker-Verernigung*, vol. 12 (1903), pp. 371-372. Translated by M. E. Szabo in *The Philosophical Review*, vol. 59 (1960), p. 12.

5. G. Leibniz, "A paper on 'Some logical difficulties'" translated in G. H. R. Parkinson, ed., *Leibniz Logical Papers: A Selection*, Clarendon Press, Oxford (1966), p. 115.

6. Geach, *op. cit.*, p. 32.

7. *Ibid.*, pp. 32-33.

8. *Ibid.*, p. 33. Both Dummett and Strawson have arguments similar to Geach's. Strawson says in *Subject and Predicate in Logic and Grammar*

> In general we can negate and compound monadic predicates in a style exactly analogous to that in which we negate and compound propositions; and the internal logic of predicate composition is exactly analogous to that of propositions. . . . It is easy to show that we cannot coherently frame compound and negative subjects in symmetrical fashion. (page 6)

while Dummett says in *Frege: Philosophy of Language* that

> Given the predicate 'wise', we may introduce a new predicate, say 'foolish' by stipulating that, for every proper name 'a', 'a is foolish' is to have the same truth-value as 'It is not the case that a is wise': but we cannot give the name 'Socrates', legitimately introduce a new name say 'Nonsocrates', by the stipulation that, for every predicate '$F(\ )$', 'F(Nonsocrates)' is to have the same truth-value as 'It is not the case that F(Socrates)'—however severely we restrict the range of predicates to which this stipulation is to be applied. (page 64)

9. F. Sommers, "Distribution matters," *Mind*, vol. 84 (1975), p. 44, see also Sommers, "Calculus of terms" in *Mind*, vol. 79 (1970), pp. 1-39.

10. Geach, *op. cit.*, p. 34.

11. Similar arguments vitiate Strawson's hypothesis and Dummett's stipulation. There is nothing in Dummett's procedure that disallows the possibility of a complex subject. We have already shown that the asymmetry Geach and now Dummett takes to exist between subject and predicate also exists for subjects and predicates as Leibniz would have us construe them. Thus Dummett's arguments fail.

    Strawson's arguments for the atomicity of singular propositions also suffers a similar fate. We have already seen that we cannot negate, i.e., obvert, the subject of a categorical proposition. But, predicates, Strawson says may also be compounded like propositions in the propositional calculus while subjects cannot. All well and good. But, once again, this is not an argument for the atomicity of singular propositions. The fact that 'some' distributes through disjunctions and not conjunctions and that 'every' distributes through conjunctions but not disjunctions was well known to logicians like Leibniz. These logicians, of course, knew that propositions like, e.g., "Every philosopher is either misguided or profound" is not equivalent to "Every philosopher is misguided or every philosopher is profound" while "Every philosopher is misguided and profound" is equivalent certainly recognition of this does not necessarily go hand-in-hand with the denial of the complexity of the logical subject, and is, in fact, consistent with it. Thus, the proposition "Socrates and Plate are philosophers" is equivalent to "Socrates is a philosopher and Plate is a philosopher" and the proposition "Socrates or Plato is right" is equivalent ti "Socrates is right and Plate is right" because singular terms, on this view, contain both signs of quantity, i.e., 'some' and 'every' and so may be distributed through both disjunctions and conjunctions. Thus, the distribution of singular terms through both conjunctions and disjunctions is maintained along with the asymmetry between the distribution of subject terms involving only one sign of quantity without forfeiting our Leibnizian program.

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