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Higher Quantity Syllogisms

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The traditional doctrine of the syllogism (cf. [2]) restricts the number of quantities to just two, universal and particular. Thompson [9] has made an insightful attempt to extend the syllogism to three more quantities (based on [6]). Though his new syllogistic rules are apparently sound, they are not complete. His rules do not generate 12 valid syllogisms in the third figure—those within the dotted circles in (6) below. Carnes (in Section 2 of [7]) devised sound and complete rules for the five quantity syllogism (and also proved them sound and complete in Section 4). Rather than the 93 syllogistic forms that Thompson claimed were valid there are 105 valid forms when the three additional quantities are added. The three new quantities are labeled by Thompson *predominant*, *majority*, and *common*. These quantities can be expressed by "few", "most", and "many", among other English quantifier words and phrases. The squares of opposition that Thompson begins with are as follows (except that I substitute the positive quantifier expressions "almost-all" for Thompson's use of "few" in predominant affirmatives and negatives, avoiding Thompson's peculiarity of stating the predominant affirmative as a kind of double negative in "Few *S* are not-*P*"):

(1)	affirmative	negative
universal	*A*: All *S* are *P*	*E*: All *S* are not-*P*
	↓	↓
predominant	*P*: Almost-all *S* are *P*	*B*: Almost-all *S* are not-*P*
	↓	↓
majority	*T*: Most *S* are *P*	*D*: Most *S* are not-*P*
	↓	↓
common	*K*: Many *S* are *P*	*G*: Many *S* are not-*P*
	↓	↓
particular	*I*: Some *S* are *P*	*O*: Some *S* are not-*P*

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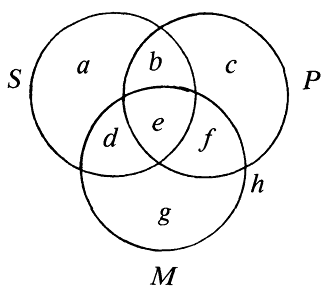
where: solid lines, ————————, connect contradictories
dashes,------------------------, connect contraries
dots,. . . . ., connect subcontraries
arrows, →→→→→→→→→→, indicate subalternations (certain immediate entailments).

I shall now show that the techniques devised by Thompson [9] as corrected and extended in [7], can be applied to six quantities, then to seven quantities, and so on to k quantities—where finite k is as *high* as you like.

To get started (and instead of re-presenting Thompson's or Carnes' rules for the 5-quantity syllogism, or my own Venn diagram methods in Section 3 of [7]), I will sketch an algebraic method for validating syllogisms with 5 quantities—a method that can be used to justify the claim that each of the 105 forms in (6) below are valid (5-quantity-wise). The method is an extension of Geach's extension of the traditional algebraic method (cf. [4], Chapter 13). First, presume that each term represents a class of objects (the S-class, the P-class, and the M-class) and that every such class has at least one member. Then label the possible subclasses as follows:

(2)



Now represent three of the five affirmative categoricals as follows:

(3) (a) All S are P: $a = 0$ and $d = 0$, where ($b \neq 0$ or $e \neq 0$)
(b) Most S are P: $b + e > a + d$, where ($b \neq 0$ or $e \neq 0$)
(c) Some S are P: $b \neq 0$ or $e \neq 0$.

Where "x" and "y" denote subclasses in (2), an expression of the type "$x + y$" denotes the cardinality of the union of x and y. Statements of the form "$x = 0$" and "$y \neq 0$" are interpreted, respectively, as "x is empty" and "y is *not* empty". The clauses prefixed with "where" in (3a) and (3b) are simply explicit recognitions of existential import—which is assumed herein, and explained further in Appendix I. Of course, the patterns displayed in (3) can be followed for negatives, with "where" clauses appropriately switched from "($b \neq 0$ or $e \neq 0$)" to "($a \neq 0$ or $d \neq 0$)".

To represent predominant and common statements (going beyond Geach), represent commons as the denials of predominants:

(4) (a) Almost-all S are P: $b + e >> a + d$, where ($b \neq 0$ or $e \neq 0$)
(b) Many S are P: $\sim(a + d >> b + e)$, where ($b \neq 0$ or $e \neq 0$).

