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Burgess on Relevance: A Fallacy Indeed

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1 John Burgess [3] replied in kind to Anderson and Belnap's diatribes on the errors of classical logicians. The bone of contention was their claim that no mathematician or layman ever uses those features of classical logic which distinguish it from relevant logic, in particular the idea that from a contradiction any proposition whatever follows (I call this *consequentia horribilis*), and the closely related inference-form, Disjunctive Syllogism.

In order to show this claim to be mistaken, Burgess adduces two examples of intuitively valid instances of

$$\text{(I)}\quad \frac{\begin{array}{l} p \text{ or } q \\ \text{not } p \end{array}}{q} \qquad \text{and} \qquad \text{(II)}\quad \frac{\begin{array}{l} \text{not both } p \text{ and } q \\ p \end{array}}{\text{not } q}$$

which he claims are in fact instances of

$$\text{(IA)}\quad \frac{\begin{array}{l} p \vee q \\ {\sim}p \end{array}}{q} \qquad \text{and} \qquad \text{(IIA)}\quad \frac{\begin{array}{l} {\sim}(p \,\&\, q) \\ p \end{array}}{{\sim}q}$$

and not of

$$\text{(IB)}\quad \frac{\begin{array}{l} p + q \\ {\sim}p \end{array}}{q} \qquad \text{or} \qquad \text{(IIB)}\quad \frac{\begin{array}{l} {\sim}(p \circ q) \\ p \end{array}}{{\sim}q}.$$

Here $\vee$ and & symbolize extensional disjunction and conjunction, and + and $\circ$ their intensional counterparts. The truth conditions of $\vee$ are that $p \vee q$ is true iff p is true or q is true; of +, that $p + q$ is true just when "if not p then q" is true, that is, iff q is true if p is not true (cf. the classical truth condition that

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$p \supset q$ is true iff either p is not true or q is). & and $\circ$ can then be introduced by definition.

Anderson and Belnap's claim (cf. [1], Sections 5.1, 15.1, 16.1, 16.3, and 25.1) was that no layman or mathematician uncorrupted by modern logic ever argues in accordance with *consequentia horribilis*, nor with (IA), which in relevant logic is accounted an invalid form, as is (IIA). When Disjunctive Syllogism is used in argument, the disjunction is intensional. The relevantly valid forms are (IB) and (IIB). Burgess therefore has to show that the examples of (I) and (II) which he presents are indeed instances of the A-forms, and not of their B-counterparts. Moreover, in order for his counterattack to be more than a token reply, he must produce examples which are not merely undeniably cases of everyday reasoning, but also reveal themselves to constitute sound inferential practice.

I shall show that Burgess' first example is not clearly an A-form, but can quite plausibly be understood as of form (IB); and that his second example exhibits a case of reasoning which, though it will lead from true premises to a true conclusion, is not a valid step.

2.1 The first example refers to a jejune card game. Let me simplify it to this case. There are three cards, of which two are face down on the table, and one is hidden from me in the hand of a truthful but uncooperative partner (in Burgess' example there are 50 other cards, and any number of partners, who are also opponents). My partner will reply with the answers "No" or "Maybe" only to questions such as "Are the cards on the table A and B?" Clearly, if he ever answers "Maybe", the question must have been right. But if not, an inference is needed. Burgess takes as an instance of such an inference,

$$\begin{array}{l} \sim(A \text{ and } C) \\ \underline{A \qquad\qquad} \\ \sim C. \end{array}$$

That is, having previously worked out that one of the cards on the table is A, and hearing my partner's reply "No" to the question, "Is it A and B?", I infer that C cannot be on the table.

But Burgess gives no clue as to how I might establish positive information, such as that A is there. Suppose the cards on the table are in fact A and B, and (therefore) I elicit the answer "No" to both "Are the cards B and C?" and "Are the cards A and C?" Accordingly, the inference I make is that the cards on the table are A and B. My inference to positive information yields knowledge of the pair of cards together. That is, my argument is

$$\begin{array}{l} \sim(B \text{ and } C) \\ \underline{\sim(A \text{ and } C)} \\ A \text{ and } B. \end{array}$$

Clearly the conjunctions here are extensional. In the premise they are inferred from $\sim C$ (since in this case C is my partner's card) and $\sim C \nvdash \sim(B \circ C)$, for example. And the conclusion supports Simplification, allowing one to conclude, e.g., that one of the cards on the table is A. So the form of argument is

$$\begin{array}{l} \sim(B \,\&\, C) \\ \underline{\sim(A \,\&\, C)} \\ A \,\&\, B. \end{array}$$

Even classically, this argument is not valid as it stands. It is an enthymeme. What is the missing premise? Clearly, that only three possibilities are available for the cards on the table, the three pairs that can be formed from three cards. That is, the additional premise is

(*) $(A \,\&\, B)$ or $(B \,\&\, C)$ or $(A \,\&\, C)$.