"Many S are P" is represented as in (5b) because it is equivalent to the *denial* of "Almost-all S are not-P" (which, in turn, is equivalent to "Few S are P") and "Almost-all S are not-P" would be represented as

(5) $(a + d >> b + e)$, where $(a \neq 0$ or $d \neq 0)$.

In both (4) and (5) ">>" is read as "greatly exceeds". (Note that since (5) is a negative, the "where" clause is switched.)

To use these definitions to demonstrate the validity of a syllogistic form, deduce the algebraic representation of the conclusion from those of the premises. Alternatively, show that the premises and the denial of the conclusion are inconsistent. To demonstrate the invalidity of a form show that there is a consistent interpretation of the algebraic representations of the premises together with the representation of the denial of the conclusion. (Appendix II contains examples.) All 12 forms that Thompson missed in the third figure are justifiable by the algebraic method (e.g., *TTI-3*—which DeMorgan ([3], p. 9) noticed—and *PKI-3*, both in (b) of Appendix 2).

All the forms in (6) are justifiable via the algebraic method, and no form not on the list is justifiable. The first claim (for soundness) can be proved by enumeration, the second (completeness) can be proved by applying the strategies adopted by Carnes in his completeness proof in [7].

(6)

AAA				
AAP	*APP*			
AAT	*APT*	*ATT*		
AAK	*APK*	*ATK*	*AKK*	
AAI	*API*	*ATI*	*AKI*	*AII*
EAE				
EAB	*EPB*			
EAD	*EPD*	*ETD*		
EAG	*EPG*	*ETG*	*EKG*	
EAO	*EPO*	*ETO*	*EKO*	*EIO*

Figure 1.

AEE				
AEB	*ABB*			
AED	*ABD*	*ADD*		
AEG	*ABG*	*ADG*	*AGG*	
AEO	*ABO*	*ADO*	*AGO*	*AOO*
EAE				
EAB	*EPB*			
EAD	*EPD*	*ETD*		
EAG	*EPG*	*ETG*	*EKG*	
EAO	*EPO*	*ETO*	*EKO*	*EIO*

Figure 2.

AAI	*PAI*	*TAI*	*KAI*	*IAI*
API	*PPI*	*TPI*	*KPI*	
ATI	*PTI*	*TTI*		
AKI	*PKI*			
AII				

EAO	*BAO*	*DAO*	*GAO*	*OAO*
EPO	*BPO*	*DPO*	*GPO*	
ETO	*BTO*	*DTO*		
EKO	*BKO*			
EIO				

Figure 3.

AAI	*AEE*	*EAO*
PAI	*AEB*	*EPO*
TAI	*AED*	*ETO*
KAI	*AEG*	*EKO*
IAI	*AEO*	*EIO*

Figure 4.

Two ideas are crucial for the development of higher and ever higher quantity syllogistic systems. First, more and more "fractional" quantities can be added to the five quantities introduced above, starting with "half" (of which "most" statements are contradictories). Second, the patterns of syllogistic forms in each figure permit an indefinite number of further valid intermediate syllogisms to be interpolated.

Rescher and Gallagher [8] suggested that there are categoricals of the form "Half or more *S* are *P*". These "half" statements are the contradictories of majority statements. In (1) above there are no contradictories of majority statements. But since "most" (as Thompson uses it, and many others would agree) simply *means* "more than half", the denial of "Most *S* are *P*" (= "More than half the *S* are *P*") ought to be "Half or more of the *S* are not-*P*". The "or more" constituent can be dropped, for it is *understood* in all of the nonuniversal quantifiers. Otherwise, none of the entailments moving down the affirmatives (or down the negatives) in (1) would be valid. To obtain the 6-quantity squares of opposition, simply substitute a new square of opposition for the row of majority statements in (1), to get:

(7)	affirmative	negative
universal	*A*: All *S* are *P*	*E*: All *S* are not-*P*
predominant	*P*: Almost-all *S* are *P*	*B*: Almost-all *S* are not-*P*
majority	*T*: Most *S* are *P*	*D*: Most *S* are not-*P*
half	*F*: Half *S* are *P*	*V*: Half *S* are not-*P*
common	*K*: Many *S* are *P*	*G*: Many *S* are not-*P*
particular	*I*: Some *S* are *P*	*O*: Some *S* are not-*P*.

Guided by this set of oppositions and entailments, it is not too hard to figure out where new intermediate syllogisms containing half statements would fall on the list of valid syllogisms. The new syllogisms fall 'in between' the others in (6) analogous to the way the new categoricals fall in between majority and common statements in (7).

Two principles help to explain the patterns of syllogisms in (6): (i) that you cannot change a valid syllogism into an invalid one by strengthening one of its premises, and (ii) that you cannot change a valid syllogism into an invalid one by weakening its conclusion. To *strengthen* a proposition replace it by one which entails it, but which it does not entail. To *weaken* one replace it by one it entails, but does not entail it. Look at the first triangular array in (6), for example (the affirmative syllogisms of Figure 1). Read along the bottom row, right to left. Each successive argument form has the second premise changed, viz., strengthened. In four such strengthening steps the traditional *AAI-1* (*Barbari*) is generated out of the traditional *AII-1* (*Darii*). So, if *AII-1* is valid, so are the rest of the forms in that row. Now observe the first column of the same array. Start at the top with *Barbara*. *AAI-1* (*Barbari*) can be generated out of *AAA-1* by four successive steps of weakening its (*Barbara*'s) conclusion. So, if *Barbara* is valid so is *AAI-1* and all the intermediate syllogisms 'in between'. For each of the four arrays in Figures 1 and 2, the same principles apply. Moving right-to-left along rows generates valid forms via strengthening a premise. And moving down columns generates valid syllogisms by weakening its conclusion. The diagonals of these four arrays must be merely hypothesized and justified some other way, such as by the algebraic method. However, once they are, the two principles confirm the remaining three members of each array.

Figures 3 and 4 are slightly different. Each array in Figure 3 has only one conclusion: *I* and *O*, respectively. So, only the strengthening-of-a-premise principle applies. Thus, in the first array of Figure 3, *AAI-3* (*Darapti*) can be generated by either successive strengthening of the first premise of *IAI-3* (*Disamis*) or successive strengthening of the second premise of *AII-3* (*Datisi*). Again the diagonal forms must be hypothesized and justified independently. In the first and third columns of Figure 4, the premise-strengthening principle applies. In the second column, it is the weakening of the conclusion, generating *AEO-4* (*Camenop*) from *AEE-4* (*Camenes*).

Now using (7) as a guide we can hypothesize where new syllogistic forms with half statements occur. For example, we can hypothesize that the first array of Figure 1 be modified to look like:

(8)

AAA					
AAP	*APP*				
AAT	*APT*	*ATT*			
AAF	*APF*	*ATF*	*AFF*		
AAK	*APK*	*ATK*	*AFK*	*AKK*	
AAI	*API*	*ATI*	*AFI*	*AKI*	*AII*

(where the new syllogistic forms containing half statements are those within the rectangular boundaries). Similarly, the whole of a new table for Figure 4 would be:

(9)

$$\begin{array}{ccc} AAI & AEE & EAO \\ PAI & AEB & EPO \\ TAI & AED & ETO \\ \hline FAI & AEV & EFO \\ \hline KAI & AEG & EKO \\ IAI & AEO & EIO. \end{array}$$

These proposed new syllogistic forms (and others like them to complete the list of syllogisms for six quantities) can all be justified by the algebraic method. Exactly on the model adopted for symbolizing common statements, symbolize half statements by taking the negation of the representation for the appropriate majority statement. Thus;

(10) Half S are P: $\sim(a + d > b + e)$.

Now by analogy to the occurrence of majority and half statements in the new six quantity syllogistic, we can introduce "third" statements and "more than two-thirds" statements. The seventh quantity—third—immediately suggests the eighth by considering the contradictories of affirmative and negative "third" statements. This square results:

(11) More-than-$\frac{2}{3}$ S are P ············ More-than-$\frac{2}{3}$ S are not-P

$\frac{1}{3}$ S are P ······· $\frac{1}{3}$ S are not-P .

If we labeled affirmative and negative "third" statements "S" and "Z" statements respectively, then the seven-quantity syllogism would have additional valid forms further interpolated into the patterns of (6). For example, Figure 4 would appear as:

(12)

$$\begin{array}{ccc} AAI & AEE & EAO \\ PAI & AEB & EPO \\ TAI & AED & ETO \\ FAI & AEV & EFO \\ \hline SAI & AEZ & ESO \\ \hline KAI & AEG & EKO \\ IAI & AEO & EIO. \end{array}$$

And we could continue on to eight quantities by interpolating "More-than-$\frac{2}{3}$" statements in analogous positions. For example, there would be a new affirmative, "More-than-$\frac{2}{3}$ S are P" entailed by the appropriate P-statements and entailing the appropriate T-statement in (7). (The algebraic method continues to apply; see (d) of Appendix II.)