Are the disjunctions here extensional or intensional? They are intensional. For the truth of the three-part disjunction does not depend solely on the truth of some single disjunct, e.g., that as a matter of fact A and B are the cards on the table. The other disjuncts are inferentially connected to $(A \,\&\, B)$ in a way in which those in

(**) $(A \,\&\, B)$ or Bach wrote the Coffee Cantata or the Van Allen belt is doughnut shaped

are not. The truth of (**) arises simply from the fact that A and B are the cards on the table; whereas that of (*) depends on the fact that there are only three possibilities (since there are only three cards), so that *if* two of them are not realised *then* the third must be. The disjunction in (*) has the force of a conditional. These are grounds therefore to treat it as intensional, so that it supports the application of Disjunctive Syllogism (*DS*) which is made of it. Burgess does not contend that one cannot deal formally with an intensional disjunction. His claim (p. 99 of [3]) is that his cases "can neither be read as nor replaced by" the intensional schemata. However, we see that there is indeed good reason to treat them so, in order to recognise the difference between (*) and (**), the one intensional, the other extensional.

The point can be generalised: if q may be legitimately inferred from not p, then "if not p then q" is true. Thus if an instance of *DS* is valid, the major premise, "p or q", must have the sense of "if not p then q". However, this conditional is not entailed (except for particular p and q) by p. Nonetheless, the rule of Addition is unquestionably valid, allowing us to infer "p or q" from p. Hence "p or q" must be ambiguous: those instances which support *DS* must differ in sense from those supported by Addition.

One is thus justified in concluding that A and B are on the table by this valid double instance of *DS*:

$$\begin{array}{l} \sim(B \,\&\, C) \\ \sim(A \,\&\, C) \\ \underline{(A \,\&\, B) + (B \,\&\, C) + (A \,\&\, C)} \\ A \,\&\, B. \end{array}$$

2.2 The second example is more complex. Burgess supposes that Ms. Zeemann has been awarded a degree for a dissertation in number theory. The core of the work is a proof by induction of $(n)(A(n) \vee B(n))$, for certain number-theoretic statements $A(n)$ and $B(n)$. Burgess is right to point out that, as in Zeemann's proof, there are occasions where $p \vee q$ is a useful consequence of p, e.g., when one thereby gains mathematical generality. That is, although we cannot, for some A and B, establish $(n)A(n)$ or $(n)B(n)$, we can establish $(n)(A(n) \vee B(n))$. Subsequently we may deduce C from this result, and so obtain a proof of C. However, it may be worth observing that whereas

$$(A \vee B) \rightarrow C$$

entails $A \rightarrow C$ and (&) $B \rightarrow C$, whence arises the utility of the satisfaction of a disjunctive condition ($\vee$-elimination), the related classically valid first-order scheme

$$\frac{(n)(A(n) \vee B(n)) \rightarrow C}{(\exists n)(A(n) \rightarrow C \;\&\; B(n) \rightarrow C)}$$

fails relevantly. Although if every number greater than 1 is divisible by 2 or by 3 then no number greater than 3 is prime; there is no number such that if it is divisible by 2 then no number greater than 3 is prime and the same if it is divisible by 3.[1] Thus the utility of $A \vee B$ in $\vee$-elimination is more limited for its universal generalisation.

Suppose then that Wyberg, as described in Burgess' Example 2b, is led to pass from a demonstration that $\sim A(1)$, together with knowledge that Zeemann has proved that $(n)(A(n) \vee B(n))$, to the conclusion $B(1)$, and so, possessing a derivation of C from $B(1)$, finally to C. Is he right to do so?