There seems to be no bar to adding as many additional quantifiers as you like one or two at a time following the same pattern. The general scheme for proposing such quantifiers (generating the appropriate categoricals for more and more intermediate syllogisms) is:

(13)
$$\begin{array}{ccc} \text{More-than-}\dfrac{n-m}{n}\ S \text{ are } P & \text{------------} & \text{More-than-}\dfrac{n-m}{n}\ S \text{ are not-}P \\ \downarrow & & \downarrow \\ m/n\ S \text{ are } P & \ldots\ldots\ldots & m/n\ S \text{ are not-}P \end{array}$$

where m and n range over positive integers such that for each case $n \geq 2m$. (A minor wrinkle is that when n is odd, there can be a so-called "middle" quantifier, e.g., "$\frac{2}{3}$", "$\frac{4}{7}$", "$\frac{11}{21}$", etc. "Middle"-quantified statements, such as "$\frac{4}{7}$ of the S are P", have the peculiarity of having their contradictories contain exactly the same quantifier: i.e., "$\frac{4}{7}$ of the S are P" is false if and only if "$\frac{4}{7}$ of the S are not-P" is true. This suggests that such middle-quantifier statements should *not* be omitted from the patterns sketched above, *and* be clearly distinguished from "more-than-$\dfrac{n-m}{n}$", when "$\dfrac{n-m}{n}$" is a middle-quantifier, e.g., "$\frac{4}{7}$" distinguished from "More-than-$\frac{4}{7}$".)

Proceeding in this fashion to increase the number of quantities and syllogistic forms produces the following formulas. First, the number of syllogistic forms (valid and invalid), s, for k quantities is (as might be suspected):

(14) $s = 32k^3$.

More interestingly, the number of *valid* syllogistic forms, v, for k quantities is given by:

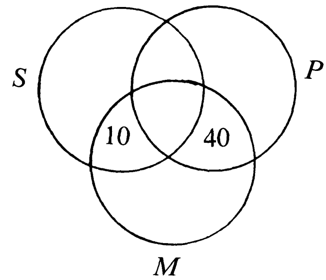
(15) $v = 3k(k + 2)$.

(Cf. Section 5.2 of [7] for an explanation of the generation of these formulas.)

Actually, there is an *apparent* bar to increasing the quantities to finite k (for k as high as you like). It turns on the question of how *exactly* to interpolate fractionally quantified statements with predominants and commons. Consider the quantifiers "$\frac{1}{4}$" and "more-than-$\frac{3}{4}$". Does "$\frac{1}{4}$ the S are P" entail "Many S are P"? Or *vice versa*? Similarly, does "Almost-all S are P" entail "More than $\frac{3}{4}$ the S are P"? Or *vice versa*? These questions do *not* prevent developing higher-quantity syllogistic systems (for finite k as high as you like). First, and most simply, one can *omit* predominant and common statements from the squares of opposition – such as (1) and (7) – and omit subsequent intermediate syllogisms containing predominant and common categoricals, and still follow the same patterns. That is, using the scheme in (13), we can develop a system of fractional quantifiers (added to universal, particular, and majority quantifiers) for as many quantifiers (and quantities) k (for finite k) as high as you like. Secondly, however, there *is* an answer to these questions about predominant and common statements which does permit them to be interpolated with any set of fractional intermediate quantifiers (and quantities). This answer develops out of handling certain proposed counterexamples to the 5-quantity syllogistic system of (1) and (6).

Here is a proposed counterexample to *PKI-3* (one of the 12 intermediate syllogisms that Thompson did not recognize). Consider that there are 50 members of the M-class. Then let 10 of them also be in the S-class and 40 of them be also in the P-class. With this distribution it is possible that *no* members of M are also in the SP-class, when they are distributed as follows:

(16)



50 *Ms*

Now if 80% of a class is sufficient to justify asserting that "almost-all" applies (i.e., "Almost-all *M* are *P*" is true) and also as few as 20% is sufficient to justifying that "many" applies (i.e., "Many *S* are *P*" is true) *then* we do have a counterexample in (16) to *PKI-3*. For on the distribution displayed in (16), "Some *S* are *P*" (the conclusion of *PKI-3*) is *false*.

What should we say now? We have on one side the elegant proposal for a 5-quantity syllogistic in (1) and (6) together with the algebraic confirmations of it (via the method introduced above, not to mention the rules and proofs from Carnes in Sections 1 and 4 of [7]). On the other side, we have this proposed counterexample involving a not extreme use of "many" and "almost-all" (that 20% is enough to count as many and 80% enough to count as almost-all). The right solution, I believe, is not to select arbitrarily some percentages to be the minimums for "many" and "almost-all" statements (say 25% for "many" and *more than* 75% for "almost-all"), *but rather* to stipulate that there are *some* percentages *or other*, i and j, such that for "Many *S* are *P*" to be true at least i percent of the *S* must be *P* and for "Almost-all *S* are *P*" to be true *more* than j percent of the *S* must be *P*, *where* $i + j \geq 100$.

What this stipulation derives from is the recognition that predominant and common statements are always *linked* via the contradictory oppositions displayed in (1). Now we have discovered what the link *is* which is needed to *defend* the 5-quantity syllogistic of (1) and (6). If the sum of i and j is permitted to be *less* than 100, then counterexamples to *PKI-3* (and some other forms in Figure 3) can be generated. But it is not unreasonable to stipulate—simply to defend (1) and (6)—that $i + j \geq 100$. In other words, the percentage has to be fairly high for an "almost-all" quantifier to apply. How high? Well, that decision is linked to how many, percentage-wise, are required to count as "many". A little reflection on these questions will result, I believe, in the acknowledgement that the stipulation is quite reasonable even if unexpected.

But with the necessity of stipulating some values or other for i and j in order to restrict the interpretation of predominant and common statements (in order to defend the 5-quantity syllogistic), a resolution of how to integrate more and more fractional quantifiers with the initial 5 quantities results. For example, if it is decided that a plausible choice of i and j is $i = 25\%$ and $j = 75\%$, then the question of where "$\frac{1}{4}$ the *S* are (not) *P*" and "More-than-$\frac{3}{4}$ the *S* are (not) *P*" fall *vis à vis* predominant and common statements is decided. For then "$\frac{1}{4}$" would be *identified* with "many" and "more-than-$\frac{3}{4}$" would be identified with "almost-all". We are not compelled to make *exactly* this selection, but for any syllogistic system with a large number, k, of quantities which *also* includes predominant and common quantities, some choice or other of i and j must be made (simply to defend against counterexamples like that proposed against *PKI-3*). But making such a choice (for any particular syllogistic system with k quantities, for $k > 3$) will solve the interpolation problem of fractionals with predominant and common quantifiers (and quantities).

In the recently popular style of intensionalistic semantic model theory, someone might now propose that "many" and "almost-all" be taken to denote certain *functions* which take i and j for arguments (respectively) to produce as values other functions, viz., fractional quantifiers of the form (respectively) "m/n" and "more-than-$\frac{n-m}{n}$", which in turn are functions that: (i) take subject-terms (perhaps) to give what Barwise and Cooper [1] call simply "quantifiers" (what I would call "quantifier phrases") or (ii) take subject-term/predicate-term pairs to give categorical propositions (where perhaps propositions are, in turn, functions from indices for worlds, times, speakers, etc. to truth-values).

Appendix I I do not represent existential import via conjoining an appropriate clause to the algebraic representation—but use "where" clauses to represent the assumption of existential import—for the following reasons.

If we represent existential import of universals via conjunction of existential claims with each universal, then the traditional square appears as follows:

(i) A: All S are P) & (There are S) E: (No S are P) & (There are S)
I: (Some S are P) $\vee$ (No S exist) O: (Some S are not-P) $\vee$ (No S exist).

Although the usual relations of contradictoriness, contrariety, subcontrariety, and subalternation (immediate entailment) all hold for (i), now the particular forms do not contain existential import. For example, the I-form categorical, represented as in (i), *can be true if no* S exist, contrary to traditional assumptions.