No, he is not. For Wyberg has no proof of $(n)(A(n) \vee B(n))$. So it is possible that its proof depends on a proof of $A(1)$, rather than $B(1)$. If so, and he has himself, it seems, proved $\sim A(1)$, he should immediately stop his progress to C, and reexamine the foundations, rather than, as he classically may, and in Burgess' example does, proceed to $B(1)$ and so to C. For $A(1) \vee B(1)$ is weaker than either disjunct. Take a similar case: suppose we know that p & q is true. We simplify this (for some reason) to p. Then we cannot infer from p that q is true. Certainly the ground for our knowledge of p supports q; but p itself does not. Nor can we in general add anything useful to p to obtain q: if we add q, for example, then certainly that supports q, but trivially so; if we add $p \rightarrow q$, then that, with p, supports q, but of course $p \rightarrow q$ is not always true. Similarly, if we know which of $A(1), B(1)$ is true, and weaken this (for, say, mathematical simplicity) to $A(1) \vee B(1)$, then we cannot infer $B(1)$ from it, even by adding $\sim A(1)$. If the ground for $A(1) \vee B(1)$ was $B(1)$, then the ground certainly supports $B(1)$ (trivially); but the ground could equally have been $A(1)$. And to adduce $\sim A(1)$ does not show that it was not: for the number theory in which Zeemann was working could have been inconsistent (e.g., Frege's).

What we can infer from

$$\sim A(1) \text{ and } (n)(A(n) \vee B(n))$$

is $B(1)$ *or* something has gone wrong (that is, there is an inconsistency). Let us formalise this clause, "something has gone wrong", by f, a sentential constant whose contradictory, t, is a theorem. So also, for any sentence A, is $A \rightarrow (t \rightarrow A)$. Then this inference is relevantly valid:

$$\frac{\sim A \quad A \vee B}{B \vee f.}$$

But we cannot use t to obtain B from this conclusion, for

$$(***) \qquad \frac{t \quad B \vee \sim t}{B}$$

is another instance of the invalid Disjunctive Syllogism for "v". Indeed, if we admit (***) into R, then we can prove $A, \sim A \vdash B$, and the system collapses into classical logic. Rather, (***) must be replaced by

$$\frac{t \quad B \vee \sim t}{B \vee f}$$

and here of course the conclusion is identical with the major premise, and such inferences can never yield B alone.

To put the same point differently, Wyberg, having shown that C follows from $B(1)$, and having shown that $\sim A(1)$, needs a further premise of the form, "if $\sim A(1)$ then $B(1)$". There is a sense of "or" in which "$A(1)$ or $B(1)$" has this force, but it is not the truth-functional sense, that is, the sense in Zeemann's theorem. For her statement, $A(1) \vee B(1)$ may, with $\sim A(1)$, amount only to $A(1)$ & $\sim A(1)$, and from this, contrary to classical beliefs, very little, other than $A(1)$ and $\sim A(1)$, can be inferred; certainly $B(1)$ cannot. Of course, her statement was not based on $A(1)$, but on $B(1)$, Burgess tells us. But one cannot just add that information; first, because, if there was a contradiction, it is not removed simply by adding a further clause (viz. $\sim A(1)$) denying it (see [2]); second, because to add $B(1)$ is to add the *conclusion* of the argument as an extra premise to ensure its validity, and then we would no longer have a case of *DS*, but of Simplification. Wyberg would be justified in proceeding nontrivially to $B(1)$ and C only if the proof of Zeemann's main result in fact supported the particular conditional $A(1) + B(1)$. That we know it does not do.

2.3 Mortensen [5] has given a different reply to Burgess' examples, namely that the applications of extensional Disjunctive Syllogism are admissible provided the context of the example makes clear that the situation is consistent and prime. (A situation or theory X is consistent if not both p and $\sim p$ belong to X, for any p; X is prime if either p or q belongs to X if $p \vee q$ does.) Mortensen's hope is that even if one cannot validly infer q from $p \vee q$ and $\sim p$ in general, one may demarcate circumstances under which the inference is acceptable: his suggested criterion is that the situation be prime and consistent. Now the assertion of primeness and consistency is not an object-level assertion, but a metalevel one. Hence Mortensen invokes primeness and consistency (as hopefully provable facts) for the metatheory (of the theory or situation X), helps himself to (IA) in that metatheory (see his Section 3, clause (3)), and so concludes that (IA) holds in the object-language.

But this is an impossible bootstraps procedure. However far one goes up the hierarchy of metalevels, one will always be invoking an instance of (IA) at a higher level for which one has no justification. Nor can one hope to find some fixed point where the assumption of primeness and consistency does not take one up a level, for again, as Belnap and Dunn [2] observed, one cannot assure oneself of consistency by the mere addition of further statements.

Burgess and Mortensen both believe that one cannot justify (IA) non-circularly. Burgess therefore relies on observation of accepted mathematical practice, claiming that it conflicts with the verdict of relevant logic—a procedure triggered by some relevantists' claim that the verdict of classical logic conflicts with that practice. Mortensen seeks to use (IA) to establish the conditions of its own applicability, that is, to discover conditions P such that, in a metatheory using (IA), we can show $P \Rightarrow$ (IA). No instance of (IA) will be used to show its very own validity, but only that of an application of the same form of argument in the theory to whose metatheory that instance of (IA) belongs.

My approach is different: we have (at least) two different formal systems, classical and relevant, with an intuitive (i.e., intended) interpretation. In one, (IA) is accounted valid, but so too is *consequentia horribilis*, from A and $\sim A$ to B. The other is paraconsistent (i.e., *horribilis* fails), but so too does (IA); however, in it another form of inference, (IB), equally interpretable as (I), is admissible. (Other formal systems have been proposed which attempt both to retain (IA) and to prohibit *horribilis*, but they generally fall down on formal grounds—e.g., in being unable to contain an implication operator.) Thus the question which arises on embracing paraconsistency is, which instances of (I) are cases of (IA) and which of (IB)? In Example 1, we saw that Wyberg's inference was of the form (IB). That form of inference was available to him since the nature of the game of Mystery Cards supplied him with a true intensional premise. In Example 2, Wyberg did indeed reason in accord with (IA), and though this might be thought to be mathematically acceptable, it is not: the reason being that if it were, so too would *consequentia horribilis*. *Horribilis* is a heuristic falsifier of the suggestion that (IA) is valid.

But (IB) really is valid. For $p + q$ is equivalent to $\sim p \rightarrow q$, and the intended interpretation of $\sim p \rightarrow q$ is "if not p then q". Now what "if not p then q" means is that if not p is true then q is true. So if "if not p then q" is true, and so is not p, then q must be true. Put another way, the validity of (IB) follows from that of

$$\frac{\text{if not } p \text{ then } q}{\text{if not } p \text{ then } q}$$

(a case of Identity), by Exportation.

One may object that the validity of (IB) has now been pushed back to that of Identity and Exportation. Of course it would be vain to suppose that one will ever escape such reliance on the validity of further forms of argument. Nonetheless, showing how the validity of a certain form follows from the meaning of its constitutent connectives leads us to a greater understanding of that form. Thus what is meant by saying that B may be validly inferred from A is that if A is true then so must be B (*necessitas consequentiae*); and clearly if A is true then indeed A must be true. So Identity is certainly valid. As regards Exportation, let us express it as follows: if "if B then C" may be inferred from A, then C may be inferred from A and B. That is, there should be some sense in which the premises of an inference are conjoined such that

(%) if A and B then C

is equivalent to

(%%) if A then if B then C.

That sense is, in fact, the conjunction in the major premise of (IIB). $p \circ q$ is equivalent to "not q does not follow from p" (just as $p + q$ is equivalent to "q follows from not p": "+", "o" and "if then" are inferential connectives, whose semantics ties them directly to inference).[2] (%%) is equivalent to

if not B does not follow from A then C

(spelling out the sense of (%)) by Contraposition and Permutation.

Hence (IB) is valid in virtue of the meaning of +: the very meaning of $p + q$ is that one may infer q from not p. Given that $p + q$ is true, then if not p is also true, q must be true.