Further, if such "where" clauses were appended to each categorical in the traditional square (to represent existential import of subjects) in the following manner

(ii) A: All S are P, where there are S E: No S are P, where there are S
I: Some S are P, where there are S O: Some S are not-P, where there are S

and also if these clauses are treated as conjunctions *throughout*, then although both universal-to-particular subalternation and contrariety between universals would hold, still

(a) the particulars aren't subcontraries (since I and O can both be false when "where" = "&")

and, more importantly,

(b) contradictoriness between A and O, and between E and I, fails.

So, I use "where" clauses to simply give an explicit reminder of the existential import assumption. Using the clauses this way does not even fully represent the existential import assumption in force, but is only a *partial* reminder. For the way I choose *herein* to interpret existential import is by means of the assumption that every class denoted by every categorical term is nonempty. So, for example, in addition to there being the assumption in effect that there are S for E-forms, there is *also* the assumption in effect that there are P (to preserve conversion *inter alia*).

I presume that my interpolation of appropriate "where" clauses (to explicitly acknowledge part of the total existential import assumption in effect) is merely a notational variant of Lemmon's [5] practice. Instead of introducing the relevant existential import claim needed in a proof on a separate line (as Lemmon does, cf. [5], p. 177), I omit adding an extra line to premises of a syllogism (sticking to the classical two premise format) and simply refer (in justifications of lines of the proof) to the line in which a reminder of existential import was appended (via appropriate "where" clause). For example, in Appendix II, see *Camenop* in (a) or see the justification for line (7) in *AAT-1* of (b).

Appendix II The following are some proofs of validity and invalidity using the algebraic method. In some of these proofs, clauses representing existential import are omitted when they are incidental. In some others, of course, the clauses are crucial. See (2) above for the subclasses denoted by "a", "b", "c", etc.

(a) Two traditional valid syllogisms.

AAA-1 (*Barbara*)

All M are P:	(1) $d = 0$ and $g = 0$ .... premise
All S are M:	(2) $a = 0$ and $b = 0$ .... premise
	(3) $a = 0$ .... from (2)
	(4) $d = 0$ .... from (1)
All S are P:	(5) $a = 0$ and $d = 0$ .... from (3) and (4)

AEO-4 (Camenop)

All P are M:	(1) $b = 0$ and $c = 0$, where $(e \neq 0$ or $f \neq 0)$ .... premise
No M are S:	(2) $d = 0$ and $e = 0$, where $(g \neq 0$ or $f \neq 0)$ and $(a \neq 0$ or $b \neq 0)$ .... premise
	(3) $b = 0$ .... from (1)
	(4) $a \neq 0$ or $b \neq 0$ .... from (2)
	(5) $a \neq 0$ .... from (3) and (4)
Some S is P:	(6) $a \neq 0$ or $d \neq 0$ .... from (5)

Camenop's second premise includes representation of existential import for both subject and predicate terms because convertibility of universal negatives coupled with the assumption of existential of import of subjects of universals results in predicate term existential import. *Camenop* is typical of syllogisms requiring existential import for validity. Clauses are extracted from "where" clauses *as if* they were regular conjuncts, but that is only an appearance. The clauses are not true conjuncts (as stated in Appendix I) and their use is a use of a background assumption, not the deductive step often labeled "simplification".

(b) Four valid intermediate syllogisms, all proved indirectly by showing that the premises and the denial of the conclusion are inconsistent.

EKG-2

No P are M:	(1) $e = 0$ and $f = 0$	premise
Many S are M:	(2) $\sim(a + b >> d + e)$	premise
$\sim$(Many S are not-P):	(3) $\sim\sim(b + e >> a + d)$	denial of conclusion
	(4) $b >> a + d$	from (1) and (3)
	(5) $\sim(a + b >> d)$	from (1) and (2)
contradiction	(6) $a + b >> 2a + d$	from (4) via addition of a's
	(7) $a + b >> d$	from (6)

AAT-1

All M are P:	(1) $d = 0$ and $g = 0$, where ($e \neq 0$ or $f \neq 0$)	premise
All S are M:	(2) $a = 0$ and $b = 0$, where ($d \neq 0$ or $e \neq 0$)	premise
$\sim$(Most S are P):	(3) $\sim(b + e > a + d)$, where ($a \neq 0$ or $d \neq 0$)	denial of conclusion
	(4) $\sim(b + e > a)$	from (3) and (1)
	(5) $\sim(e > a)$	from (4) and (2)
	(6) $\sim(e > 0)$	from (5) and (2)
	(7) $e = 0$	from (6)
	(8) $d \neq 0$ or $e \neq 0$	from (2)
contradiction	(9) $d \neq 0$	from (7) and (8)
	(10) $d = 0$	from (1)