3 So Mortensen and Burgess are both mistaken in their different attempts to rehabilitate (IA). What unites their approaches is their common belief that relevance is a unitary, separate, and identifiable phenomenon. In Burgess' case this belief was taken from Anderson and Belnap, his main object of criticism, and manifests itself in his repeated references to whether one state of affairs is relevant or not to another. In Mortensen's case it shows itself in his desire to find features of any deductive situation which constrain it to be a *DS*-theory, that is, to be a situation in which (IA) is valid. But it is a mistake to suppose that relevance is some additional criterion by which we narrow the class of classically valid inferences to those of relevant logic. This is to get the cart before the horse. One cannot challenge a purported derivation of q from p by the assertion that q is not in some sense relevant to p. If q has indeed been derived from p, what greater connection of relevance could a logician desire? None.[3] Relevant logic is indeed misnamed if it encourages us to suppose that relevance is a notion which should be explained and used as a criterion of the validity of inferences supplementary to consideration of the truth values of their constituent sentences. To be sure, mere computation of truth values is not sufficient to show validity; but there is no single other notion which may be added to that computation to yield validity either. Rather, the relevantist criterion for validity must actually replace the classical one of "Safety First."

In places Anderson and Belnap appear to have shared the belief that the relevantist notion of consequence results from classical consequence by a relevance restriction. But it cannot be right, since—and it is worth repeating—to deduce q from p in itself shows a logically relevant connection between p and q. For example, take the inference pattern

(II) not both p and q
p
―――――――
not q.

Given a nonmodal reading of "if", this is equivalent (by Conditionalisation and Modus Ponens) to

$$\text{(II}') \quad \frac{\text{not both } p \text{ and } q}{\text{if } p \text{ then not } q.}$$

But the conclusion states that we may infer (that is, relevantly infer) not q from p. Hence if the premise is not grounded on any relevance between p and q (or not q), then it cannot "contain" the conclusion, and the form, so understood, is invalid (cf. IIA). Whereas if the form is valid, then the premise must import a relevant connection of the sort given in the conclusion, and so the conjunction must be intensional (cf. IIB). There are everyday instances of reasoning of the form (II′) (see [8]), and the same point applies *mutatis mutandis* to (I). The valid forms are indeed (IB) and (IIB), where the major premise has the force of a conditional;

in the case of (I): and in the case of (II):

$$\frac{\begin{array}{l}\text{if } {\sim}p \text{ then } q\\ {\sim}p\end{array}}{q} \qquad \frac{\begin{array}{l}\text{if } p \text{ then } {\sim}q\\ p\end{array}}{{\sim}q.}$$

Consequently, the central issue between classical and relevant logicians is the criterion for validity: if $p \vee q$ and ${\sim}p$ are true, does it follow that q must be true? It does so only in the classical sense of "follow"; so the most important task now outstanding for the relevantist is to provide a nonclassical modeling for the notion of "consequence".[4]

Better grounds than Burgess' must be given for granting that any apparently acceptable application of (I) and (II) is of form (IA) or (IIA), or is valid. Wyberg's support for von Eckes' conjecture rested not on mathematics but on the pathology of classical logic, that $B(1)$, and indeed anything whatever, follows from $A(1)$ and ${\sim}A(1)$. It would not vindicate Wyberg for it to turn out that von Eckes' conjecture was correct, and could moreover be shown to be so by adding a derivation of it from $B(1)$, prefaced by Zeemann's demonstration of $B(1)$, for not every argument with true premises and a true conclusion is valid; one must pass from premises to conclusion by valid steps.

NOTES

1. A suitable 4 X 4-matrix over a 2-element domain provides a formal countermodel.
2. I explored form (II) and its connection with a popular criterion for validity in [8].
3. For an early glimpse of this fact, see [6], p. 140. See also [7], p. 196.
4. Consider the following passage in Mortensen's paper: "[If Burgess is right,] then . . . it must always be that B in every deductive situation which contains both A and ${\sim}A \vee B$. Furthermore, if Burgess is wrong . . . , then for some A, B, it must be that B fails to be in some deductive situation containing A and ${\sim}A \vee B$" (pp. 36-37). That does not follow: "it is not the case that if P [i.e., $A \in X$ and ${\sim}A \vee B \in X$] then Q [i.e., $B \in X$]" is not equivalent to "P and not Q", unless "if" is understood classically, or "and" is understood as in Note 2. See, also, [4], p. 201: since (IA) is invalid in the relevantist's sense (as she says), then (as she does not realise) he *must* deny that if $p \vee q$ is true and ${\sim}p$ is true, then, necessarily, q is true. That would be true only if "if" were understood in the classical sense.

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