TTI-3

Most M are P:	(1) $e + f > d + g$	premise
Most M are S:	(2) $d + e > g + f$	premise
$\sim$(Some S are P):	(3) $b = 0$ and $e = 0$	denial of conclusion
	(4) $f > d + g$	from (1) and (3)
	(5) $d > g + f$	from (2) and (3)
contradiction	(6) $f > d$	from (4)
	(7) $d > f$	from (5)

PKI-3

Almost-all M are P:	(1) $e + f >> d + g$ .... premise
Many M are S:	(2) $\sim(g + f >> d + e)$ .... premise
$\sim$(Some S are P):	(3) $b = 0$ and $e = 0$ .... denial of conclusion
	(4) $f >> d + g$ .... from (1) and (3)
	(5) $\sim(g + f >> d)$ .... from (2) and (3)
contradiction	(6) $g + f >> d + 2g$ .... from (4)
	(7) $g + f >> d$ .... from (6)

(c) Two invalid syllogisms (traditional and intermediate).

IAO-3

Some M are P .... $e \neq 0$ or $f \neq 0$
All M are S .... $g = 0$ and $f = 0$, where ($d \neq 0$ or $e \neq 0$)
$\sim$(Some S are not-P) .... $\sim(a \neq 0$ or $d \neq 0)$
But $a = 0$, $d = 0$, $e \neq 0$, $f = 0$, and $g = 0$ permit all of these propositions (i.e., the premises and the denial of the conclusion) to be consistently true.

TAT-3

Most M are P:	(1) $e + f > d + g$, where ($e \neq 0$ or $f \neq 0$) ..... premise
All M are S:	(2) $g = 0$ and $f = 0$, where ($d \neq 0$ or $e \neq 0$) .... premise
$\sim$(Most S are P):	(3) $\sim(b + e > a + d)$, where ($a \neq 0$ or $d \neq 0$) .... denial of conclusion

But $g = 0$, $f = 0$, $e = 7$, $d = 5$, $a = 20$, and $b = 10$ permit all of these propositions to be consistently true.

(d) Two valid syllogisms with higher quantities (half and third):

AFK-1 (cf. (8) in text)

All M are P:	(1) $d = 0$ and $g = 0$ .... premise
$\frac{1}{2}$ the S are M:	(2) $\sim(a + b > d + e)$ .... premise
$\sim$(Many S are P):	(3) $\sim\sim(a + d >> b + e)$ ... denial of conclusion
	(4) $a >> b + e$ .... from (3) & (1)
	(5) $a + b >> 2b + e$ .... from (4)
	(6) $a + b >> e$ .... from (5)
contradiction	(7) $\sim(a + b > e)$ .... from (1) & (2)
	(8) $a + b > e$ .... from (6)

ESO-4 (cf. (12) in text)

No P are M:	(1) $e = 0$ and $f = 0$, where ($b \neq 0$ or $c \neq 0$) and ($d \neq 0$ or $g \neq 0$) .... premise

$\frac{1}{3}$ the M are S:	(2) $\sim(g + f > 2(e + d))$ [1] where ($d \neq 0$ or $e \neq 0$)	premise
$\sim$(Some S are not-P):	(3) $\sim(a \neq 0$ or $d \neq 0)$	denial of conclusion
	(4) $a = 0$ and $d = 0$	from (3)
	(5) $d \neq 0$ or $e \neq 0$	from (2)
contradiction	(6) $e \neq 0$	from (4) & (5)
	(7) $e = 0$	from (1)

NOTE

1. Because: $\sim$(More-than-$\frac{2}{3}$ M are not-S) $\equiv \sim(g + f > \frac{2}{3}(e + f + d + g)) \equiv \sim(g + f > 2(e + d))$. Thus, $\sim(g + f > 2(e + d)) \equiv \frac{1}{3}$ the M are S, since $\sim$(More-than-$\frac{2}{3}$ the M are not-S) $\equiv \frac{1}{3}$ the M are S.

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